Rupayan Gupta

Addendum to Lecture 8: An example of a flexible prices model

The environment of the economy

1. Let there be 10 firms and 10 households in the economy.
2. The production function of each firm is given by: \( y_S = l_D^{1/2} \)
   (Think of \( y \) as ‘bread’ and \( l \) as labor input. Output is increasing in labor usage).
3. Let \( p \) be the price level in the market for goods & services.
4. Let \( W \) be the money wage rate in the labor market.
5. Let the utility function of each household depend on the amount of leisure they consume (we assume each day to have 24 hours in it) and the amount of bread they eat:
   \[ U = U = y_D^{1/2} \cdot (24 - l_S)^{1/4}, \]
   (Where \( l_S \) is the amount of labor supplied by the household. Utility increases for the consumption of more bread and/or leisure).
6. For simplicity, we assume that all profits are retained by the firms and not redistributed to households. (This will change the following analysis only marginally from what we saw in lecture 8).

How the model works

The household’s utility maximization problem:

Maximize: \( U = y_D^{1/2} \cdot (24 - l_S)^{1/4} \)

Such that: \( p \cdot y_D = W \cdot l_S \) (the household’s income constraint, i.e., the amount spent on bread should equal the household’s income).

The budget constraint can be written as: \( y_D = (W/p) \cdot l_S \)

Substituting for \( y_D \) in the utility function, the household’s maximization can be written as a single variable problem:

Maximize: \( U = [(W/p) \cdot l_S]^{1/2} \cdot (24 - l_S)^{1/4} \)

So, in the above problem the only thing the household has to decide on is how much labor to supply in order to maximize its utility.

(The next couple of lines involve the use of calculus and may be skipped. I am including them to show you that there is an actual exercise, from which the rest of what I am going to do follows. You will not be required to know the maximization technique: in an exam I will give you the results of the maximization exercise).
The utility function is maximized by setting the first derivative of the utility function with respect to \( l_s \) equal to 0 & solving the resultant equation:

\[
\frac{\partial U}{\partial l_s} = 0
\]

\[
\Rightarrow \left( \frac{W}{p} \right)^{1/2} \left[ \frac{1}{2} l_s^{-1/2} (24 - l_s)^{1/4} - \frac{1}{4} (24 - l_s)^{-3/4} l_s^{1/2} \right] = 0
\]

Solution to the above equation gives us the labor supply function:

\( l_s^* = 16 \)

(This is a special case, where labor supply is constant & independent of the real wage rate. This is due to the simple utility function we have taken in this problem. Usually labor supply would be a function of the real wage rate).

So, aggregate labor supply (for all 10 households) is:

\( L_s^* = 10 \cdot l_s^* = 160 \)

And the output demand function of the household (substituting \( l_s^* \) into the budget constraint of the household) is:

\( y_d^* = 16(W/p) \)

And the aggregate demand function of output (bread) of the household sector is:

\( Y_d^* = 160(W/p) \)

Now, let us look at the profit maximization problem of the firm:

Maximize: \( \pi = p y_s - W \cdot l_d = p \cdot l_d^{1/2} - W \cdot l_d \)

The above follows from the production function: \( y_s = l_d^{1/2} \)

So the firm’s profit is the revenue it makes by selling bread minus the total wage payments to its workers.

\textit{(Again, the solution to this problem will require the use of calculus. You may skip the next couple of lines if you want to).}

The profit function is maximized by setting the first derivative of the profit function with respect to \( l_d \) equal to 0 & solving the resultant equation:

\[
\frac{\partial \pi}{\partial l_d} = 0 \Rightarrow \left[ \frac{1}{2} p l_d^{-1/2} - W \right] = 0
\]
Solution to the above equation gives us the labor demand function:

\[ l_D^* = \frac{1}{4} (p/W)^2 \]

Hence the aggregate labor demand function (for all 10 firms) is:

\[ L_D^* = (10/4)(p/W)^2 \]

Plugging the firm’s labor demand into its production function, we get its output (bread) supply function:

\[ y_S^* = \frac{1}{2}(p/W) \]

And the aggregate output supply function for the economy (all 10 firms) is:

\[ Y_S^* = (10/2)(p/W) \]

**Lastly, we calculate the profit functions firms** (plugging their output supply & labor demand functions into the profit equation):

\[ \pi^* = p.y_S^* - W.l_D^* = p.\frac{1}{2}(p/W) - W. \frac{1}{4}(p/W)^2 = \frac{1}{4}(p^2/W) \]

So, aggregate profits (for all 10 firms) is:

\[ \Pi^* = 10 \pi^* = (10/4)(p^2/W) \]

**Now, we impose the following equilibrium conditions:**

1. Labor market: Aggregate supply = Aggregate demand
   
   \[ L_S^* = L_D^* = L^* \]

2. Goods & services market: Aggregate demand = Aggregate Supply
   
   \[ Y_D^* + (\Pi^*/p) = Y_S^* = Y^* \], where \((\Pi^*/p)\) is the ‘real profit’ for firms, i.e., their profits accounted in units of ‘bread’.

**Note:** As we have assumed that firms retain profits and do not distribute them to households, the aggregate demand of the economy is the sum of the demands of the households and the firms. Think of firms’ real profits as the amount of bread they demand (and consume) in the economy.

From the labor market clearing condition, we get:

\[ 160 = (10/4)(p/W)^2 \]

And from the goods & services market we get:

\[ 160(W/p) + (10/4) (p/W) = (10/2)(p/W) \]

**We see from the above equations that we get \( W^*/p^* = 1/8 \) as the real wage rate which simultaneously clears both the labor & the goods market at the full-
employment ($L^*$) and potential output ($Y^*$) levels. (Verify that the solution to each market clearing condition gives $W^*/p^* = 1/8$).

For fully flexible prices and money wages, if the economy is off-equilibrium, then money wages and prices will self-adjust and converge to this equilibrium level of real wages. This will lead to full-employment and potential level of output for the economy. Notice that we can only solve for relative prices in this model (the real wage rate), and not for the money wage rate or price level in the goods & services market, separately.