ECONOMICS 207
FALL 2006
EXAM 3

Problem 1 (25 points)**.**

Consider the following matrices.

$$A = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 2 \\ -4 & -3 \end{bmatrix}, \quad C = \begin{bmatrix} -2 & 1 & 3 \\ 3 & 4 & -4 \end{bmatrix}$$

$$D = \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}, \quad F = \begin{bmatrix} 2 & 1 \\ -3 & -2 \\ 1 & 2 \end{bmatrix}$$

$$a = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

Compute the following

(i) A + B

$$A + B = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ -4 & -3 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ -1 & -1 \end{bmatrix}$$

(ii) AB

$$AB = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix} * \begin{bmatrix} 3 & 2 \\ -4 & -3 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

(iii) C′B

$$C'B = \begin{bmatrix} -2 & 3 \\ 1 & 4 \\ 3 & -4 \end{bmatrix} * \begin{bmatrix} 3 & 2 \\ -4 & -3 \end{bmatrix} = \begin{bmatrix} -18 & -13 \\ -13 & -10 \\ 25 & 18 \end{bmatrix}$$

Date: 2 November 2006.

(iv) a′F

$$a'F = \begin{bmatrix} 2 & 1 & -1 \end{bmatrix} * \begin{bmatrix} 2 & 1 \\ -3 & -2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -2 \end{bmatrix}$$

(v) FA

$$FA = \begin{bmatrix} 2 & 1 \\ -3 & -2 \\ 1 & 2 \end{bmatrix} * \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 11 & 8 \\ -18 & -13 \\ 10 & 7 \end{bmatrix}$$

Problem 2 (15 Points).

Solve the following system of equations for x_1, x_2, and x_3 by writing the system as an augmented matrix and performing row reduction.

$$4x_1 + x_2 + x_3 = 2$$

$$-2x_1 + x_2 + x_3 = -1$$

$$4x_1 + x_2 + 2x_3 = 1$$

$$\begin{bmatrix} 4 & 1 & 1 & 2 \\ -2 & 1 & 1 & -1 \\ 4 & 1 & 2 & 1 \end{bmatrix} \quad \begin{matrix} R2 \to R1 + 2R2 \\ R3 \to R1 - R3 \end{matrix}$$

$$\begin{bmatrix} 4 & 1 & 1 & 2 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \quad \begin{matrix} R1 \to \frac{1}{4}R1 \\ R2 \to \frac{1}{3}R2 \\ R3 \to -1R3 \end{matrix}$$

$$\begin{bmatrix} 1 & \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} R1 \to R1 - \frac{1}{4}R2$$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} R2 \to R2 - R3$$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Hence $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ 1 \\ -1 \end{bmatrix}$

Problem 3 (20 Points)**.**

For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise. Also find the points of inflection for each function.

(i) $f(x) = x^3 - 2x^2 + x + 1$

To find the critical points we need to set $f' = 0$ and solve for x :

$$\begin{aligned} f'(x) &= 3x^2 - 4x + 1 = 0 \\ x &= \frac{4 \pm \sqrt{16 - 4(3)(1)}}{6} \\ x &= 1 \text{ or } \frac{1}{3} \end{aligned}$$

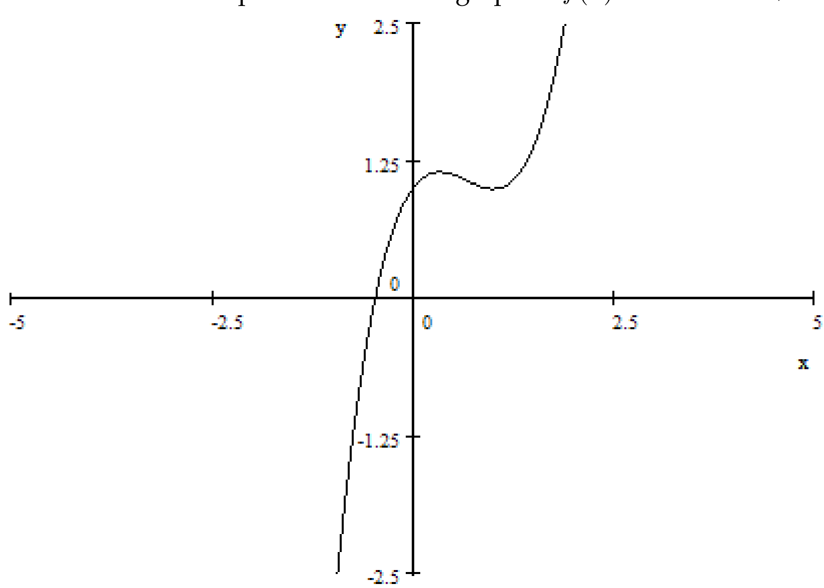
Now we need to determine if these critical points are maxima, minima, or inflection points.

$$\begin{aligned} f''(x) &= 6x - 4 \\ f''(1) &= 2 > 0 \quad {}^{+}\smile^{+} \\ f''\left(\frac{1}{3}\right) &= -2 < 0 \quad {}^{-}\frown^{-} \end{aligned}$$

Hence, $x = 1,$ is a local minimum, and $x = \frac{1}{3}$ is a local maximum. Now what about other inflection points?

$$\begin{aligned} f''(x) &= 6x - 4 = 0 \\ x &= \frac{2}{3} \end{aligned}$$

$x = \frac{2}{3}$ is another inflection point. Here is the graph of $f(x) = x^3 - 2x^2 + x + 1$



(ii) $f(x) = 3x^4 - 4x^3 + 2$

To find the critical points we need to set $f' = 0$ and solve for x :

$$\begin{aligned} f'(x) &= 12x^3 - 12x^2 = 0 \\ 0 &= 12x^2(x-1) \\ \text{So, } x &= 0 \text{ or } x = 1 \end{aligned}$$

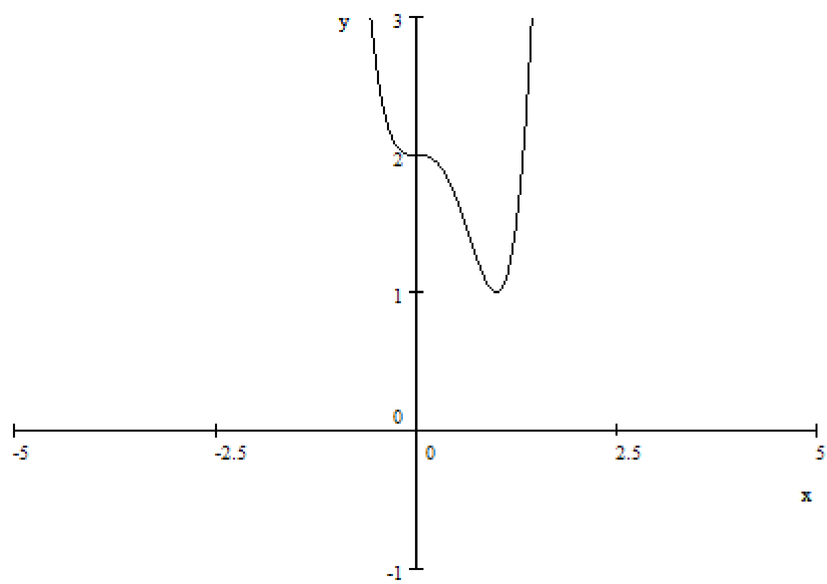
Now we need to determine if these critical points are maxima, minima, or inflection points.

$$\begin{aligned} f''(x) &= 36x^2 - 24x \\ f''(0) &= 0 \\ f''(1) &= 36 - 24 > 0 \end{aligned}$$

$x = 0$ is an inflection point, and $x = 1,$ is a local minimum. What about other inflection points?

$$\begin{aligned} f''(x) &= 36x^2 - 24x = 0 \\ 0 &= 12x(3x-2) \\ x &= 0 \text{ or } x = \frac{2}{3} \end{aligned}$$

So, we find that $x = \frac{2}{3}$ is also an inflection point. Here is the graph of $f(x) = 3x^4 - 4x^3 + 2$



Problem 4 (10 points)**.** Calculate the definate integral.

(i) $\int_1^4 18x^2 + \frac{1}{x} - e^{2x} dx$

$$\begin{aligned}\int_1^4 18x^2 + \frac{1}{x} - e^{2x} dx &= 6x^3 + \ln(x) - \frac{1}{2}e^{2x}|_1^4 \\ &= \left(6 * 4^3 + \ln(4) - \frac{1}{2}e^8\right) - \left(6 + \ln(1) - \frac{1}{2}e^2\right) \\ &= 378 + \ln(4) + \frac{1}{2}e^2 - \frac{1}{2}e^8\end{aligned}$$

Note: $\ln(1) = 0$

Problem 5 (30 points)**.** The cost function for a firm is a rule or mapping that tells the total cost of production of any output level produced by the firm. If the variable y represents the output of the firm, then the cost function is given by c(y). Marginal cost represents the change in the cost of production for the firm as output changes and is given by the derivative of the cost function with respect to output,i.e., Marginal Cost(MC) = $c'(y)$.

(i) Find the profit maximizing level of output for the following firm. Demonstrate that the level you choose maximizes profit.

$$price \;=\; p \;= \$300$$

$$cost \;=\; c(y) \;= 1000 \;+\; 27y \;-\; 9y^2 \;+\; y^3$$

$$\begin{aligned}
\pi &= 300y - \left(1000 + 27y - 9y^2 + y^3\right) \\
\pi' &= 300 - 27 + 18y - 3y^2 = 0 \\
0 &= -3y^2 + 18y + 273 \\
0 &= -y^2 + 6y + 91 \\
y &= \frac{-6 \pm \sqrt{36 - 4\,(-1)\,(91)}}{-2} \\
&= \frac{-6 \pm 20}{-2} = -7 \;\text{ or }\; 13
\end{aligned}$$

Now we will need to see which one is the profit maximizing output level and not the profit minimizing output level!

$$\begin{aligned}
\pi'' &= 18 - 6y \\
\pi''\,(13) &= 18 - 6 * 13 < 0
\end{aligned}$$

So, $y = 13$ is the profit maximizing output level.

(ii) What is revenue minus variable cost for this firm when price is \$300?

$$\begin{aligned}
\text{Rev} - \text{VC} &= 300 * 13 - \left(27 * 13 - 9 * 13^2 + 13^3\right) \\
&= 3900 - 351 + 1521 - 2197 \\
&= \$2873
\end{aligned}$$

(iii) Find producer surplus for this firm assuming you are only given the following marginal cost function: $MC(y) \;=\; 27 - 18y + 3y^2$ and a price of \$300.

$$\begin{aligned}
PS &= \int_0^{13} 300 - \left(27 - 18y + 3y^2\right) dy \\
&= 300y - \left(27y - 9y^2 + y^3\right) |_0^{13} \\
&= 3900 - 351 + 1521 - 2197 \\
&= \$2873
\end{aligned}$$