Appendix to Lecture 7: Full Employment Equilibrium

A simple flexible price model of the economy (Neoclassical model)

Let there be 10 firms and 5 households in the economy. The production function of each firm is given by \( y_S = f(l_D) \) (think of \( y \) as ‘bread’ and \( l \) is the labor input of the firm). Let \( p \) be the price level in the market for goods & services. Let \( W \) be the money wage rate in the labor market. Let the utility function of each household depend on the amount of leisure they consume (we assume each day to have 24 hours in it, so the amount of leisure is 24 minus the hours worked) and the amount of bread they eat:

\[
U = U(y_D, 24 - l_S),
\]

where \( l_S \) is the amount of labor supplied by the household. Utility increases for the consumption of more bread and/or leisure.

The household’s utility maximization problem:

Maximize: \( U = U(y_D, 24 - l_S) \)

Such that: \( p \cdot y_D = W \cdot l_S + \pi_H \) (the household’s income constraint, where \( \pi_H \) is the household’s profit income, i.e. the amount they earn by being shareholders in firms).

(Note: The utility function could look like \( U = y_D^{1/2} \cdot (24 - l_S)^{1/4} \))

The household maximizes its utility by equating the marginal product of its leisure to negative of the real wage rate (marginal product of leisure: it is the additional amount of utility derived from consuming an additional unit of leisure). The marginal product of leisure is falling for a larger amount of consumption of leisure. Basically, households will consume leisure till the point where the additional benefit of consuming more leisure (its marginal utility for that amount of leisure) equals the opportunity cost of consuming more leisure (negative of the real wage rate of labor).

The moot point is that the solution to this problem by the household gives us its labor supply equation, \( l_S^* = l_S(W/p) \), and as \( W/p \) will rise, \( l_S^* \) will rise too.

Once we get the labor supply function of the household, we can find the output demand function, \( y_D = y_D(W/p) \) using its budget constraint (which is \( p \cdot y_D = W \cdot l_S + \pi_H \)). The amount of output demanded will rise with a rise in the real wage rate.

Example. If I were asking you a question in an exam, I could probably write a labor supply equation like:

\[
l_S^* = 10 + 2 \cdot W / p
\]
As there are 10 households, the aggregate labor supply of the economy (equation 1) is:

\[ L_s^* = 10 \cdot l_s^* \]

Example.

If the labor supply equation is:

\[ l_s^* = 10 + 2 \cdot W / p , \]

Then,

\[ L_s^* = 10(10 + 2 \cdot W / p) \]

Coming back to our model,

The aggregate demand for bread (equation 2) is:

\[ Y_d^* = 10 \cdot y_d^* \]

Example.

If the aggregate labor supply equation is:

\[ l_s^* = (10 + 2 \cdot W / p) , \]

Then,

\[ Y_d^* = 10 \cdot y_d^* = 10 \cdot (W / p)l_s^* - \pi_H / p \]

\[ = 10 \cdot (W / P) (10 + 2 \cdot W / p) - \pi_H / p \]

Now, let us look at the profit maximization problem of the firm:

Maximize: \( \pi_F = p \cdot y_S - W \cdot l_D = p \cdot f(l_D) - W \cdot l_D \)

(Recalling the equation for the production function: \( y_S = f(l_D) \))

In order to maximize profits the firm hires labor till the point where the marginal product of a laborer equals its real wage (where the additional benefit of employing more labor equals the cost of employing that additional labor, the latter being the real wage rate).

This exercise gives us the labor demand equation of the firm:

\[ l_D^* = l_D(W / p) \]

Example.

\[ l_D^* = \frac{1}{4} (p/W)^2 \]
As real wages fall, the demand for labor rises.

The aggregate demand for labor for all 5 firms (equation 3) is:

\[ L_D^* = 5 * I_D^* \]

Plugging \( I_D^* \) into the production function, the supply of output for each firm (from the production function) is:

\[ y_S^* = f(I_D^*) = y_S(W/p) \]

Output supply falls with a rise in the real wage rate.

Example.

For

\[ y_S^* = (I_D^*)^{1/2} \]

and

\[ I_D^* = \frac{1}{4} (p/W)^2, \]

\[ y_S^* = \frac{1}{2} (p/W) \]

The aggregate production of the economy (equation 4) is:

\[ Y_S^* = 5 * y_S^* \]

Example.

Given the example above, the aggregate production function would be:

\[ Y_S^* = 5 * y_S^* = 5 * (1/2)(p/W) \]

Digression: *The number of firms producing in the economy matters.* One firm producing with 10 units of inputs in an economy is not the same as 5 producing with 2 units each. The aggregate productive output of the economy is usually not the same for both (barring some special cases). *In our context, this means that we must sum together the production of different firms to arrive at aggregate production, and not write the aggregate production function of the economy as just a ‘blown-up version’ of the individual firm’s production function.* The aggregate production for the firms (that is the sum of production for the firms), and the production of the economy for the aggregate amount of input used
by the firms is usually not the same, even if input usage is at the same level for each firm (if we take the aggregate production function of the economy to have the same functional form as those of the firms). Consider an economy where the only productive input is labor. This is easy to see. Suppose there are 5 firms in the economy each having the production function $Y_S = l_D^{1/2}$. Let each of these firms employ 4 workers. So each of these firms produces 2 units of output, and the total output of the economy is $5 \times 2 = 10$ units. However, if we take the aggregate production function of the economy to be $Y_S = L_D^{1/2}$, with $L_D$ being the aggregate labor usage, $Y_S = (20)^{1/2}$, which is 4.47 and not 10.

**Going back to the model:**
As already observed, we have the equations for aggregate demand & supply in both the goods & services market, as well as the labor market (from the respective decision processes of households & firms in the economy).

*We now impose the following equilibrium conditions:*

1. Labor market clearing: $L_S^* = L_D^*$
2. Goods & services market clearing: $Y_S^* = Y_D^*$
3. Complete distribution of aggregate firms’ profits to households: $5^* \pi_F = 10^* \pi_H$
   (the total profit earnings of all households through their shares they own in firms should equal the sum of profits for all firms in the economy).

**Imposing these conditions, if the prices & wages in all markets are fully flexible, we will be able to find an equilibrium real wage rate $(W/p)^*$ that will clear both the labor and the goods & services market, and there will be full employment in the economy. Production will take place at the potential level of output for the economy. If the economy is ‘off-equilibrium’, flexibility ensures ‘self-adjustment’ of the economy to the full employment level through changes in money wages and prices.**

(See the diagrams from class notes:
Diagrams: The labor & goods markets for flexible prices & money wages
What happens for inflexible prices & money wages if you are off equilibrium?)

This completes the discussion of the self-adjusting ‘full employment’ flexible price model. From the next lecture we shall start discussing a model of ‘underemployment equilibrium’, when the economy experiences sticky prices.