When a firm can vary more than one input, it can usually choose to produce any given output level in different ways. For example, in the long run, a car wash can vary both its labor and the number of automatic car-washing machines it uses. If it acquires more machines, it can wash the same number of cars with less labor. Even in the short run, a firm can often vary more than one input. An artichoke farmer might be able to achieve the same total yield using less land if he hires more labor to care for and harvest the crop.

How does a manager choose among different methods of production when she can vary more than one input? She will choose to produce any given level of output in the cheapest way possible. This appendix presents a graphical technique to help you understand how a firm with two variable inputs finds the lowest-cost production methods. The technique can be used over any time horizon—short run or long run—as long as there are two inputs whose quantities can be varied within that horizon. Finally, at the end of this appendix, the technique will be generalized to the case of more than two variable inputs.

**ISOQUANTs**

Imagine that you own an artichoke farm, and you are free to vary two inputs: labor and land. Your output is measured in “boxes of artichokes per month.” Your farm’s production function tells us the maximum possible number of boxes you could produce in a given month using different combinations of labor and land. Alternatively, it tells us all the different input mixes that could be used to produce any given quantity of output.

Table A1 lists some of the information we could obtain from your production function. Notice that, to produce each of the three output levels included in the table, there are many different combinations of inputs you could use. For example, the table tells us that your farm could produce 4,000 boxes of artichokes using 2 hectares of land and 18 workers, or 3 hectares and 11 workers, or 5 hectares and 5 workers, and so on.

(Note: If it seems to you that no artichoke farm would ever use some of these combinations—such as 14 hectares of land and 2 workers—you are right. But that
is because of the relative costs of labor and land, which we haven’t discussed yet. Table A1 simply tells us what is possible for the firm, not what is sensible.

The information in the table can also be illustrated with a graph. In Figure A1, the quantity of land is plotted along the horizontal axis, and the number of workers on the vertical axis. Each combination of the two inputs is represented by a point. For example, the combination 3 hectares, 11 workers is represented by the point labeled B, while the combination 5 hectares, 12 workers is represented by point F.

Now let’s focus on a single output level: 4,000 boxes per month. Table A1 lists 5 of the different input combinations that can be used to produce this output level, each represented by a point in Figure A1. When we connect all 5 points with a smooth line, we get the curve labeled “Q = 4,000” in Figure A1. This curve is called an isoquant (“iso” means “same,” and “quant” stands for “quantity of output”).

Every point on an isoquant represents an input mix that produces the same quantity of output.

<table>
<thead>
<tr>
<th></th>
<th>2,000 Boxes of Artichokes per Month</th>
<th>4,000 Boxes of Artichokes per Month</th>
<th>6,000 Boxes of Artichokes per Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hectares of Land</td>
<td>Number of Workers</td>
<td>Hectares of Land</td>
<td>Number of Workers</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>14</td>
<td>2</td>
</tr>
</tbody>
</table>

1 If you read the appendix to Chapter 5, you will recognize that isoquants are similar to indifference curves. But while an indifference curve represents different combinations of two goods that give same level of consumer satisfaction, an isoquant represents different combinations of two inputs that give the same level of firm output.
Figure A1 also shows two additional isoquants. The higher one is drawn for the output level $Q = 6,000$, and the lower one for the output level $Q = 2,000$. When these curves are shown together on a graph, we have an \textit{isoquant map} for the firm.

\textbf{Things to Know about Isoquants}

As we move along any isoquant, the quantity of output remains the same, but the combination of inputs changes. More specifically, as we move along an isoquant, we are \textit{substituting one input for another}. For example, as we move from point B to point C along the isoquant labeled $Q = 4,000$, the quantity of land rises from 3 to 5 hectares, while the number of workers falls from 11 to 5. You are substituting land for labor, while maintaining the same level of output. Since each of the two inputs contributes to production, every time you increase one input, you must decrease the other in order to maintain the same level of output.

\textit{An increase in one input requires a decrease in the other input to keep total production unchanged. This is why isoquants always slope downward.}

What happens as we move from isoquant to isoquant? Whenever we move to a higher \textit{isoquant} (moving northeasterly in Figure A1), the quantity of output increases. Moving directly northward means you are using more labor with the same amount of land, and moving directly eastward means you are using more land with the same amount of labor. When you move both north and east simultaneously (as in the move from point B to point F), you are using more of \textit{both} inputs. For all of these movements, output increases. For the same reason, if we move southwestward, output decreases.

\textit{Higher isoquants represent greater levels of output than lower isoquants.}

Finally, notice something else about Figure A1: as we move rightward along any given isoquant, it becomes flatter. To understand why, we must take a closer look at an isoquant’s slope.

\textbf{The Marginal Rate of Technical Substitution}

The (absolute value of the) slope of an isoquant is called the \textit{marginal rate of technical substitution (MRTS)}. As the name suggests, it measure the rate at which a firm can substitute one input for another while keeping output constant. In our example, the MRTS tells us how many \textit{fewer} workers you can employ each time you use \textit{one more hectare of land}, and still maintain the same level of output.

For example, if you move from point A to point B along isoquant $Q = 4,000$, you use 1 more hectare and 7 fewer workers, so the MRTS $= 7/1 = 7$ for that move. Going from point B to point C, you use 2 more hectares of land, and 6 fewer workers, so the MRTS $= 6/2 = 3$.

Using this new term, the changing slope of an isoquant can be expressed this way:

\textit{as we move rightward along any given isoquant, the marginal rate of technical substitution decreases.}
But why does the MRTS decrease? To answer this question, it helps to understand the relationship between the MRTS and the marginal products of land and labor. You've already learned that the marginal product of labor (MPL) is a firm’s additional output when one more worker is hired and all other inputs remain constant. The marginal product of land (MPN, using “N” for land) is defined in a similar way: it’s the additional output a firm can produce with one additional unit of land (one more hectare, in our example), holding all other inputs constant.

Suppose that, starting from a given input mix, you discover that your MPN is 21 boxes of artichokes, and your MPL is 7 boxes. Then conduct the following mental experiment: add one hectare of land, with no change in labor, and your output increases by 21 boxes. Then, give up 3 workers, with no change in land, and your output decreases by $3 \times 7 = 21$ boxes. In this case, adding 1 hectare of land, and hiring 3 fewer workers leaves your output unchanged. The slope of the isoquant for a move like this would be $\frac{\Delta L}{\Delta N} = \frac{-3}{1} = -3$.

More generally, each time we change the amount of labor (L), the firm’s output will change by $\Delta L \times MPL$. Each time we change the firm’s land (N), the change in output will be $\Delta N \times MPN$. If we want the net result to be zero change in output, we must have

$$\Delta L \times MPL + \Delta N \times MPN = 0$$

or

$$\Delta L \times MPL = -\Delta N \times MPN.$$

Rearranging this equation gives us:

$$\frac{\Delta L}{\Delta N} = \frac{-MPN}{MPL}.$$

The left-hand side is the ratio of the change in labor to the change in land needed to keep output unchanged, that is, the slope of the isoquant. The right-hand side tells us that this slope is equal to the ratio of the marginal products of land and labor, except for the sign, which is negative. That is,

*at each point along an isoquant with land measured horizontally, and labor measured vertically, the (absolute value of the) slope of the isoquant, which we call the MRTS, is the ratio of the marginal products, $MPN/MPL$.*

Now, what does this have to do with the shape of the isoquant? As we move rightward and downward along an isoquant, the firm is acquiring more and more land, and using less and less labor. The marginal product of land will decrease—since land is becoming more plentiful—and the marginal product of labor will increase—since labor is becoming more and more scarce. Taken together, these changes tell us that the ratio $MPN/MPL$ must fall and so must the slope of the isoquant.

*An isoquant becomes flatter as we move rightward because the MPN decreases, while the MPL increases, so the ratio—$MPN/MPL$—decreases.*

**ISOCAST LINES**

An isoquant map shows us the different input mixes capable of producing different amounts of output. But how should the firm choose among all of these input mixes? In order to answer that question, we must know something about input prices. After all, if you own an artichoke farm, you must pay for your land and labor.
To keep the math simple, let’s use round numbers. We’ll suppose that the price of labor—the wage—is $500 per month ($P_L = $500), and the price of land—what you must pay in rent to its owner, or your implicit cost if you own the land yourself—is $1,000 per hectare per month ($P_N = $1,000). An isocost line (“same cost” line) tells us all combinations of the two inputs that would require the same total outlay for the firm. It is very much like the budget line you learned about in Chapter 5, which showed all combinations of two goods that resulted in the same cost for the consumer. The difference is that an isocost line represents total cost to a firm rather than a consumer, and is based on paying for inputs rather than goods.

Figure A2 shows three isocost lines for your artichoke farm. The middle line (labeled TC = $7,500) tells us all combinations of land and labor that would cost $7,500 per month. For example, point G represents the combination 3 hectares, 9 workers, for a total cost of 3 x $1,000 + 9 x $500 = $7,500.

**Things to Know about Isocost Lines**

Notice that all three isocost lines in figure A2 slope downward. Why is this? As you move rightward in the figure, you are using more land. If you continued to use an unchanged amount of labor, your cost would therefore increase. But an isocost line shows us input combinations with the same cost. Thus, to keep your cost unchanged as you use more land (move rightward), you must also employ fewer workers (move downward).

*If you use more of one input, you must use less of the other input in order to keep your total cost unchanged. This is why isocost lines always slope downward.*

Notice, though, that the slope of the isocost line remains constant as we move along it. That is, isocost lines are straight lines. Why? Let’s find an expression for the slope of the isocost line. Each time you change the number of workers by ΔL, your total cost will change by $P_L \times \Delta L$. Each time you change the amount of land you
use by $\Delta N$, your total cost will change by $P_N \times \Delta N$. In order for your total cost to remain the same as you change the amounts of both land and labor, the changes must satisfy the equation:

$$P_L \times \Delta L + P_N \times \Delta N = 0,$$

or,

$$P_L \times \Delta L = -P_N \times \Delta N$$

which can be rearranged to

$$\frac{\Delta L}{\Delta N} = -\frac{P_N}{P_L}.$$

The term on the left is the change in labor divided by the change in land that leaves total cost unchanged—the slope of the isocost line. The term on the right is the (negative of the) ratio of the inputs’ prices. In our example, with $P_N = $1,000 and $P_L = $500, the slope of the isocost line is -$1,000/$500 = -2.

Now you can see why the isocost line is a straight line: As long as the firm can continue to buy its inputs at unchanged prices, the ratio -$P_N/P_L$ will remain constant. Therefore, the slope of the isocost line will remain constant as well.

**The slope of an isocost line with land (N) on the horizontal axis and labor (L) on the vertical axis is -$P_N/P_L$. This slope remains constant as we move along the line.**

Finally, there is one more thing to note about isocost lines. As you move in a northeasterly direction in figure A2, to higher isocost lines, you are paying for greater amounts of land and labor, so your total cost must rise. For the same reason, as you move in a southwesterly direction, you are paying for smaller amounts of land and labor, so your total costs fall.

**Higher isocost lines represent greater total costs for the firm than lower isocost lines.**

In Figure A2, the highest line represents all inputs combinations with a total cost of $10,000, and the lowest line represents all combinations with a total cost of $5,000.

**THE LEAST-COST INPUT COMBINATION**

Now we are ready to combine what we know about a firm’s production—represented by its isoquants—with our knowledge of the firm’s costs—represented by its isocost lines. Together, these will allow us to find the least-cost input combination for producing any level of output a firm might choose to produce.

Suppose you want to know what is the best way to produce 4,000 boxes of artichokes per month. Figure A3 reproduces the isocoun labeled $Q = 4,000$ from Figure A1, along with the three isocost lines from Figure A2. You would like to find the input combination that is capable of producing 4,000 boxes (an input combination on the isocost $Q = 4,000$), with the lowest possible cost (an input combination on the lowest possible isocost line). As you can see in the diagram, there is only
one input combination that satisfies both requirements: point C. At this point, the
firm uses 5 hectares of land, and 5 workers, for a total cost of $5 \times 1,000 + 5 \times
$500 = $7,500. As you can see, while there are other input combinations that can
also produce 4,000 boxes, such as point J or point K, each of these lie on a higher
isocost line (TC = $10,000), and will require a greater total outlay than the least-
cost combination at point C.

The least-cost combination will always be found where the isocost line is tan-
gent to the isoquant. This is the where the two lines touch each other at a single
point, and both lines have the same slope.

This result will prove very useful. We already know that the slope of the iso-
quant at any point is equal to $-\frac{\text{MPN}}{\text{MPL}}$. And we know that the slope of the iso-
cost line is equal to $-\frac{\text{PN}}{\text{PL}}$. Putting the two together, we know that when you have
found the least-cost input combination for any output level,

\[-\frac{\text{MPN}}{\text{MPL}} = -\frac{\text{PN}}{\text{PL}}\]

or

\[\frac{\text{MPN}}{\text{MPL}} = \frac{\text{PN}}{\text{PL}}.\]

The term on the left-hand side is just the MRTS between land and labor. We
conclude that:

When a firm is using the least-cost combination of inputs for a particular
output level, the firm’s MRTS between the two inputs (MRTS) will
equal the ratio of its input prices (P_N/P_L):
In our example, $P_N/P_L = $1,000/$500 = 2. This tells us that, at point C, the ratio $MPN/MPL = 2$ as well.

Finally, we can rearrange the equation $MPN/MPL = P_N/P_L$ to get:

$$\frac{MPN}{P_N} = \frac{MPL}{P_L}.$$  

This form of the equation gives us another insight. It says that when you have found the least-cost input mix for any output level, the marginal product of land divided by the price of land will be equal to the marginal product of labor divided by the price of labor.

How can we interpret the marginal product of an input divided by its price? It gives us the additional output from spending one more dollar on the input. For example, if the (monthly) price of a hectare of land is $1,000, and using one more hectare increases your output by 21 boxes ($MPN = 21$), then an additional dollar spent on land will give you $1/1,000$ of a hectare, which, in turn, will increase your output by $(1/1,000) \times 21 = 21/1,000$ or .021 boxes. So, $MPN/P_N = 21/1,000$ is the additional output from one more dollar spent on land. As a kind of shorthand, we’ll call $MPN/P_N$ the “marginal product per dollar” of land.

Using this language, we can state our result this way:

When a firm is using the least-cost combination of inputs for any output level, the marginal product per dollar of land ($MPN/P_N$) must equal the marginal product per dollar of labor ($MPL/P_L$).

In the next section, where this result is stated more generally, you will learn the intuition behind it.

**Generalizing to the Case of More than Two Inputs**

When a firm can vary three or more inputs, we cannot illustrate isoquants and isocost lines on a two-dimensional graph. Nevertheless, the conclusions we reached for the two-input case can be generalized to any number of inputs.

Suppose a firm has several variable inputs, which we can label $A, B, C, \ldots$, with marginal products $MP_A, MP_B, MP_C, \ldots$ and input prices $P_A, P_B, P_C, \ldots$ Then for any level of output, the least-cost combination of all of these inputs will always satisfy:

$$\frac{MP_A}{P_A} = \frac{MP_B}{P_B} = \frac{MP_C}{P_C} = \ldots$$

That is,

When a firm with many variable inputs has found its least-cost input mix, the marginal product per dollar of any input will be equal to the marginal product per dollar of any other input.

How do we know this must always be true? First, remember that $MP_A/P_A$ tells us the additional output the firm will produce per additional dollar spent on input $A$. Next, suppose we have two inputs, $A$ and $B$, for which $MP_A/P_A$ is not equal to $MP_B/P_B$. Then we can show that the firm can always shift its spending from one input to another, lowering its cost while leaving its output unchanged.
Let’s take a specific example. Suppose that $\frac{MPA}{PA} = 2$, and $\frac{MPB}{PB} = 3$. Then the firm can easily save money by shifting dollars away from input A toward input B. Each dollar shifted away from A causes output to decrease by 2 units, while each dollar shifted toward input B causes output to rise by 3 units. Thus, the firm could shift dollars away from input A, and use only some of those dollars to increase the amount of input B, and still keep its production unchanged.

The same holds for any other two inputs we might compare: whenever the marginal product per dollar is different for any two inputs, the firm can always shift its spending from the input with the lower marginal product per dollar to the input with the higher marginal product per dollar, achieving lower total cost with no change in output.
How Firms Make Decisions: Profit Maximization

CHAPTER OUTLINE

The Goal of Profit Maximization
Understanding Profit
  Two Definitions of Profit
  Why Are There Profits?

The Firm’s Constraints
  The Demand Constraint
  The Cost Constraint

The Profit-Maximizing Output Level
  The Total Revenue and Total Cost Approach
  The Marginal Revenue and Marginal Cost Approach
  Profit Maximization Using Graphs
  What About Average Costs?
  The Marginal Approach to Profit

Dealing with Losses
  The Short Run and the Shutdown Rule
  The Long Run: The Exit Decision

Using the Theory: Getting It Wrong and Getting It Right
  Getting It Wrong: The Failure of Franklin National Bank
  Getting It Right: The Success of Continental Airlines

In 2001, the managers of Nintendo America, Inc., knew that they had another winner on their hands: the Game Boy Advance handheld game machine. The new product ran 17 times faster than its predecessor (Game Boy Color), and displayed 32,768 colors on its 2.9 inch screen. It was sure to dazzle consumers, displaying images more spectacular and faster-moving than any competing product.

Then came the hard questions. Where should the new product be produced: Japan, the United States, or perhaps Hong Kong? How should the company raise the funds to pay the costs of production? When should it bring the product to market? How much should it spend on advertising, and in which types of media? And finally, what price should the company charge, and how many units should it plan to produce?

These last decisions—how much to produce and what price to charge—are the focus of this chapter. In the end, Nintendo planned to produce 23 million units for the first year, and decided to charge $99.95. But why didn’t it charge a lower price that would allow it to sell more units? Or a higher price that would give it more profit on each unit sold?