ECONOMICS 207
SPRING 2010
PRACTICE PROBLEM SET

Problem 1: Sove the following equations for x

a. $\dfrac{3x-2}{4x+5} = \dfrac{3}{2}$

b. $\dfrac{6x^2+7x-3}{3x^2+2x-1} = 0$

c. $\dfrac{8x^2-1}{2} = 1-x$

d. $\left(15x^{2/3} - 2x^{1/3}\right)^{1/2} - 1 = 0$

e. $4x^{2/3} = 100$

g. $9x^{2/5} = 72x^{-1/5}$

Problem 2: Solve the following system of equations using **Cramer's rule.**

$$\begin{aligned} x - y + z &= 4 \\ 2x + 3y + z &= 1 \\ x + 2y - 2z &= -5 \end{aligned}$$

Problem 3: Solve the following system of equations using **substitution.**
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$$\begin{aligned} x + y + z &= 1 \\ 3x - 2y - z &= 6 \\ 2x + y + z &= 3 \end{aligned}$$

Problem 4: Solve the following systems of equations using substituion

$$\begin{aligned} 2x^{1/2}y^{1/2} &= 70 \\ 250xy^{-3/2} &= 98 \end{aligned}$$

Problem 5: Consider the following function

$$y = 2x^3 - \frac{11}{2}x^2 + 3x + 4$$

a. Find all the critical values of x.

b. Use the second derivative test to classify the critical points as local maximum, or minimum.

Problem 6: Consider the following function

$$y = 3x^4 - \frac{11}{3}x^3 + x^2 - 7$$

a. Find all the critical values of x.

b. Use the second derivative test to classify the critical points as local maximum, or minimum.

Problem 7: Consider the following function

$$y = 2x^4 + 35$$

a. Find all the critical values of x.

b. Is the critical point that you got in (a) a local max, a local min or a point of inflection? Why?

Problem 8: The cost function for a firm is a rule that tells the total cost of production of any output level produced by the firm. Suppose that the **cost function** for a competitive firm is given by

$$C(y) = cost\ = 1000 + 600y - 80y^2 + 4y^3$$

where y stands for the level of output. Price per unit of the output is \$712.

a. Write down the expression of the **marginal cost**

a. Write down the expression for the **revenue** function.

b. Write down the expression for the **profit** function.

c. How much output (y) should the firm produce to **maximize profit**. Use the **second order condition** to verify that profit is indeed maximized.

Problem 9: Find the determinants of the following matrices:

$$\begin{pmatrix} 2 & 6 \\ 2 & 3 \end{pmatrix}, \begin{pmatrix} -1 & 3 \\ 5 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & -1 \\ 3 & 2 & 2 \\ 2 & 0 & 4 \end{pmatrix}, \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 2 & 3 & 1 \end{pmatrix}$$

Problem 10: Differentiate the following functions:

a.

$$e^{3x} + \left(x^2 + 3x + 2\right)^5$$

b.

$$\ln\left(x^3\right)$$

c.

$$\left[\ln\left(5x^2 + 6\right)\right]^{1/3}$$

d.

$$e^{3x^2+5}\,(2x+1)^{1/4}$$

e.

$$e^{4x} + \left(\ln x + x^3 + 5x^2\right)^3$$

f.

$$(\ln x)^3$$

g.

$$\ln\left[\left(5x^2+1\right)^{1/3}\right]$$

h.

$$\frac{x^2+3x+2}{2e^x}$$

Problem 11: Find the first and second order partial derivatives of each of the following functions. You do not need to simplify a.

$$z = 4x^2 - 6y^2 + 2x^{1/2}y^{1/2}$$

$\frac{\partial z}{\partial x} =$

$\frac{\partial z}{\partial y} =$

$\frac{\partial^2 z}{\partial x^2} =$

$\frac{\partial^2 z}{\partial y^2} =$

$\frac{\partial^2 z}{\partial y \partial x} =$

$\frac{\partial^2 z}{\partial x \partial y} =$

b.

$$z = x^{2/3}y^{1/3} + 2x^2y^2 - xy$$

$\frac{\partial z}{\partial x} =$

$\frac{\partial z}{\partial y} =$

$\frac{\partial^2 z}{\partial x^2} =$

$\frac{\partial^2 z}{\partial y^2} =$

$\frac{\partial^2 z}{\partial y \partial x} =$

$\frac{\partial^2 z}{\partial x \partial y} =$

c.

$$z = e^{x^2+y}$$

$\frac{\partial z}{\partial x} =$

$\frac{\partial z}{\partial y} =$

$\frac{\partial^2 z}{\partial x^2} =$

$\frac{\partial^2 z}{\partial y^2} =$

$\frac{\partial^2 z}{\partial y \partial x} =$

$\frac{\partial^2 z}{\partial x \partial y} =$

d.

$$z = 5x^{1/3}y^{3/5} - x - 2y$$

$\frac{\partial z}{\partial x} =$

$\frac{\partial z}{\partial y} =$

$\frac{\partial^2 z}{\partial x^2} =$

$\frac{\partial^2 z}{\partial y^2} =$

$\frac{\partial^2 z}{\partial y \partial x} =$

$\frac{\partial^2 z}{\partial x \partial y} =$

Problem 12: Consider the following production function of a competitive firm

$$y = 40L + 10K - 2L^2 + 2LK - K^2$$

where y denotes output; L and K denote labor and capital input respectively.

$$\begin{aligned} p &= 1 \\ w_L &= 34, w_K = 2 \end{aligned}$$

where p ,w_L and w_K stand for price per unit of output, labor and capital input respectively.

a. What is the expression for the **revenue** function

b.What is the expression for the **cost** function

c. What is the expression for the **profit** function

d. How much labor (L) and capital (K) must the firm employ to maximize its output. **Verify** using the second derivative test that the profit is indeed maximized.

e. How much output (y) will the firm produce.

Problem 13: [You need not Do this problem. I have worked out the steps in the file containing the hints. So go through the steps and use them to do the next problem]

Consider the following production function of a competitive firm which uses two inputs viz., labor and capital to produce its output,

$$y = output = 5L + 4K - 2L^2 + LK - K^2$$

where, L and K stand for labor and capital respectively. We are also given the following information:

$$\begin{aligned} price\ per\ unit\ of\ \textbf{labor} &= 6 \\ price\ per\ unit\ of\ \textbf{capital} &= 2 \end{aligned}$$

However, the firm **has a production constraint**. It has to produce 9 units of the output. How much labor (L) and capital (K) should the firm potentially employ if it wants to **minimize its cost of producing 9 units of the output.**

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Problem 15: Consider a consumer who consumes two commodities: x_1 and x_2. The utility that he gets from consuming these commodities is given by:

$$U(x_1, x_2) = utility = 30x_1 + 20x_2 - 2x_1^2 + 2x_1x_2 - 3x_2^2$$

Also let

$$p_1 = 20; p_2 = 40$$

where p_1 and p_2 represent price per unit of x_1 and x_2 respectively. The consumer's **income is 440 dollars**. How many units of x_1 and x_2 should he consume if he wants to **maximize his utility subject to his income constraint**.

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