

Profit Maximization

$$\begin{aligned}
 \text{Profit } (\pi) \quad \text{Profit} &= \pi = \text{Revenue} - \text{Cost} \\
 &= py - \sum_{i=1}^n w_i x_i, \quad y \in P(x) \\
 &= pf(x_1, x_2, \dots, x_n) - \sum_{i=1}^n w_i x_i \\
 &= py - C(y, w_1, w_2, \dots, w_n)
 \end{aligned}$$

$$\begin{aligned}
 \text{Profit max problem} \quad \pi(p, w_1, w_2, \dots) &= \max_{x, y} \left[py - \sum_{i=1}^n w_i x_i \right] \text{ such that } x \in P(x) \\
 &= \max_x pf(x_1, x_2, \dots, x_n) - \sum_{i=1}^n w_i x_i \\
 &= \max_y py - C(y, w_1, w_2, \dots)
 \end{aligned}$$

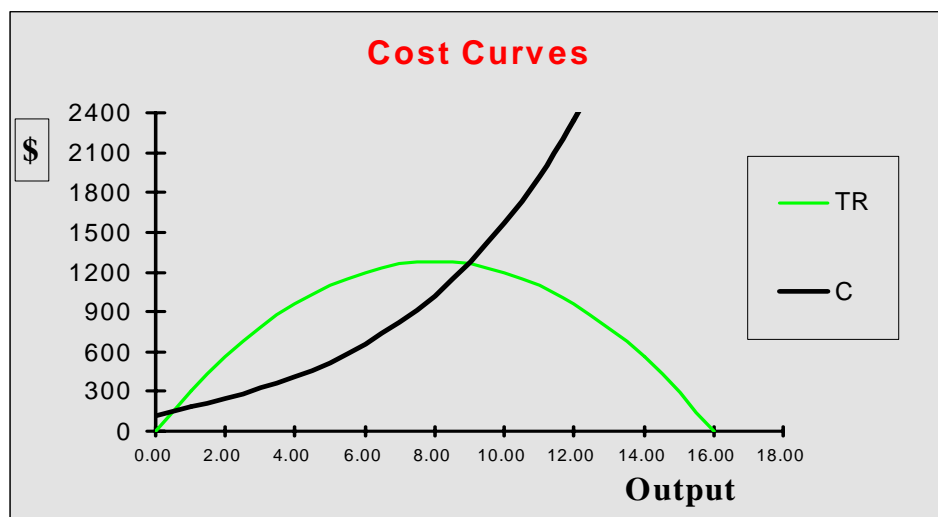
$$\text{Total revenue (TR)} \quad \text{Revenue} = R = TR = py = pf(x_1, x_2, \dots, x_n)$$

$$\text{Total Cost (TC)} \quad TC = C(x_1, x_2, \dots, w_1, w_2, \dots) = \sum_{i=1}^n w_i x_i = C(y, w_1, w_2, \dots)$$

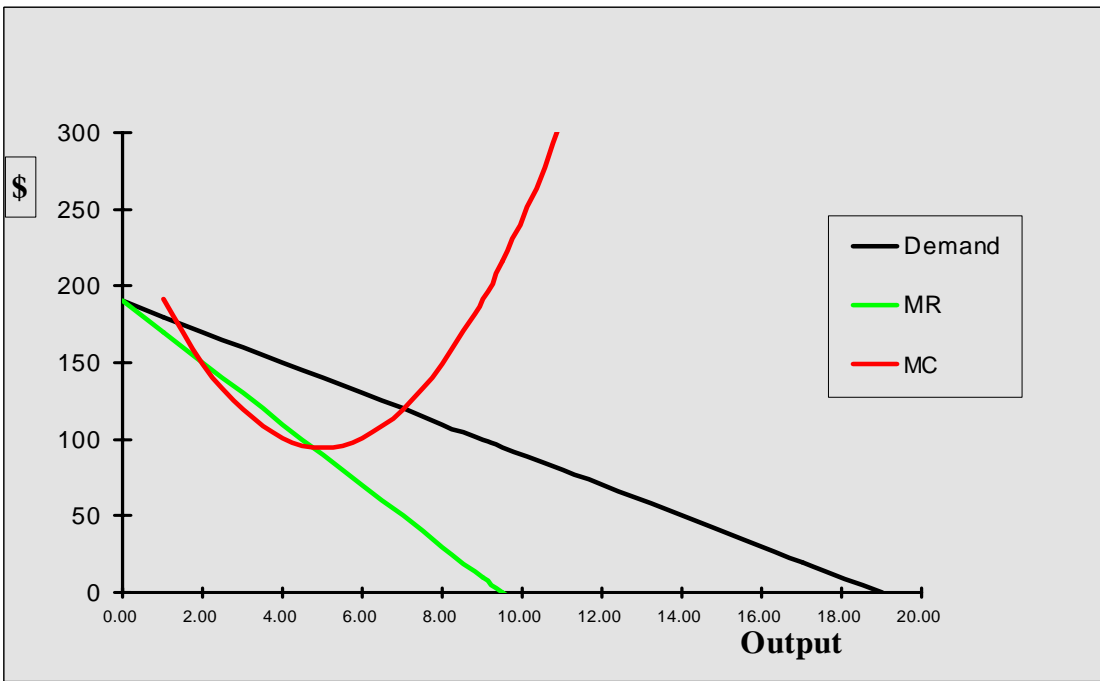
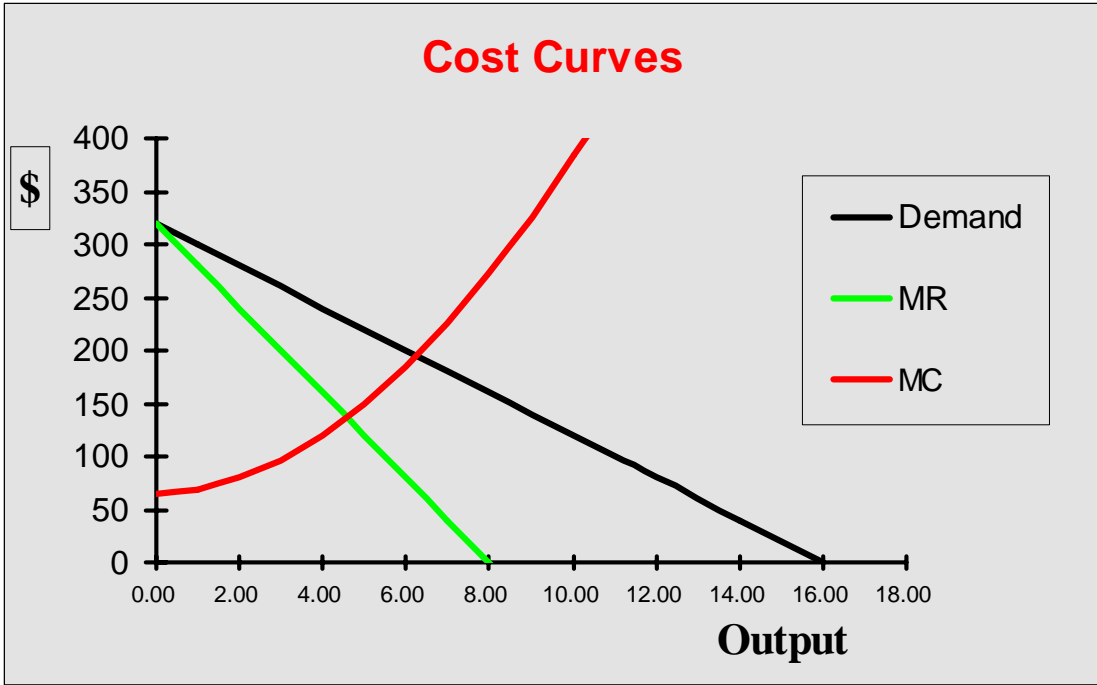
$$\text{Marginal cost (MC)} \quad MC = \frac{\Delta C(y, w)}{\Delta y} = \frac{\Delta TC(y, w)}{\Delta y}$$

$$\text{Marginal revenue (MR)} \quad MR = \frac{\Delta R(y, p)}{\Delta y} = \frac{\Delta TR(y, p)}{\Delta y}$$

Optimal output π is maximized; TR - TC is maximized; MR = MC



y	Price	FC	VC	C	AFC	AVC	ATC	MC	TR	MR	Profit
0.00	320	120	0.00	120.00					0		-120.00
								64.00		300.00	
1.00	300	120	64.00	184.00	120.00	64.00	184.00		300		116.00
								66.00		260.00	
2.00	280	120	130.00	250.00	60.00	65.00	125.00		560		310.00
								74.00		220.00	
3.00	260	120	204.00	324.00	40.00	68.00	108.00		780		456.00
								88.00		180.00	
4.00	240	120	292.00	412.00	30.00	73.00	103.00		960		548.00
								108.00		140.00	
5.00	220	120	400.00	520.00	24.00	80.00	104.00		1100		580.00
								134.00		100.00	
6.00	200	120	534.00	654.00	20.00	89.00	109.00		1200		546.00
								166.00		60.00	
7.00	180	120	700.00	820.00	17.14	100.00	117.14		1260		440.00
								204.00		20.00	
8.00	160	120	904.00	1024.00	15.00	113.00	128.00		1280		256.00
								248.00		-20.00	
9.00	140	120	1152.00	1272.00	13.33	128.00	141.33		1260		-12.00
								298.00		-60.00	
10.00	120	120	1450.00	1570.00	12.00	145.00	157.00		1200		-370.00
								354.00		-100.00	
11.00	100	120	1804.00	1924.00	10.91	164.00	174.91		1100		-824.00
								416.00		-140.00	
12.00	80	120	2220.00	2340.00	10.00	185.00	195.00		960		-1380.00
								521.00		-200.00	
14.00	40	120	3262.00	3382.00	8.57	233.00	241.57		560		-2822.00
								681.00		-280.00	
16.00	0	120	4624.00	4744.00	7.50	289.00	296.50		0		-4744.00



Example of Operating Versus Shutting Down

Suppose at 6 units of output, $TFC = \$50$, $TVC = \$60$ and $TC = \$110$. Suppose also at 6 units of output that total revenue is equal to $\$95$. By operating the firm can make $\$35$ over variable cost ($\$95 - 60$). This will help cover the fixed costs of $\$50$. The firm's net loss is just $\$15.00$ ($35 - 50$ or $95 - 110$). This is larger than the net loss of $\$50.00$ if they do not produce.

But suppose that only $\$10$ of the fixed costs are sunk, so that by shutting down the firm can recover $\$40.00$ of fixed cost. The firm's net loss by shutting down is then only $\$10.00$. The firm is better off by shutting down. A table summarizes.

	Fixed Cost	Variable Cost	Total Cost	Sales Revenue	Asset Disposal Revenue	Total Revenue	Net Profit or Loss
Operate	50	60	110	95	0	95	-15
Shut-down	50	0	50	0	40	40	-10