Problem 1: Find the first and second order partial derivatives of each of the following functions

a. \[ z = 6(20x + 50y - x^2 + xy - 3y^2) - 4x - 9y \]

\[
\frac{\partial z}{\partial x} = \quad \frac{\partial z}{\partial y} =
\]

\[
\frac{\partial^2 z}{\partial x^2} = \quad \frac{\partial^2 z}{\partial y^2} =
\]

\[
\frac{\partial^2 z}{\partial y \partial x} = \quad \frac{\partial^2 z}{\partial x \partial y} =
\]

b. \[ z = 15x^{2/3} y^{1/5} - 5x - 3y \]

\[
\frac{\partial z}{\partial x} = \quad \frac{\partial z}{\partial y} =
\]

\[
\frac{\partial^2 z}{\partial x^2} = \quad \frac{\partial^2 z}{\partial y^2} =
\]

\[
\frac{\partial^2 z}{\partial y \partial x} = \quad \frac{\partial^2 z}{\partial x \partial y} =
\]
c. 

\[ z = 336x^{1/3}y^{1/2} - 49x - 96y \]

\[ \frac{\partial z}{\partial x} = \quad \frac{\partial z}{\partial y} = \]

\[ \frac{\partial^2 z}{\partial x^2} = \quad \frac{\partial^2 z}{\partial y^2} = \]

\[ \frac{\partial^2 z}{\partial y \partial x} = \quad \frac{\partial^2 z}{\partial x \partial y} = \]

d. 

\[ z = 100x + 40y - 2x^2 + 3xy - 2y^2 - 5(81x + 7y - 50) \]

\[ \frac{\partial z}{\partial x} = \quad \frac{\partial z}{\partial y} = \]

\[ \frac{\partial^2 z}{\partial x^2} = \quad \frac{\partial^2 z}{\partial y^2} = \]

\[ \frac{\partial^2 z}{\partial y \partial x} = \quad \frac{\partial^2 z}{\partial x \partial y} = \]
e. 
\[ z = 150x + 200y - 4(10x + 4y - 2x^2 + 3xy - 2y^2 - 5) \]

\[ \frac{\partial z}{\partial x} = \]

\[ \frac{\partial^2 z}{\partial x^2} = \]

\[ \frac{\partial^2 z}{\partial y \partial x} = \]

\[ \frac{\partial^2 z}{\partial x \partial y} = \]

\[ \frac{\partial^2 z}{\partial y^2} = \]

f. 
\[ z = 27x + 16y - 10(x^{1/3}y^{1/4} - 5) \]

\[ \frac{\partial z}{\partial x} = \]

\[ \frac{\partial^2 z}{\partial x^2} = \]

\[ \frac{\partial^2 z}{\partial y \partial x} = \]

\[ \frac{\partial^2 z}{\partial x \partial y} = \]

\[ \frac{\partial^2 z}{\partial y^2} = \]
g. \[ z = x^{1/4} y^{1/2} - 3(6x + 7y - 16) \]

\[
\frac{\partial z}{\partial x} = \quad \frac{\partial z}{\partial y} =
\]

\[
\frac{\partial^2 z}{\partial x^2} = \quad \frac{\partial^2 z}{\partial y^2} =
\]

\[
\frac{\partial^2 z}{\partial y \partial x} = \quad \frac{\partial^2 z}{\partial x \partial y} =
\]

h. \[ z = (2x + 5y)^2 \]

\[
\frac{\partial z}{\partial x} = \quad \frac{\partial z}{\partial y} =
\]

\[
\frac{\partial^2 z}{\partial x^2} = \quad \frac{\partial^2 z}{\partial y^2} =
\]

\[
\frac{\partial^2 z}{\partial y \partial x} = \quad \frac{\partial^2 z}{\partial x \partial y} =
\]
i. 

\[ z = (3x^2 + 4y^2 - 3xy)^2 \]

\[ \frac{\partial z}{\partial x} = \quad \frac{\partial z}{\partial y} = \]

\[ \frac{\partial^2 z}{\partial x^2} = \quad \frac{\partial^2 z}{\partial y^2} = \]

\[ \frac{\partial^2 z}{\partial y \partial x} = \quad \frac{\partial^2 z}{\partial x \partial y} = \]
Problem 2: Consider the following function:

\[ y = \frac{4}{3}x^3 - \frac{11}{2}x^2 - 3x + 4 \]

a. Find the critical points of the function. Use the second derivative test to classify the critical points into local maximum and local minimum.

b. Find the potential points of inflection of this function.
Problem 3: Consider the following function:

\[ y = 2x^4 - \frac{14}{2}x^2 + \frac{5}{2}x^2 + 3 \]

a. Find the critical points of the function. Use the second derivative test to classify the critical points into local maximum and local minimum.

b. Find the potential points of inflection of this function.
Problem 4: Consider the following function:

\[ y = \frac{2}{3}x^3 + \frac{1}{2}x^2 - 15x + 2 \]

a. Find the critical points of the function. Use the second derivative test to classify the critical points into local maximum and local minimum.

b. Find the potential points of inflection of this function.
Problem 5: Consider the following function:

\[ y = \frac{2}{3}x^3 - \frac{13}{2}x^2 + 6x + 4 \]

a. Find the critical points of the function. Use the second derivative test to classify the critical points into local maximum and local minimum.

b. Find the potential points of inflection of this function.
Problem 6: Consider the following function:

\[ y = \frac{x^4}{4} - \frac{5}{3}x^3 - x^2 + 24x + 4 \]

a. Find the critical points of the function. (Hint: \( x = 3 \) is a critical point.) Use the second derivative test to classify the critical points into local maximum and local minimum.

b. Find the potential points of inflection of this function.
Problem 7: Consider the following function:

\[ y = \frac{1}{2}x^4 - \frac{1}{3}x^3 - x^2 + x + 4 \]

a. Find the critical points of the function. (Hint: \( x = -1 \) is a critical point). Use the second derivative test to classify the critical points into local maximum and local minimum.

b. Find the potential points of inflection of this function.
Problem 8: Find the determinants of the following matrices.

a. \[
\begin{bmatrix}
5 & 2 \\
1 & 2 \\
\end{bmatrix}
\]

b. \[
\begin{bmatrix}
3 & 0 \\
1 & -7 \\
\end{bmatrix}
\]

c. \[
\begin{bmatrix}
5 & 2 \\
-1 & 6 \\
\end{bmatrix}
\]

d. \[
\begin{bmatrix}
1 & -2 & 1 \\
-3 & 1 & 2 \\
0 & 1 & 2 \\
\end{bmatrix}
\]

e. \[
\begin{bmatrix}
1 & 1 & 3 \\
2 & 1 & -1 \\
1 & 0 & -1 \\
\end{bmatrix}
\]
Problem 9:  
\[
\begin{align*}
2x + 3y + z &= 5 \\
x - 4y + 3z &= -6 \\
x + 2y - z &= 2
\end{align*}
\]

a. Solve the given system using substitution.
b. Solve the system using Cramer’s rule.