ECONOMICS 207
FALL 2010
PROBLEM SET 6
14^{th} April 2010

Problem 1: Consider the following function:

$$y = -2x^2 + 12x + 2$$

Find the critical values of x. Use the second derivative test to classify the critical points into local maximum and local minimum.

Problem 2: Consider the following function:

$$y = 2x^2 + 5x + 4$$

Find the critical value(s) of x. Use the second derivative test to classify the critical points into local maximum and local minimum.

Problem 3: Consider the following function:

$$y = 2x^3 - \frac{17}{2}x^2 + 5x + 4$$

Find the critical values of x. Use the second derivative test to classify the critical points into local maximum and local minimum.

Problem 4: Consider the following function:

$$y = x^3 + \frac{13}{2}x^2 + 4x + 3$$

Find the critical values of x. Use the second derivative test to classify the critical points into local maximum and local minimum.

Problem 5: Consider the following function:

$$y = \frac{5}{3}x^3 - 8x^2 + 3x - 40$$

Find the critical values of x. Use the second derivative test to classify the critical points into local maximum and local minimum.

Problem 6: Consider the following function:

$$y = \frac{x^2}{x^2 + 2}$$

Find the critical values of x. Use the second derivative test to classify the critical points into local maximum and local minimum.

Problem 7: Consider the following function:

$$y = x^3 - \frac{23}{2}x^2 - 36x - 45$$

Find the critical values of x. Use the second derivative test to classify the critical points into local maximum and local minimum

Problem 8: Consider the following function:

$$y = 5 - \frac{1}{4}x^4$$

a. Find the critical points of the function.

b. Is the function at a local maximum or a local minimum at the critical point.

Problem 9: Consider the following function:

$$y = 5 - 3x^5$$

a. Find the critical points of the function.

b. Is the function at a local maximum or a local minimum at the critical point.

Problem 10: Consider the following function:

$$y = 2 + 3x^6$$

a. Find the critical points of the function.

b. Is the function at a local maximum or a local minimum at the critical point.

Problem 11: Find the determinants of the following sqaure matrices

a. $\begin{bmatrix} 2 & -1 & 3 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix}$, b. $\begin{bmatrix} -1 & 3 \\ 4 & 6 \end{bmatrix}$, c. $\begin{bmatrix} 2 & -5 \\ -2 & 2 \end{bmatrix}$

Problem 12: Find the first and second order partial derivatives of each of the following functions. You do not need to simplify

a.

$$z = 45x^3 - 26y^{2/3} + 5x^2y$$

$\dfrac{\partial z}{\partial x} =$ $\dfrac{\partial z}{\partial y} =$

$\dfrac{\partial^2 z}{\partial x^2} =$ $\dfrac{\partial^2 z}{\partial y^2} =$

$\dfrac{\partial^2 z}{\partial y \partial x} =$ $\dfrac{\partial^2 z}{\partial x \partial y} =$

b.

$$z = 2x^2y + 2xy^2 - 3xy$$

$\dfrac{\partial z}{\partial x} =$ $\dfrac{\partial z}{\partial y} =$

$\dfrac{\partial^2 z}{\partial x^2} =$ $\dfrac{\partial^2 z}{\partial y^2} =$

$\dfrac{\partial^2 z}{\partial y \partial x} =$ $\dfrac{\partial^2 z}{\partial x \partial y} =$

c.

$$z = e^{x^2 y}$$

$\frac{\partial z}{\partial x} =$ $\frac{\partial z}{\partial y} =$

$\frac{\partial^2 z}{\partial x^2} =$ $\frac{\partial^2 z}{\partial y^2} =$

$\frac{\partial^2 z}{\partial y \partial x} =$ $\frac{\partial^2 z}{\partial x \partial y} =$

d.

$$z = 15x^{2/3}y^{3/5} - 2x - 4y$$

$\frac{\partial z}{\partial x} =$ $\frac{\partial z}{\partial y} =$

$\frac{\partial^2 z}{\partial x^2} =$ $\frac{\partial^2 z}{\partial y^2} =$

$\frac{\partial^2 z}{\partial y \partial x} =$ $\frac{\partial^2 z}{\partial x \partial y} =$

Problem 13: Find the critical points for each of the following functions. Use the Hessian Matrix to classify them as a local maxima, minima or saddle point.

a.

$$f(x_1, x_2) = 16x_1 + 12x_2 - x_1^2 - x_2^2$$

b.

$$f(x_1, x_2) = 26x_1 + 18x_2 - x_1^2 - x_2^2$$

c.

$$y = 16x_1 + 12x_2 + x_1^2 + x_2^2$$

Problem 14: Yearly profits for a firm are given by

$$P(x, y) = -x^2 - y^2 + 22x + 18y - 102$$

where x is the amount spent on research and y is the amount spent on advertising. Find the values of x and y that can maximise profit. Find the corresponding (maximum) profit.

Problem 15: Consider a competitive firm with the following **cost function** and **output price**

$$C(y) = 1000 + 800y - 80y^2 + 4y^3$$

$$p = 748$$

where y represents output.

a. Write down the expression for the **profit function**. Find the **potential** levels of output that maximize profit (**i.e., the critical values** of y)

c. Use the second derivative test to determine which of the levels of output from part (a) above actually maximizes profits.

Problem 16: Consider a competitive firm with the following *production function*

$$y = 189x + 90x^2 - 3x^3$$

where y denotes output and x denotes the single variable input that is used in the production process. Furthermore, $p_y = 10$ and p_x =5130, where p_y is the price per unit of the output and p_x is the price per unit of the input.

Find the **potential** levels of x that maximize **output**. Use the second derivative test to determine which of these values of x actually maximize **output.**