ECONOMICS 207
SPRING 2006
EXAM 2

Problem 1 (18 Points)**.**
Find the derivatives of each of the following functions with respect to x.

(i) $y \ = \ 10x^3 \ - \ 5x^2 + \ln(x) - 4x \ + \ 5$

$$y' = 30x^2 - 10x + \frac{1}{x} - 4$$

(ii) $y \ = \ (2x \ - \ 5x^2)(6x^2 \ - \ 2).$

$$y' = (2x \ - \ 5x^2)(12x) + (6x^2 \ - \ 2)(2 - 10x)$$

(iii) $y \ = \ \frac{2x^5 \ + \ 4x^3 \ - \ 11x}{4x^2}$

$$y' = \frac{(4x^2)(10x^4 + 12x^2 - 11) - (2x^5 \ + \ 4x^3 \ - \ 11x)(8x)}{(4x^2)^2}$$

(iv) $y \ = \ ln\left[\,(2x^{-3} \ + \ 7x)^5 \ + \ 2e^{3x}\,\right]$

$$y' = \frac{1}{(2x^{-3} \ + \ 7x)^5 \ + \ 2e^{3x}} * \left[5\left(2x^{-3} \ + \ 7x\right)^4\left(-6x^{-4} + 7\right) + \frac{2}{3}e^{3x}\right]$$

(v) $y \ = \ 15x^{5/8} \ - \ 25x$

$$y' = \frac{75}{8}x^{\frac{-3}{8}} - 25$$

(vi) $y \ = \ 9x^4e^{7x^5-3x+2}$

$$y' = \left(9x^4\right)\left(e^{7x^5-3x+2} * \left(35x^4 - 3\right)\right) + e^{7x^5-3x+2} * \left(36x^3\right)$$

Date: 4 October 2006.

Problem 2 (16 Points)**.**

Solve the following system of equations for x_1, x_2, and x_3.

$$x_1 - 3x_2 + 2x_3 = -5$$
$$3x_1 - 7x_2 - 2x_3 = 9$$
$$-x_1 + 4x_2 - 5x_3 = 3$$

$$\begin{array}{c} -3x_1 + 9x_2 - 6x_3 = 15 \\ \underline{3x_1 - 7x_2 - 2x_3 = 9} \\ 2x_2 - 8x_3 = 24 \end{array} \quad \begin{array}{c} x_1 - 3x_2 + 2x_3 = -5 \\ \underline{-x_1 + 4x_2 - 5x_3 = 3} \\ x_2 - 3x_3 = -2 \end{array}$$

$$\begin{array}{c} 2x_2 - 8x_3 = 24 \\ -2x_2 + 6x_3 = 4 \\ -2x_3 = 28 \\ x_3 = -14 \end{array}$$

$$x_2 = -44,\ x_3 = -109$$

Problem 3 (15 points)**.**

Solve the following system of equations.

$$30\,x_1^{-2/3}\,x_2^{1/3}\;-\;10\;=0$$
$$20\,x_1^{1/3}\,x_2^{-2/3}\;-\;25\;=0$$

$$\begin{aligned} 30x_2^{\frac{1}{3}} &= 10x_1^{\frac{2}{3}} \\ x_2 &= \left(\frac{1}{3}x_1^{\frac{2}{3}}\right)^3 \end{aligned}$$

$$\begin{aligned} x_1^{\frac{1}{3}}\left(\left(\frac{1}{3}x_1^{\frac{2}{3}}\right)^3\right)^{\frac{-2}{3}} &= \frac{5}{4} \\ 9x_1^{\frac{1}{3}}x_1^{\frac{-4}{3}} &= \frac{5}{4} \\ x_1 &= \frac{36}{5} \end{aligned}$$

$$x_2 = \left(\frac{1}{3}\left(\frac{36}{5}\right)^{\frac{2}{3}}\right)^3 = \frac{48}{25}$$

Problem 4 (15 points)**.** Find the profit maximizing level of output for a firm who faces output price, p, and has the following cost function.

$$price \;=\; p \;= 400$$

$$cost \;=\; c(y) \;= 300 \;+\; 300y \;-\; 25y^2 \;+\; 2y^3$$

$$\begin{aligned}
\pi &= 400y - \left(300 + 300y - 25y^2 + 2y^3\right) \\
\pi' &= 400 - \left(300 - 50y + 6y^2\right) \\
0 &= 100 + 50y - 6y^2 \\
y &= -\frac{5}{3} \; or \; 10
\end{aligned}$$

$$\begin{aligned}
\pi'' &= 50 - 12y \\
\pi''\,(10)\,50 - 120 &< 0 \text{ So } y = 10 \text{ is the profit max output level.}
\end{aligned}$$

Problem 5 (20 Points).

In the following problem you are given a production function for a firm where y is the variable representing the level of output and x is the level of the variable input. Assume that the price of output for this firm is price = p = 1. Assume that the price of the input = w = 85. The function representing output as a function of input is given by

$$output = y = f(x) = 210x + 35x^2 - x^3$$

(i) Write a function representing the revenue of the firm as a function of price and input level.

Revenue = $1 * \left(210x + 35x^2 - x^3 \right)$

(ii) Write a function representing the cost of the firm as a function of input price and input level.

Cost = $85x$

(iii) Write a function representing the profit of the firm as a function of input price, output price and input level.
Profit = $210x + 35x^2 - x^3 - 85x$

(iv) Find the profit maximizing level of input, x.
$\pi' = 210 + 70x - 3x^2 - 85$
$0 = 125 + 70x - 3x^2$
$x = 25 \; or \; \frac{-5}{3}$

$\pi'' = 70 - 6x$
$\pi''(25) = 70 - 6 * 25 < 0$ So, $y = 25$ is the profit max input level.