Economics 101
Spring 2001
Section 4 - Hallam
Problem Set #8

Due date: April 11, 2001

1. Choose 3 of the 11 markets listed below. To what extent do they satisfy the 7 conditions for perfect competition? In each case give reasons for your conclusion.

    1. Market for fresh vegetables in Madison, WI
    2. Market for seed corn in Iowa
    3. Market for delivered pizza in Ames, IA
    4. Market for baseball players
    5. Market for unskilled farm labor in California
    6. World market for wheat
    7. Secondary market for treasury bills (3-month)
    8. Market for combines in the United States
    9. Market for sport utility vehicles
    10. Market for live cattle in western Iowa
    11. Market for running shoes

2. Assume a firm with the following cost function

$$cost = 100 + 40y - 10y^2 + y^3$$

where y is the level of output for the firm. Assume that the firm is a price taker and that the price of y is 72, i.e., $p_y = 72$.

a. What is an expression for the firm's profit in terms of y?

$\pi =$

b. Create a table listing output levels from 4 to 10, the price of the good at each level (it will equal 72 at all levels), the revenue ($p_y y$) for each level, the cost at each level, the marginal cost between each level, and the profit at each level. You can use a spreadsheet if you like.

c. Create a graph of revenue and cost for each output level so that both curves are in the same graph. Label the curves and title the graph. At what level of y does profit seem to be maximized?

d. Create another graph with price and marginal cost plotted against output. Label the curves and title the graph. At what level of y does profit seem to be maximized?

3. Assume a firm with the following cost function

$$cost = 80 + 50y - 5y^2 + 0.25y^3$$

where y is the level of output for the firm. An exact equation for marginal cost is MC = 50 - 10y + 0.75 y^2. Assume that the firm is a price setter and that inverse demand is given by P = 50 - 2y. Assume that marginal revenue is given by MR = 50 - 4y.

a. What is an expression for the firm's profit in terms of y? (Multiply the first term out).

π =

b. Create a table listing output levels from 0 to 10, the price of the good at each level, the revenue ($p_y y$) for each level, the marginal revenue at each level, the variable cost at each level, the total cost at each level, the marginal cost at each level, and the profit at each level. You should probably use a spreadsheet to make this simpler. A table would look like this.

y	Price	TR	MR	FC	VC	TC	MC	Profit
0.00	50	0	50	80.00	0.00	80.00	50.00	-80.00
1.00	48		46	80.00	45.25	125.25		
2.00	46					162.00		
3.00							26.75	
4.00		168			136.00		22.00	-48.00
5.00		200	30		156.25		18.75	-36.25
5.75	39		27					
6.00	38	228				254.00		
7.00		252			190.75	270.75	16.75	
8.00			18		208.00	288.00		-16.00
8.33			17	80.00		294.12		-16.34
9.00		288		80.00				
9.50	31	295		80.00		318.09	22.69	
10.00	30	300	10	80.00	250.00	330.00	25.00	-30.00

c. Create a graph of revenue, variable cost, and total cost for each output level so that all three curves are in the same graph. Label the curves and title the graph. At what level of y does profit seem to be maximized?

d. Create another graph with price, marginal revenue, and marginal cost plotted against output. Label the curve. At what level of y does profit seem to be maximized?

e. Should this firm produce in the long run?

f. Should this firm produce in the short run assuming all fixed costs are sunk?

4. Suppose in problem 3 that $40 of the $80 of fixed costs are sunk so that if the firm shuts down it can obtain $40 of asset disposal revenue. Should the firm produce or shut-down in the short run?

5. Suppose in problem 3 that $10 of the $80 of fixed costs are sunk so that if the firm shuts down it can obtain $70 of asset disposal revenue. Should the firm produce or shut-down in the short run?

6. Assume a firm with the following cost function

$$cost = 80 + 50y - 5y^2 + 0.25y^3$$

where y is the level of output for the firm. An exact equation for marginal cost is MC = 50 - 10y + 0.75 y^2. Assume that the firm is a price setter and that inverse demand is given by P = 38 - 2y. Assume that marginal revenue is given by MR = 38 - 4y.

a. What is an expression for the firm's profit in terms of y? (Multiply the first term out).

π =

b. Create a table listing output levels from 0 to 10, the price of the good at each level, the revenue ($p_y y$) for each level, the marginal revenue at each level, the variable cost at each level, the total cost at each level, the marginal cost at each level, and the profit at each level. You should probably use a spreadsheet to make this simpler. A table would look like this.

y	Price	TR	MR	FC	VC	TC	MC	Profit
0.00	38	0	38	80.00	0.00	80.00	50.00	-80.00
1.00		36		80.00	45.25	125.25		-89.25
2.00						162.00		
3.00		96					26.75	
4.00		120	22		136.00		22.00	
5.00	28		18		156.25		18.75	-96.25
5.75								
6.00						254.00		
7.00		168	10		190.75	270.75	16.75	
8.00		176	6		208.00	288.00		
8.33		178		80.00		294.12		-116.34
9.00	20			80.00				-127.25
9.50	19		0	80.00		318.09	22.69	
10.00	18	180	-2	80.00	250.00	330.00	25.00	-150.00

c. Create a graph of revenue, variable cost, and total cost for each output level so that all three curves are in the same graph. Label the curves and title the graph. At what level of y does profit seem to be maximized?

d. Create another graph with price, marginal revenue, and marginal cost plotted against output. Label the curves. At what level of y does profit seem to be maximized?

e. Should this firm produce in the long run?

f. Should this firm produce in the short run assuming all fixed costs are sunk?

7. Assume that the manufacturing of biking socks is a perfectly competitive industry. The market demand for biking socks is described by a linear demand function $Q^D = 400 - 2P$. The inverse demand is $P = 200 - \frac{1}{2} Q^D$. There are 30 manufacturers of biking socks. Each manufacturer has the same production costs. These are described in the long-run total and marginal cost functions below.

   $TC(q) = 200 + 10q + 2q^2$

   $MC(q) = 10 + 4q$.

   a. Show that an individual firm in this industry maximizes profit by producing $q = \frac{P - 10}{4} = \frac{1}{4} P - 2.5$.

   b. Derive the industry supply curve and show that it is $Q^S = 7.5P - 75$.

   c. Find the equilibrium market price by setting supply equal to demand. The answer is P = $50.

   d. Find the aggregate quantity traded in equilibrium. The answer is Q = 300.

   e. How much output does each firm produce? The answer is 10.

   f. Show that each firm earns zero profit in equilibrium.

8. Consider the following market where there are only two firms. Assume that they behave competitively even though they might behave in a non-competitive manner. Assume that there is a market demand curve given by

$$Q = 200 - p$$

The cost functions for the two firms in the industry are given by

$$cost(y_1) = 1000 + 20y_1 + y_1^2$$
$$cost(y_2) = 500 + 20y_2 + .5y_2^2$$

The marginal cost functions for the two firms in the industry are given by

$$MC(y_1) = 20 + 2y_1$$
$$MC(y_2) = 20 + y_2$$

In equilibrium the total supplied by both firms will equal the market demand

$$Q = y_1 + y_2$$

a. Find an equation representing the market supply of firm 1 as a function of price. (Hint: The answer is $y_1 = \frac{1}{2} p - 10$.

b. Find an equation representing the market supply of firm 2 as a function of price. (Hint: The answer is $y_2 = p - 20$.

c. What is the market supply curve assuming both firms produce? The answer is 3/2p - 30.

d. What is the market equilibrium price assuming both firms produce? The answer is P = $92.

e. What is the profit for firm 1?

f. What is the profit for firm 2?

g. Will other firms want to enter this industry?

9. Consider a firm with the following long run cost function.

$$cost(y_1) = 36 + 10y_1 + 0.25y_1^2$$

Assume that of the fixed cost of \$36, \$20 is sunk (at least in the short run), and \$16 is avoidable. Assume that in the long run, all costs are avoidable. Marginal cost is given by

$$MC(y_1) = 10 + 0.5y_1$$

Average cost reaches its minimum at the point where it is equal to marginal cost.

a. From a long-run perspective, calculate the level of y at which average cost is minimized. (I will do it for you.)

$$AC(y_1) = \frac{36 + 10y_1 + 0.25y_1^2}{y} = 10 + 0.5y_1 = MC(y_1)$$
$$\Rightarrow 36 + 10y_1 + 0.25y_1^2 = 10y + 0.5y_1^2$$
$$\Rightarrow 36 + 0.25y_1^2 = 0.5y_1^2$$
$$\Rightarrow 36 = 0.25y_1^2$$
$$\Rightarrow 144 = y_1^2$$
$$\Rightarrow 12 = y_1$$

b. In the long run, how high does the price need to be for the firm to continue operating? To find this plug the answer to a in the marginal cost equation.

c. What is an expression for avoidable cost? (Again, I will do this for you.)

$$\text{Avoidable cost}(y_1) = 16 + 10y_1 + 0.25y_1^2$$

d. What is an expression for average avoidable cost?

e. From a short-run perspective, calculate the level of y at which average avoidable cost is minimized.

f. In the short run, how high does the price need to be for the firm to continue operating? To find this plug the answer to e in the marginal cost equation.

g. What is the supply function for this firm assuming that it chooses to produce? Hint: You get this by setting marginal cost equal to p, and then solving the equation to get y_1 on the left hand side and p on the right hand side. Second Hint: The answer is $y_1 = 2p - 20$.

h. What is this firm's long-run supply function? (It will have two parts.)

i. What is this firm's short-run supply function? (It will have two parts.) Remember that the short run supply function is the marginal cost function above the minimum of average avoidable cost.

10. Consider a firm with the following long run cost function.

$$cost(y_2) = 16 + 7y_2 + y_2^2$$

Assume that of the fixed cost of \$16, \$7 is sunk (at least in the short run), and \$9 is avoidable. Assume that in the long run, all costs are avoidable. Marginal cost is given by

$$MC(y_2) = 7 + 2y_2$$

Average cost reaches its minimum at the point where it is equal to marginal cost.

a. From a long-run perspective, calculate the level of y at which average cost is minimized.

b. In the long run, how high does the price need to be for the firm to continue operating?

c. What is an expression for avoidable cost?

d. What is an expression for average avoidable cost?

e. From a short-run perspective, calculate the level of y at which average avoidable cost is minimized.

f. In the short run, how high does the price need to be for the firm to continue operating?

g. What is the supply function for this firm assuming that it chooses to produce? Hint: You get this by setting marginal cost equal to p, and then solving the equation to get y_2 on the left hand side and p on the right hand side.

h. What is this firm's long-run supply function? (It will have two parts.)

i. What is this firm's short-run supply function? (It will have two parts.) Remember that the short run supply function is the marginal cost function above the minimum of average avoidable cost.

11. Consider a firm with the following cost function.

$$cost(y_3) = 8 + 8y_3 + 0.5y_3^2$$

Assume that of the fixed cost of \$8, \$6 is sunk (at least in the short run), and \$2 is avoidable. Assume that in the long run, all costs are avoidable. Marginal cost is given by

$$MC(y_3) = 8 + y_3$$

Average cost reaches its minimum at the point where it is equal to marginal cost.

a. From a long-run perspective, calculate the level of y at which average cost is minimized.

b. In the long run, how high does the price need to be for the firm to continue operating?

c. What is an expression for avoidable cost?

d. What is an expression for average avoidable cost?

e. From a short-run perspective, calculate the level of y at which average avoidable cost is minimized.

f. In the short run, how high does the price need to be for the firm to continue operating?

g. What is the supply function for this firm assuming that it chooses to produce? Hint: You get this by setting marginal cost equal to p, and then solving the equation to get y_3 on the left hand side and p on the right hand side.

h. What is this firm's long-run supply function? (It will have two parts.)

i. What is this firm's short-run supply function? (It will have two parts.) Remember that the short run supply function is the marginal cost function above the minimum of average avoidable cost.

12. Now consider a market containing the first two firms (those in problems 9 and 10). Assume that they behave competitively (are price takers) even though they might behave in a non-competitive manner. Assume that there is a market demand curve given by

$$Q = 36 - p$$

In equilibrium the total supplied by both firms will equal the market demand

$$Q = y_1 + y_2$$

a. Find the long run market supply equation. It will have 3 parts, one for when there is zero output, one for when only firm 2 produces, and one for when both firms produce. Write it in the following form

$$y = \begin{cases} \quad , & p \geq 16 \\ \quad , & 15 \leq p \leq 16 \\ 0, & p < 15 \end{cases}$$

b. Find the market equilibrium price.

c. Find the equilibrium quantity supplied for each firm.

d. What is the profit for firm 1?

e. What is the profit for firm 2?

f. Now consider the situation if the third firm (problem 11) enters the market. What is the long-run market supply function? It will have 4 parts. Write it in the same form as part a.

g. Find the market equilibrium price if all firms participate in the market.

h. Find the equilibrium quantity supplied for each firm.

i. What is the profit for firm 1?

j. What is the profit for firm 2?

k. What is the profit for firm 3?

l. Is this a stable equilibrium for this market? Why?

m. What is the long run equilibrium price in this market? How many firms will participate?

n. What is the short-run market supply function for the market with all three firms participating? It will have 4 parts. Write it in the same form as parts a and f.

o. What is the short run equilibrium price in this market?

p. How much does each firm produce?

13a. Consider the following production function

$$y = 20x_1 + 15x_2 - 0.5x_1^2 - 0.5x_2^2$$

The price of x_1 is \$40 and the price of x_2 is \$20. You are trying to find which of the following sets of points is the cost minimizing way to produce 250 units of output. For each of the input combinations in question, verify that it will produce 250 units (or close with rounding), compute its cost, find the marginal rate of substitution and the price ratio. Then decide which point is minimum cost.

x_1	x_2	y	Cost	MPP_1	MPP_2	MRS_{12}	$\frac{-w_2}{w_1}$
13.367	6.000	250.000	654.67	6.633	9.000	-1.357	
12.190	7.000	250.000	627.59	7.810	8.000		
2.000	8.000	126.000		18.000	7.000		
10.566	9.000	250.000		9.434	6.000		
10.000	10.000			10.000	5.000	-0.500	
9.560	11.000		602.3877	10.440	4.000	-0.383	
9.230	12.000	250.000	609.1868	10.770	3.000	-0.279	
9.000	13.000	250.000		11.000	2.000	-0.182	
0.500	14.000	121.875	300	19.500	1.000	-0.051	

13b. Consider the following production function

$$y = 20x_1 + 15x_2 - 0.5x_1^2 - 0.5x_2^2$$

The price of x_1 is \$40 and the price of x_2 is \$20. You are trying to find which of the following sets of points is the cost minimizing way to produce 272.5 units of output. For each of the input combinations in question, verify that it will produce 272.5 units (or close with rounding), compute its cost, find the marginal rate of substitution and the price ratio. Then decide which point is minimum cost.

x_1	x_2	Cost	y	MPP_1	MPP_2	MRS_{12}	$\frac{-w_2}{w_1}$
14.432	8.000	737.2894		5.568	7.000	-1.257	
13.000	9.000	700		7.000	6.000	-0.857	
12.584	10.000	703.3521	272.500	7.416	5.000	-0.674	
12.000	11.000		272.500	8.000	4.000	-0.500	
11.574	12.000		272.500	8.426	3.000	-0.356	-0.500
11.282	13.000	711.2881	272.500	8.718	2.000	-0.229	-0.500
11.112	14.000	724.4722	272.500	8.888	1.000		-0.500
11.056	15.000	742.2291		8.944	0.000		-0.500
11.000	16.000	760		9.000	-1.000	0.111	-0.500

13c. Consider the following production function

$$y = 20x_1 + 15x_2 - 0.5x_1^2 - 0.5x_2^2$$

The price of x_1 is \$40 and the price of x_2 is \$20. You are trying to find which of the following sets of points is the cost minimizing way to produce 296.875 units of output. For each of the input combinations in question, verify that it will produce 296.875 units (or close with rounding), compute its cost, find the marginal rate of substitution and the price ratio. Then decide which point is minimum cost.

x_1	x_2	Cost	y	MPP_1	MPP_2	MRS	$\frac{-w_2}{w_1}$
17.500	10.000	900	296.875	2.500	5.000	-2.000	-0.500
16.683	10.500	877.335		3.317	4.500	-1.357	-0.500
16.000	11.000	860		4.000	4.000	-1.000	-0.500
15.641	11.500	855.644	296.875	4.359	3.500	-0.803	-0.500
15.283	12.000	851.3204	296.875	4.717	3.000	-0.636	-0.500
15.000	12.500			5.000	2.500	-0.500	-0.500
14.780	13.000	851.1939	296.875	5.220	2.000	-0.383	-0.500
14.615	13.500		296.875	5.385	1.500	-0.279	-0.500
14.500	14.000		296.875	5.500	1.000	-0.182	-0.500

13d. Consider the following production function

$$y = 20x_1 + 15x_2 - 0.5x_1^2 - 0.5x_2^2$$

The price of x_1 is \$40 and the price of x_2 is \$20. You are trying to find which of the following sets of points is the cost minimizing way to produce 302.5 units of output. For each of the input combinations in question, verify that it will produce 302.5 units (or close with rounding), compute its cost, find the marginal rate of substitution and the price ratio. Then decide which point is minimum cost.

x_1	x_2	Cost	y	MPP_1	MPP_2	MRS	$\frac{-w_2}{w_1}$
16.683	12.000	907.335	302.500	3.317	3.000	-0.905	-0.500
16.292	12.500		302.500	3.708	2.500	-0.674	-0.500
16.000	13.000		302.500	4.000	2.000		-0.500
15.787	13.500	901.477	302.500	4.213	1.500		-0.500
15.641	14.000	905.644	302.500	4.359	1.000	-0.229	-0.500
15.556	14.500	912.2361		4.444	0.500	-0.113	-0.500
15.528	15.000	921.1146	302.500	4.472	0.000	0.000	-0.500
15.556	15.500	932.2361	302.500	4.444	-0.500	0.113	-0.500
15.641	16.000	945.644	302.500	4.359	-1.000	0.229	-0.500

14. Consider the following production function

$$y = 20x_1 + 15x_2 - 0.5x_1^2 - 0.5x_2^2$$

The price of x_1 is \$40 and the price of x_2 is \$20. The following table contains the minimum cost ways to produce various levels of y along with their marginal cost.

Output	x1	x2	w1	w2	Cost	MC
250	10	10	40	20	600	4
251	10.08	10.04	40	20	604.02	4.03239
252	10.16	10.08	40	20	608.07	4.06558
255	10.41	10.2	40	20	620.42	4.17029
256	10.49	10.25	40	20	624.61	4.20703
257	10.58	10.29	40	20	628.83	4.24476
258	10.66	10.33	40	20	633.1	4.28353
260	10.83	10.42	40	20	641.74	4.36436
261	10.92	10.46	40	20	646.13	4.40653
265	11.28	10.64	40	20	664.11	4.58831
266	11.37	10.69	40	20	668.72	4.63739
269	11.66	10.83	40	20	682.87	4.79463
270	11.75	10.88	40	20	687.69	4.85071
271	11.85	10.93	40	20	692.57	4.90881
272	11.95	10.98	40	20	697.51	4.96904
272.5	12	11	40	20	700	5
273	12.05	11.03	40	20	702.51	5.03155
274	12.15	11.08	40	20	707.57	5.09647
275	12.25	11.13	40	20	712.7	5.16398
280	12.79	11.39	40	20	739.44	5.547
281	12.9	11.45	40	20	745.04	5.63436
282	13.01	11.51	40	20	750.72	5.72598
285	13.37	11.68	40	20	768.34	6.03023
286	13.49	11.74	40	20	774.42	6.14295
287	13.61	11.81	40	20	780.63	6.26224
290	14	12	40	20	800	6.66667
291	14.13	12.07	40	20	806.74	6.81994
292	14.27	12.14	40	20	813.64	6.9843
293	14.41	12.21	40	20	820.72	7.16115
294	14.56	12.28	40	20	827.97	7.35215
295	14.71	12.35	40	20	835.42	7.55929
296	14.86	12.43	40	20	843.1	7.78499
296.875	15	12.5	40	20	850	8
297	15.02	12.51	40	20	851	8.03219
298	15.18	12.59	40	20	859.17	8.30455
300	15.53	12.76	40	20	876.39	8.94427
301	15.71	12.86	40	20	885.52	9.32505
302	15.9	12.95	40	20	895.06	9.759
302.5	16	13	40	20	900	10
303	16.1	13.05	40	20	905.06	10.2598
306	16.78	13.39	40	20	938.75	12.4035
309	17.63	13.82	40	20	981.68	16.9031
310	18	14	40	20	1000	20
311	18.45	14.23	40	20	1022.5	25.8199
312	19.11	14.55	40	20	1055.3	44.7214
312.5	20	15	40	20	1100	3163373

a. If the price of output is $4.00, how much should the firm produce?

b. If the price of output is $5.00, how much should the firm produce?

c. If the price of output is $4.63739, how much should the firm produce?

d. If the price of output is $8.00, how much should the firm produce?

e. If the price of output is $10.00, how much should the firm produce?

f. If the price of output is $20.00, how much should the firm produce?

g. Explain why input levels in part a and 13a are the same?

h. Explain why input levels in part b and 13b are the same?

i. Explain why input levels in part d and 13c are the same?

j. Explain why input levels in part e and 13d are the same?

15. Work question 5 from Skills and Tools in Chapter 7.
16. Work question 1 from Skills and Tools in Chapter 8.
17. Work question 2 from Skills and Tools in Chapter 8.
18. Work question 3 from Skills and Tools in Chapter 8.
19. Work question 4 from Skills and Tools in Chapter 8.
20. Work question 5 from Skills and Tools in Chapter 8.