**Economics 101**
**Spring 2001**
**Section 4 - Hallam**
**Problem Set #5**

**Due date: March 2, 2001**

1. Consider the following data on quantities of $q_1$ and $q_2$ and utility. In the table $q_2$ is held fixed at 3 units. Compute marginal utility for each change in $q_1$. Rather than placing it between each value of $q_1$ and the subsequent one, put it next to the subsequent one.

| $q_1$ | $q_2$ | **u, $q_2$ = 3** | **$MU_1$, $q_2$ = 3** |
|---|---|---|---|
| 0 | 3 | 0.00 | NA |
| 1 | 3 | 6.93 | 6.93 |
| 2 | 3 | 8.24 | |
| 3 | 3 | 9.12 | |
| 4 | 3 | 9.80 | |
| 5 | 3 | 10.37 | |
| 6 | 3 | 10.85 | |
| 7 | 3 | 11.27 | |
| 8 | 3 | 11.66 | 0.39 |
| 9 | 3 | 12.00 | |
| 10 | 3 | 12.33 | |
| 11 | 3 | 12.62 | |
| 12 | 3 | 12.90 | 0.28 |

2. Consider the following hypothetical data on cloth and wine production in England and Portugal. Cloth is measured in 1,000 yards, wine is measured in 1,000 liters. Assume that capital is freely mobile so only labor costs matter. Also assume that real wages will tend to equalize so that only labor quantities matter. The data below gives the number of **workers required per unit of output**.

| | Cloth | Wine |
|---|---|---|
| England | 100 workers | 75 workers |
| Portugal | 60 workers | 30 workers |

a. Who has a comparative advantage in cloth? (Show work)
b. Who has a comparative advantage in wine? (Show work)
c. Convert the data to **output per worker** and make a new table.
d. Who has the absolute advantage in cloth?
e. Using the new data, show who has the comparative advantage in cloth.

3. In the table below are listed a number of combinations of $q_1$ and $q_2$. Based on this data draw this individual's indifference curves for u = 12, u = 14, u = 16 and u = 18. You can use just 5 or 6 points for each indifference curve. **Use graph paper.**

| $q_1$ | $q_2$ | cost | **u bar** |
|---|---|---|---|
| 41.569 | 0.75 | 143.89 | 12.00 |
| 27.000 | 1 | 135.00 | 12.00 |
| 9.546 | 2 | 181.09 | 12.00 |
| 5.196 | 3 | 253.39 | 12.00 |
| 6.831 | 2.5 | 216.16 | 12.00 |
| 5.196 | 3 | 253.39 | 12.00 |
| 4.123 | 3.5 | 291.75 | 12.00 |
| 3.375 | 4 | 330.75 | 12.00 |
| 2.415 | 5 | 409.83 | 12.00 |
| 1.837 | 6 | 489.67 | 12.00 |
| 1.458 | 7 | 569.92 | 12.00 |
| 42.875 | 1 | 166.75 | 14.00 |
| 23.338 | 1.5 | 168.18 | 14.00 |
| 15.159 | 2 | 192.32 | 14.00 |
| 10.847 | 2.5 | 224.19 | 14.00 |
| 8.251 | 3 | 259.50 | 14.00 |
| 6.548 | 3.5 | 296.60 | 14.00 |
| 5.359 | 4 | 334.72 | 14.00 |
| 3.835 | 5 | 412.67 | 14.00 |
| 2.917 | 6 | 491.83 | 14.00 |
| 2.315 | 7 | 571.63 | 14.00 |
| 64.000 | 1 | 209.00 | 16.00 |
| 34.837 | 1.5 | 191.17 | 16.00 |
| 22.627 | 2 | 207.25 | 16.00 |
| 16.191 | 2.5 | 234.88 | 16.00 |
| 12.317 | 3 | 267.63 | 16.00 |
| 9.774 | 3.5 | 303.05 | 16.00 |
| 8.000 | 4 | 340.00 | 16.00 |
| 5.724 | 5 | 416.45 | 16.00 |
| 4.355 | 6 | 494.71 | 16.00 |
| 3.456 | 7 | 573.91 | 16.00 |
| 91.125 | 1 | 263.25 | 18.00 |
| 49.602 | 1.5 | 220.70 | 18.00 |
| 32.218 | 2 | 226.44 | 18.00 |
| 23.053 | 2.5 | 248.61 | 18.00 |
| 17.537 | 3 | 278.07 | 18.00 |
| 13.917 | 3.5 | 311.33 | 18.00 |
| 11.391 | 4 | 346.78 | 18.00 |
| 8.150 | 5 | 421.30 | 18.00 |
| 6.200 | 6 | 498.40 | 18.00 |
| 4.920 | 7 | 576.84 | 18.00 |

4. Assume that $p_1 = 4$, $p_2 = 12$, and I = 80. Draw in a budget line for Problem 3. What are the optimum levels of $q_1$ and $q_2$?

5. Now assume that $p_1 = 2$ and $p_2 = 81$ and I = 135. Draw in a budget line for Problem 3. What are the optimum levels of $q_1$ and $q_2$? (You may guess).

6. For this problem $p_1 = 16$, $p_2 = 20$ and I = 240. Below is a table of alternative consumption choices and the utility and the marginal utility they provide. Which is the optimal choice?

| $q^2$ | $q^1$ | **u** | $MU_1$ | $MU_2$ |
|---|---|---|---|---|
| 0 | 15 | 0.000 | 0.000 | ∞ |
| 1 | 13.75 | 19.257 | 0.351 | 9.629 |
| 2 | 12.5 | 26.592 | 0.532 | 6.648 |
| 4 | 10 | 35.566 | 0.890 | 4.446 |
| 6 | 7.5 | 40.537 | 1.352 | 3.379 |
| 8 | 5 | 42.295 | 2.115 | 2.644 |
| 10 | 2.5 | 39.764 | 3.977 | 1.989 |

Show and explain the answer using the three following equilibrium conditions.

$$\frac{p_2}{p_1} = \frac{MU_{q_2}}{MU_{q_1}}$$

$$\frac{-p_2}{p_1} = MRS_{q_1 q_2} = \frac{\Delta q_1}{\Delta q_2}$$

$$\frac{MU_{q_1}}{p_1} = \frac{MU_{q_2}}{p_2}$$

7. For this problem $p_1 = 6$, $p_2 = 4$ and I = 58. Below is a table of alternative consumption choices and the utility and the marginal utility they provide. Which is the optimal choice?

| $q^1$ | $q^2$ | cost | u | $MU_1$ | $MU_2$ |
|---|---|---|---|---|---|
| 9.666667 | 0 | 58 | 5.369 | 0.151003 | 0.939575 |
| 8.333333 | 2 | 58 | 6.851 | 0.220211 | 0.799283 |
| 7 | 4 | 58 | 8 | 0.3 | 0.7 |
| 5.666667 | 6 | 58 | 8.855 | 0.398475 | 0.61985 |
| 4.333333 | 8 | 58 | 9.409 | 0.529256 | 0.548858 |
| 3 | 10 | 58 | 9.615 | 0.721125 | 0.48075 |
| 1.666667 | 12 | 58 | 9.348 | 1.05165 | 0.408975 |
| 0 | 14.5 | 58 | 7.71 | 2.313 | 0.29173 |

Show and explain the answer using the three following equilibrium conditions.

$$\frac{p_2}{p_1} = \frac{MU_{q_2}}{MU_{q_1}}$$

$$\frac{-p_2}{p_1} = MRS_{q_1 q_2} = \frac{\Delta q_1}{\Delta q_2}$$

$$\frac{MU_{q_1}}{p_1} = \frac{MU_{q_2}}{p_2}$$

8. Consider the following utility function

$$u(q_1,\ q_2) = 20q_1 + 14q_2 - q_1^2 - q_2^2$$

which is valid for low levels for $q_1$ and $q_2$. Complete the following table where $MU_1$ is computed holding $q_2$ constant.

| $q_1$ | $q_2$ | u | $MU_1$ |
|---|---|---|---|
| 0 | 1 | 13 | — |
| 1 | 1 | 32 | 19 |
| 2 | 1 | 49 | 17 |
| 3 | 1 | 64 | 15 |
| 5 | 1 | | |
| 7 | 1 | | |
| 0 | 2 | | — |
| 1 | 2 | | |
| 2 | 2 | | |
| 3 | 2 | | |
| 5 | 2 | | |
| 7 | 2 | | |
| 0 | 3 | | — |
| 1 | 3 | | |
| 2 | 3 | | |
| 3 | 3 | | |
| 5 | 3 | | |
| 7 | 3 | | |
| 0 | 5 | | — |
| 1 | 5 | 64 | |
| 2 | 5 | 81 | 17 |
| 3 | 5 | | |
| 5 | 5 | | |
| 7 | 5 | | |

9. An indifference curve shows combinations of goods that all lead to the same utility level. For the two good case, we can find the indifference curve from the utility function by specifying the level of utility and then solving the utility function equation for $q_1$ as a function of $q_2$. Consider then the indifference curve for the utility function in problem 8. Letting the given level of utility be denoted $u^0$, the function is given by

$$u^0 = 20q_1 + 14q_2 - q_1^2 - q_2^2$$

We can write this as a function of $q_1$ as follows

$$\begin{aligned} u^0 &= 20q_1 + 14q_2 - q_1^2 - q_2^2 \\ \Rightarrow q_1^2 + q_2^2 + u^0 &= 20q_1 + 14q_2 \\ \Rightarrow q_1^2 - 20q_1 + u^0 + q_2^2 - 14q_2 &= 0 \\ \Rightarrow q_1^2 - 20q_1 + (u^0 + q_2^2 - 14q_2) &= 0 \end{aligned}$$

This is a quadratic equation in $q_1$ that can be solved using the quadratic formula where

$$a = 1,\ b = -20,\ and\ c = u^0 + q_2^2 - 14q_2$$

Solving we obtain

$$\begin{aligned} q_1 &= \frac{20 \pm \sqrt{(-20^2) - (4)(1)(u^0 + q_2^2 - 14q_2)}}{2} \\ &= \frac{20 \pm \sqrt{400 - 4(u^0 + q_2^2 - 14q_2)}}{2} \end{aligned}$$

For example, if q = 120, and $q_2 = 3$, then $q_1 = 6.3945$. This is clear from

$$\begin{aligned} q_1 &= \frac{20 \pm \sqrt{400 - 4(120 + 9 - 42)}}{2} \\ &= \frac{20 \pm \sqrt{52}}{2} \\ &= \frac{20 \pm 7.211}{2} \\ \Rightarrow q_1 &= 13.6055,\ or\ q_1 = 6.3945 \end{aligned}$$

We chose the more efficient level of $q_1 =$ 6.3945.

For a utility level of 120 and $q_2 = 2$, find the level of $q_1$ that puts the consumer on the indifference curve.

10. For a utility level of 120 and levels of $q_2$ of 1, 2, 3, 4, 5, 6, 6.5 and 7, find the level of $q_1$ puts the consumer on the indifference curve. Make a table with these points.

| $q_1$ | $q_2$ | Utility |
|---|---|---|
| | 1 | 120 |
| | 2 | 120 |
| 6.3944 | 3 | 120 |
| | 4 | 120 |
| | 5 | 120 |
| 4.7085 | 6 | 120 |
| | 6.5 | 120 |
| 4.6148 | 7 | 120 |

11. Graph these points in a diagram with $q_1$ on the vertical axis. **Use graph paper.**

12. Assume the price of $q_1$ = 10 and the price of $q_2$ = 4 for problem 11.

    a. Show on the same graph all combinations of $q_1$ and $q_2$ that cost $50.00.

    b. Show on the same graph all combinations of $q_1$ and $q_2$ that cost $80.00.

    c. By shifting this line in a parallel fashion, find the combination of $q_1$ and $q_2$ that has the lowest cost among those that allow for 120 units of utility.

    How many units of $q_1$ and $q_2$ are chosen?

    How much income do you think is spent to obtain this utility level?

13. Work question 3 from Skills and Tools in Chapter 5.

14. Work question 4 from Skills and Tools in Chapter 5.

15. Work question 6 from Skills and Tools in Chapter 5.