Task: generalize the concept of the expenditure multiplier

- more variables
- more parameters
- more slopes
- more intercepts
- more relationships
- more even finer
- bigger model
- more realistic

Key: know your appendix in Chapter 10

p. 252
p. 253
\[ t = \frac{27}{\text{AR.GDP}} = 0.22 \leq 0.72 \]

**How come?**

**Public Sector**

<table>
<thead>
<tr>
<th>Income</th>
<th>Expenditure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taxes</td>
<td></td>
</tr>
<tr>
<td>income</td>
<td></td>
</tr>
<tr>
<td>federal</td>
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<tr>
<td>state</td>
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<tr>
<td>local</td>
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<tr>
<td>sales</td>
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<tr>
<td>federal</td>
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<tr>
<td>state</td>
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<tr>
<td>local</td>
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<tr>
<td>property</td>
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<tr>
<td>federal</td>
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<tr>
<td>state</td>
<td></td>
</tr>
<tr>
<td>local</td>
<td></td>
</tr>
</tbody>
</table>

\[ \text{Currently a deficit} \]

\[ \text{Total} \leq \text{Total} \]
Sec. Sec. (t)
Remark (t)

\[ \text{total} \Rightarrow \text{Economy of Scope} \]

Text:
All ideas, except the federal common tax, are targeted to on fundraising.
The multiplier

- define it
  \[ \frac{\text{DROP}}{\text{AE}} \geq +1.0 \]
  
- tell a convincing story

- derive it
  - algebraically
  - geometrically

- apply it
  - estimate the parameters
  - examples of changes in \( \overline{AE} \)
Last lecture:

\[ \frac{\Delta GDP}{\Delta I} \leq \frac{1}{1 - b} > 10 \]

Investment multiplier

This lecture:

\[ \frac{\Delta Y}{\Delta \bar{A}} = \frac{1}{1 - (b(1 - \epsilon) - \gamma)} > 10 \]

Questions:

- What is \( \Delta Y \)?
- What is \( \Delta \bar{A} \)?
- What is \( b \)?
- What is \( \epsilon \)?
- What is \( \gamma \)?
- How do we get the formula?
The Algebra of the Multiplier

Net taxes equal real GDP ($Y$) multiplied by the marginal tax rate ($t$). That is,

$$T = tY.$$  

Use this equation in the previous one to obtain

$$C = a + b(1 - t)Y.$$  

This equation describes consumption expenditure as a function of real GDP.

**Import Function**

Imports depend on real GDP, and the import function is

$$M = mY.$$  

**Aggregate Expenditure Curve**

Use the consumption function and the import function to replace $C$ and $M$ in the aggregate planned expenditure equation. That is,

$$AE = a + b(1 - t)Y + I + G + X - mY.$$  

Collect the terms on the right side of the equation that involve $Y$ to obtain

$$AE = [a + I + G + X] + [b(1 - t) - m]Y.$$  

Autonomous expenditure ($A$) is $[a + I + G + X]$, and the slope of the $AE$ curve is $[b(1 - t) - m]$. So the equation for the $AE$ curve, which is shown in the following figure, is

$$AE = A + [b(1 - t) - m]Y.$$  

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**Aggregate Expenditure**

Aggregate planned expenditure ($AE$) is the sum of the planned amounts of consumption expenditure ($C$), investment ($I$), government purchases ($G$), and exports ($X$) minus the planned amount of imports ($M$). That is

$$AE = C + I + G + X - M.$$  

**Consumption Function**

Consumption expenditure ($C$) depends on disposable income ($YD$), and we write the consumption function as

$$C = a + bYD.$$  

Disposable income ($YD$) equals real GDP minus net taxes ($Y - T$). So by replacing $YD$ with ($Y - T$), the consumption function becomes

$$C = a + b(Y - T).$$  

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**Know It**
Equilibrium Expenditure occurs when aggregate planned expenditure (\( AE \)) equals real GDP (\( Y \)). That is,

\[ AE = Y \]

In the figure below, the scales of the x-axis (real GDP) and the y-axis (aggregate planned expenditure) are identical, so the 45° line shows the points at which aggregate planned expenditure equals real GDP. That is, the 45° line is the line along which \( AE = Y \).

The figure shows the point of equilibrium expenditure at the intersection of the \( AE \) curve and the 45° line.

To calculate equilibrium expenditure and real GDP, we solve the equations for the \( AE \) curve and the 45° line for the two unknown quantities \( AE \) and \( Y \). So, starting with

\[ AE = A + [b(1 - t) - m]Y \]

\[ AE = Y, \]

replace \( AE \) with \( Y \) in the \( AE \) equation to obtain

\[ Y = A + [b(1 - t) - m]Y. \]

The solution for \( Y \) is

\[ Y = \frac{1}{1 - [b(1 - t) - m]} A. \]

The Multiplier

The multiplier equals the change in equilibrium expenditure and real GDP (\( \Delta Y \)) that results from a change in autonomous expenditure (\( A \)) divided by the change in autonomous expenditure.

A change in autonomous expenditure (\( \Delta A \)) leads to a change in equilibrium expenditure and real GDP (\( \Delta Y \)), which is given by

\[ \Delta Y = \frac{1}{1 - [b(1 - t) - m]} \Delta A. \]

Multiplier = \[ \frac{1}{1 - [b(1 - t) - m]} \]

The size of the multiplier depends on the slope of the \( AE \) curve \( b(1 - t) - m \). The larger the slope, the larger is the multiplier. So the multiplier is larger,

- The greater the marginal propensity to consume (\( b \))
- The smaller the marginal tax rate (\( t \))
- The smaller the marginal propensity to import (\( m \))

An economy with no imports and no marginal taxes has \( m = 0 \) and \( t = 0 \). In this special case, the multiplier equals 1/(1 - b). If \( b \) is 0.75, then the multiplier is 4, as shown in the following figure. In an economy with \( b = 0.75 \), \( t = 0.2 \), and \( m = 0.1 \), the multiplier is 1 divided by 1 minus 0.75(1 - 0.2) - 0.1, which equals 2. Make up some more examples to show the effects of \( b \), \( t \), and \( m \) on the multiplier.

\[ Y = \frac{1}{1 - [b(1-t)-m]} (\hat{a} + \hat{I} + \hat{G} + \hat{X}) \]

A theory of the business cycle
AE: aggregate expenditure
RGDP: real gross domestic product
Y: national income

with equilibrium $AE = RGDP = Y$

Symbols will be used interchangeably

Output (in eq. B) = OABC = RGDP
Expenditure (in eq. B) = OABC = AE
Income (in eq. B) = OABC = Y
- autonomous expenditure

- current expenditure that does not change if current income changes

- mortgage payments
- credit card payments
- car loan payments
- Stafford Loan repayments
- Room and Board at I.S.U

- pretty much all of your current income is precommitted in the short run
À: submit your expenditure

\bar{\alpha}\begin{pmatrix}
\bar{\eta} \\
\bar{\xi} \\
\bar{\chi}
\end{pmatrix}

\bar{\alpha} = \left[ \bar{\alpha} + \bar{\xi} + \bar{\eta} + \bar{\chi} \right]

Real consumption expenditure (1992 $b.) \( "C" \)

\[ b = \frac{\Delta C}{\Delta GDP} = \frac{11000}{1500} = 0.73 \]

1998

1100

1500

6000

7500

8000

Real GDP (1992 $b.)

\( b \) = marginal propensity to consume


Gov't receipts ($b.) \( "T" \) (includes sec. sec.)

\[ t = \frac{\Delta T}{\Delta GDP} = \frac{13000}{1800} = 0.72 \]

1998

13000

1500

1800

2500

2000

1500

1000

500

1968

1970

1972

1974

1976

1978

1980

1982

1984

1986

1988

1990

1992

1994

1996

1998

Government receipts
This chart shows the percent of an average worker's income taken by direct taxes. These taxes include Federal, state and local income taxes, worker paid Social Security taxes, and sales taxes. (See the Tax Foundation.)

With 35.4% of your earnings going to taxes, you must earn $1.55 to spend $1.00.

**Hidden Taxes Consumers Pay**

But, there are other taxes that are not so obviously removed from your pay. These taxes are not intended to be readily observable. Some of these taxes are paid by your company and add to the price of products or services you and your company charge for the products you produce. Your company, for example, pays an equal amount of Social Security taxes for you. Your company also pays a variety of other taxes such as unemployment taxes, workers compensation taxes, property taxes, corporate income taxes, energy taxes, pollution taxes, just to name a few. These taxes are added to the prices of goods and services your company provides to consumers.

As a consumer, you pay not only your taxes, but the taxes of all the manufacturing companies that had a hand in producing the final products you are purchasing. These taxes are rolled into the price of the product and are passed on to you. Americans for Tax Reform calculated that these hidden taxes increase the cost of goods and services by 26% to 75%. (See Americans for Tax Reform.) For example, taxes take:

<table>
<thead>
<tr>
<th>Hidden Costs of Taxes</th>
</tr>
</thead>
<tbody>
<tr>
<td>• 26% of the cost of electricity</td>
</tr>
<tr>
<td>• 28% of the cost of a restaurant meal</td>
</tr>
<tr>
<td>• 31% of the cost of bread</td>
</tr>
<tr>
<td>• 38% of the cost of a pizza</td>
</tr>
<tr>
<td>• 40% of the cost of an airline ticket</td>
</tr>
<tr>
<td>• 46% of the cost of a firearm</td>
</tr>
<tr>
<td>• 54% of the cost of gasoline</td>
</tr>
</tbody>
</table>

Families pay both obvious taxes (35.4%) and hidden taxes (at least 26% of purchases). If 80% of "after tax" disposable income is spent, hidden taxes add about 20% to the family's tax burden, bringing the total tax burden to over 55%.
Marginal propensity to import

\[ m = \frac{\Delta M}{\Delta RGDP} = \frac{575}{1300} \approx 0.44 \]
Autonomous Expenditure Multiplier.

\[ Y_t = \frac{1}{1 - [b(1-\epsilon) - m]} \times \bar{A} \]

\[ b = 0.75 \]
\[ \epsilon = 0.25 \]
\[ m = 0.15 \]

\[ Y^2 = \frac{1}{1 - [0.75(1-0.25) - 0.15]} \times \bar{A} \]

\[ Y^2 = 0.875 \]

\[ Y^2 = 1.70 \times \bar{A} \]

\[ \Delta Y^2 = 1.70 \times \Delta \bar{A} \]

\[ \frac{\Delta Y^2}{\Delta \bar{A}} = 1.70 \]
Key Concepts

consumption function \rightarrow \text{slope of the consumption function} \rightarrow \text{prop. to consume}

import function \rightarrow \text{slope of the import function} \rightarrow \text{prop. to import}

tax function \rightarrow \text{slope of the tax function} \rightarrow \text{marginal tax rate}

Quick Test

b = ?
m = ?
e = ?

ΔC/ΔAGDP = ?

ΔT/ΔAGDP = ?

Δ%AGDP = ?

Δ%AGDP = ?