ECONOMICS 207
SPRING 2010
LABORATORY EXERCISE 13
19^{th} April 2010

Problem 1: A firm's production function is given below:

$$y = 90x_1 + 45x_2 - 3x_1^2 + 3x_1x_2 - 2x_2^2$$

where, y denotes output. x_1 and x_2 represent the variable inputs used in the production process. We also have the following information:

$$p = 4, w_1 = 120, w_2 = 80$$

where, p, w_1 and w_2 represent the price per unit of y, x_1 and x_2 respectively.
a. Write down the **total cost** function

b. Write down the **revenue** function.

c. Write down the **profit** function.

b. How much x_1 and x_2 should the firm employ in order to **maximize profit**. Verify using the Hessian matrix that profit is indeed maximized.

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Problem 2: Consider the following function in two variables

$$y = 2x_1^2 - x_1x_2 + x_2^2 + 7x_2$$

Find the **critical values** of x_1 and x_2. Use the Hessian matrix to determine whether the function is at a local maximum or a local minimum this point.

Problem 3: Given below is the production function of a competitive firm that uses two inputs in the production process, namely x_1 and x_2.

$$y = 40x_1 + 10x_2 - 2x_1^2 + 2x_1x_2 - x_2^2$$

where y represents output. Also we have

$$p = 1; w_1 = 34; w_2 = 2$$

where p, w_1, w_2 represent the price per unit of y, x_1 and $x_2$respectively.

Write down the expression for the **profit function**. Find the levels of x_1 and x_2 that could potentially maximize **profit.** Use the Hessian matrix to verify whether they actually maximize **profits.**

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Problem 4: Differentiate the following functions with respect to x

a.

$$y = e^{x^2}$$

b.

$$y = \ln(3x^2 + 4x + 1)$$

c

$$y = (3x^2 + 15x - 1)^3$$

d.

$$y = 4e^{x^2+3x}$$

e.

$$y = x^2 e^x$$

f.

$$\left(x^2 + 3x + 1\right)(2x + 1)^2$$

g.

$$y = x^{1/5} \ln x$$

h.

$$y = 4x^{1/3}a^{-1/2} + 3$$

Problem 5: A firm's production function is given below:

$$y = 30x_1 + 15x_2 - 2x_1^2 + x_1x_2 - x_2^2$$

where, y denotes output. x_1 and x_2 represent the variable inputs used in the production process. The price per unit of x_1 and x_2 is 60 and 12 respectively. The firm **cannot spend more than $468** on inputs. The firm wants to **maximize its output** subject to its **budget constraint**.

a. Write down the **objective function**

b. Write down the **constraint**.

c. Now write down the **Lagrangean** function.

d. Now use the Lagrangean function to find out how much x_1 and x_2 the firm should use if it wants to maximize output subject to its budget constraint.

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