Economics 101
Spring 2001
Problem Set #7

Due date: March 30, 2000

1. Consider the following production function

$$y = 40x_1 + 20x_2 - 2x_1^2 - x_2^2$$

The price of x_1 is given by w_1 = $600 and the price of x_2 is w_2 = $100. You are trying to find which of the following sets of points is the cost minimizing way to produce 129 units of output. For each of the input combinations in question, verify that it will produce 129 units (or close with rounding), compute its cost, find the marginal rate of substitution and the input price ratio. Then decide which point is minimum cost.

x_1	x_2	Cost	y	MPP_1	MPP_2	MRS_{12}	$\frac{-w_2}{w_1}$
3.292	1.000	2075.078	129.000	26.833	18.000		
2.686	2.000	1811.378	129.000	29.257	16.000		
2.000	3.000			32.000	14.000		
1.784	4.000	1470.497		32.863	12.000		
1.456	5.000	155.1512	129.000	34.176	10.000		
1.197	6.000	1317.955	129.000	35.214	8.000		
1.000	7.000		129.000	36.000	6.000		
0.862	8.000	1317.300		36.551	4.000		
0.500	9.000			38.000	2.000		

2. Consider the following data for a problem like problem 1. Fill in the missing data and determine the minimum cost way to produce 400 units of output?

x_1	x_2	y	MPP_1	MPP_2	MRS_{12}	$\frac{-w_2}{w_1}$
6.784	1.000	400.000	46.433	38.000		-0.528
6.000	2.000	400.000	48.000	36.000		-0.528
2.000	3.000	227.000	56.000	34.000		-0.528
4.623	4.000	400.000	50.754	32.000		-0.528
4.019	5.000	400.000	51.962	30.000		-0.528
3.467	6.000	400.000	53.066	28.000		-0.528
2.963	7.000	400.000	54.074	26.000		-0.528
2.505	8.000	400.000	54.991	24.000		-0.528
0.500	9.000	308.750	59.000	22.000		-0.528

3. Consider the following data on a production function. The price of x_1 is w_1 = \$2.00 and the price of x_2 is w_2 = \$2.00. You are trying to find which of the following sets of points is the cost minimizing way to produce 40 units of output. The pairs listed will all produce 40 units. Compute the cost of each input pair. Fill in the marginal rate of substitution and the MPP/\$ ratios. Also fill in the slope of the isocost line. Show in 2 different ways the optimum input combination.

x_1	**x_2**	**Cost**	**y**	**MPP_1**	**MPP_2**	**MRS**	**MPP_1 / w_1**	**MPP_2 / w_2**	**Slope of Isocost**
16.00	4.00		40.00	0.63	5.00	-8.00	0.31		
12.64	4.50	34.28	40.00	0.79	4.44				
10.24	5.00	30.48	40.00	0.98	4.00			2.00	
8.46	5.50		40.00	1.18	3.64			1.82	
7.11	6.00	26.22	40.00	1.41	3.33		0.70	1.67	
6.06	6.50	25.12	40.00	1.65	3.08	-1.86	0.83	1.54	
5.22	7.00		40.00	1.91	2.86	-1.49	0.96		
4.55	7.50		40.00	2.20	2.67				
4.00	8.00		40.00	2.50	2.50			1.25	
3.54	8.50		40.00	2.82	2.35		1.41		
3.16	9.00		40.00	3.16	2.22	-0.70	1.58		
2.84	9.50	24.67	40.00	3.53	2.11		1.76		
2.56	10.00		40.00	3.91	2.00	-0.51		1.00	

4. Consider the following data on a production function. The price of w_1 is \$4.00 and the price of w_2 is \$2.00. You are trying to find which of the following sets of points is the cost minimizing way to produce 160 units of output. The pairs listed will all produce 160 units. Compute the cost of each input pair. Fill in the marginal rate of substitution and the MPP/\$ ratios. Also fill in the slope of the isocost line. Show in 2 different ways the optimum input combination.

x_1	x_2	Cost	y	MPP_1	MPP_2	MRS	MPP_1 /w_1	MPP_2/w_2	Slope of Isocost
18.204	60.000	192.818	160.000	2.197	1.333	-0.607	0.549		
17.905	60.500	192.619	160.000	2.234	1.322				
17.612	61.000		160.000	2.271	1.311				
17.327	61.500		160.000	2.309	1.301				
17.049	62.000		160.000	2.346	1.290		0.587	0.645	
16.777	62.500		160.000	2.384	1.280		0.596	0.640	
16.512	63.000		160.000	2.422	1.270			0.635	
16.253	63.500		160.000	2.461	1.260	-0.512			
16.000	64.000		160.000	2.500	1.250				
15.753	64.500		160.000	2.539	1.240		0.635	0.620	
15.511	65.000	192.046	160.000	2.579	1.231	-0.477		0.615	
15.276	65.500		160.000	2.619	1.221			0.611	
15.045	66.000		160.000	2.659	1.212			0.606	
14.820	66.500	192.278	160.000	2.699	1.203	-0.446	0.675	0.602	
14.599	67.000	192.397	160.000	2.740	1.194	-0.436	0.685	0.597	

5. Consider the following data on a production function. The price of w_1 is \$3.75 and the price of w_2 is \$10.00. You are trying to find which of the following sets of points is the cost minimizing way to produce 80 units of output. The pairs listed will all produce 80 units. Compute the cost of each input pair. Fill in the marginal rate of substitution and the MPP/\$ ratios. Also fill in the slope of the isocost line. Show in 2 different ways the optimum input combination.

x_1	x_2	Cost	y	MPP_1	MPP_2	MRS	MPP_1/w_1	MPP_2/w_2	Slope of Isocost
29.9419	5		80	0.66796	5.333333	-7.9845	0.178122	0.53333	
26.368	5.5	153.883	80	0.75847	4.848485	-6.3924		0.48485	
23.4803	6		80	0.85178	4.444444			0.44444	
21.1035	6.5	144.138	80	0.94771	4.102564			0.41026	
19.1180	7	141.693	80	1.04613	3.809524	-3.6415			
17.4377	7.5		80	1.14693	3.555556				
16	8		80	1.25	3.333333				
14.7575	8.5		80	1.35524	3.137255				
13.6746	9	141.28	80	1.46256	2.962963		0.390016		
12.7235	9.5		80	1.57189	2.807018	-1.7858	0.41917	0.2807	
11.8824	10		80	1.68315	2.666667	-1.5843	0.448841	0.26667	

6. Consider the following data on a production function. The price of w_1 is \$5.00 and the price of w_2 is \$10.00. You are trying to find which of the following sets of points is the cost minimizing way to produce 268 units of output. The pairs listed will all produce 268 units. Fill in the marginal rate of substitution and the MPP/\$ ratios. Also fill in the slope of the isocost line. Show in 2 different ways the optimum input combination.

x_1	x_2	y	MPP_1	MPP_2	MRS	MPP_1/w_1	MPP_2/w_2	Slope of Isocost
10.707	5.75	268	4.42295	19.1203				
10	6	268	8	16				
9.5947	6.25	268	10.3712	13.7841				
9.3106	6.5	268	12.2577	11.9318		2.452		
9.0978	6.75	268	13.8587	10.2935		2.772		
8.9339	7	268	15.2643	8.80175		3.053		
8.8063	7.25	268	16.5246	7.41905	-0.449	3.305	0.742	
8.7074	7.5	268	17.6706	6.12205	-0.3465	3.534	0.612	
8.6317	7.75	268	18.7233	4.89501	-0.2614	3.745	0.490	
8.5756	8	268	19.6977	3.72671			0.373	
8.536	8.25	268	20.6049	2.60881			0.261	
8.5116	8.5	268	21.4534	1.53492	-0.0715	4.291	0.153	
8.5	8.75	268	22.25	0.5	-0.0225	4.450	0.050	
8.5	9	268	23	-0.5	0.02174	4.600	-0.050	
8.5105	9.25	268	23.7079	-1.4684		4.742	-0.147	
8.5307	9.5	268	24.3772	-2.4079		4.875	-0.241	
8.5597	9.75	268	25.0112	-3.3209	0.13278			
8.5969	10	268	25.6125	-4.2094	0.16435			

7. On a piece of graph paper where you let each square represent 0.125, graph the isoquant (x_1 on vertical and x_2 on horizontal) and the isocost line that is tangent to it.

8. Which of the points on the isoquant from problem 7 are not relevant?

9. Consider the following production function

$$y = 22x_1 + 14x_2 - x_1^2 - x_2^2$$

which is valid for low levels for x_1 and x_2. An isoquant shows combinations of inputs that all lead to the same level of output. For the two good case, we can find the isoquant from the production function by specifying the level of output and then solving the production function equation for x_1 as a function of x_2. Letting the given level of output be denoted y*, the function is given by

$$y^* = 22x_1 + 14x_2 - x_1^2 - x_2^2$$

We can write this as a function of x_1 as follows

$$y^* = 22x_1 + 14x_2 - x_1^2 - x_2^2$$

$$\Rightarrow x_1^2 + x_2^2 + y^* = 22x_1 + 14x_2$$

$$\Rightarrow x_1^2 - 22x_1 + y^* + x_2^2 - 14x_2 = 0$$

$$\Rightarrow x_1^2 - 22x_1 + (y^* + x_2^2 - 14x_2) = 0$$

This is a quadratic equation in x_1 that can be solved using the quadratic formula where

$$a = 1,\ b = -22,\ and\ c = y^* + x_2^2 - 14x_2$$

Solving we obtain

$$x_1 = \frac{22 \pm \sqrt{(-22^2) - (4)(1)(y^* + x_2^2 - 14x_2)}}{2}$$

$$x_1 = \frac{22 \pm \sqrt{484 - 4(y^* + x_2^2 - 14x_2)}}{2}$$

$$x_1 = 11 - \frac{1}{2}\sqrt{484 - 4(y^* + x_2^2 - 14x_2)}$$

For example, if y = 80, and $x_2 = 3$, then $x_1 = 2.3977$. This is clear from

$$x_1 = \frac{22 - \sqrt{484 - 4(80 + 9 - 42)}}{2}$$

$$= \frac{22 - \sqrt{296}}{2}$$

$$= \frac{22 - 17.2046}{2}$$

$$\Rightarrow x_1 = 2.3977$$

For a production level 80 and a level of x_2 of 4 find the level of x_1 that puts the firm on the isoquant.

10. For an output utility level of 80 and levels of x_2 of 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10, find the level of x_1 puts the firm on the isoquant. Make a table with these points.

y	x_1	x-2
80	4.5969	0
80		1
80		2
80	2.3977	3
80		4
80	1.7264	5
80	1.566	6
80		7
80	1.566	8
80		9
80		10

11. Graph these points on graph paper in a diagram with x_1 on the vertical axis.

12. Assume the price of $x_1 = 6$ and the price of $x_2 = 2$ in the problem 11 and draw in isocost lines in the diagram.

a. Show on the same graph all combinations of x_1 and x_2 that cost \$16.00.

b. Show on the same graph all combinations of x_1 and x_2 that cost \$24.00.

c. By shifting this line in a parallel fashion, find the combination of x_1 and x_2 that has the lowest cost among those that allow for 80 units of production.

d. About how much does it cost?

13. Assume a firm with the following cost function

$$cost = 128 + 69y - 14y^2 + y^3$$

where y is the level of output for the firm. Assume that the price of y is 60, i.e., $p_y = 60$.

a. What is an expression for the firm's profit in terms of y?

$\pi =$

b. Create a table listing output levels from 6 to 12, the price of the good at each level (it will equal 60 at all levels), the revenue ($p_y y$) for each level, the cost at each level, the marginal cost between each level, and the profit at each level.

c. Create a graph of revenue and cost for each output level so that both curves are in the same graph. Label the curves and title the graph. At what level of y does profit seem to be maximized?

d. Create another graph with price and marginal cost plotted against output. Put the marginal cost of going from 4 to 5 units of output above 4.5 on the horizontal axis, etc. Label the curves and title the graph. At what level of y does profit seem to be maximized?

14. In the diagram on the next page there is an increase in the price of good 1. The initial situation is $p_1 = 1$, $p_2 = 1$, and income = 54. Then the price of p_1 rises to 2. In the diagram show the income and substitution effect of the price change where the real income is evaluated at the initial utility level. That is, consider an income change that takes the consumer from the initial indifference curve to the new one after a change in prices.
    a. Show the initial equilibrium and label it as point A.
    b. Show the subsequent equilibrium and label it as point B.
    c. Show where the consumer would move due to the substitution effect along the initial indifference curve and label it point C.
    d. Use an arrow to show the substitution effect.
    e. Show the income effect with an arrow.
    f. Is good 1 a normal or inferior good?

15. In the diagram two pages hence is a decrease in the price of good 2. The initial situation is $p_1 = 4$, $p_2 = 12$, and income = 208. Then the price of p_2 falls to 2. In the diagram show the income and substitution effect of the price change where the real income is evaluated at the initial utility level. That is, consider an income change that takes the consumer from the initial indifference curve to the new one after a change in prices.
    a. Show the initial equilibrium and label it as point A.
    b. Show the subsequent equilibrium and label it as point B.
    c. Show where the consumer would move due to the substitution effect along the initial indifference curve and label it point C.
    d. Use an arrow to show the substitution effect.
    e. Show the income effect with an arrow.
    f. Is good 2 a normal or inferior good?

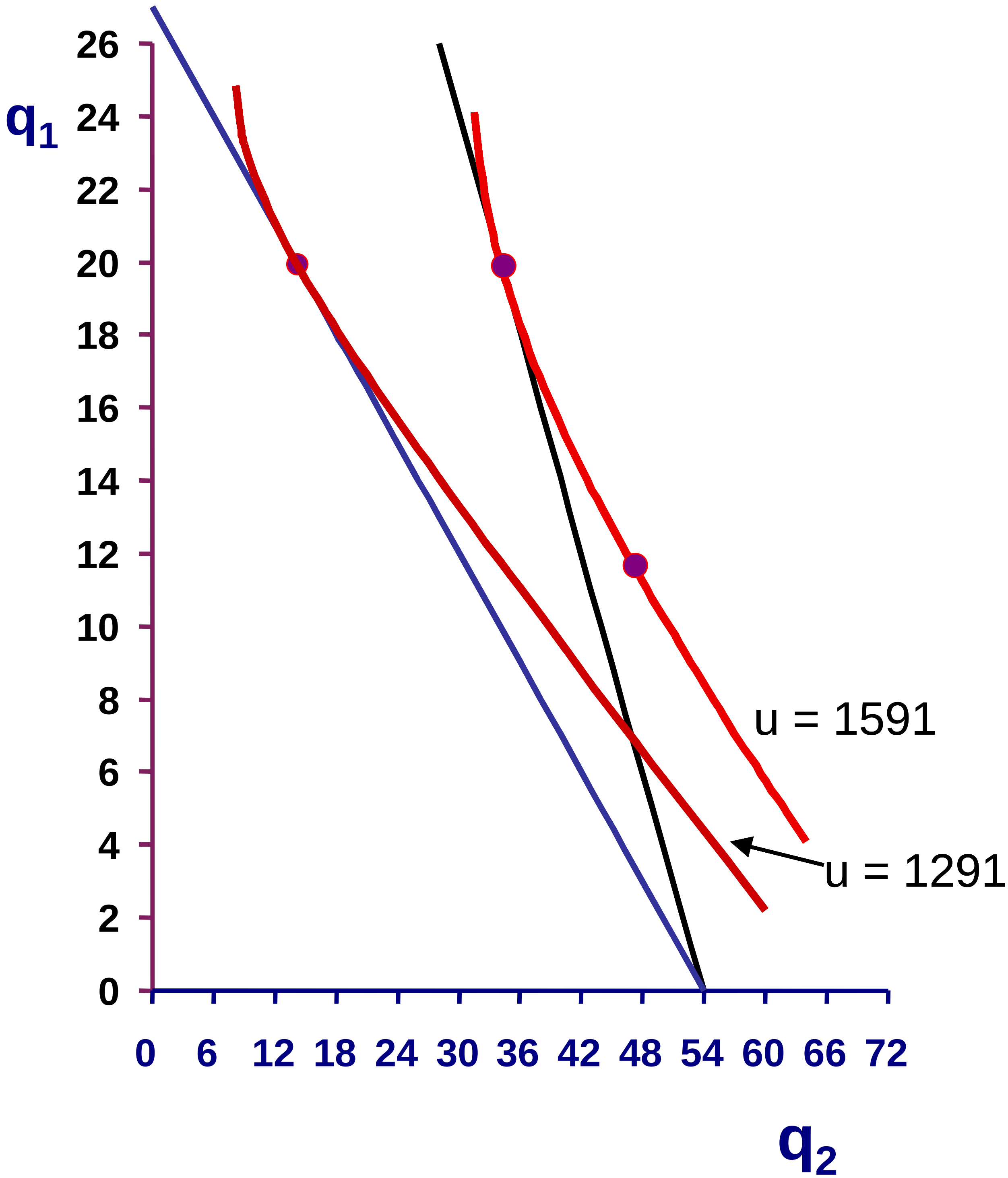
16. Work question 1 from Skills and Tools in Chapter 7.

17. Work question 2 from Skills and Tools in Chapter 7.

18. Work question 3 from Skills and Tools in Chapter 7.

19. Work question 4 from Skills and Tools in Chapter 7.

Increase in p_1



Decrease in the price of good 2