ECONOMICS 207
FALL 2006
EXAM 4

Problem 1. (17 Points) Find the inverse of the matrix $A = \begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$.

$$A^{-1} = \frac{1}{1} \begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$$

Problem 2. (17 Points) Use the inverse matrix you found above to solve the following system of equations:

$$\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = A^{-1} * \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix} * \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 12 \\ -7 \end{bmatrix}$$

Date: November 15, 2006.

Problem 3. (17 Points) Calculate the determinant of the matrix $A = \begin{bmatrix} -5 & 2 & 1 \\ 2 & -1 & 4 \\ -3 & 1 & 7 \end{bmatrix}$

$$\begin{aligned} |A| &= -2\begin{vmatrix} 2 & 4 \\ -3 & 7 \end{vmatrix} + (-1)\begin{vmatrix} -5 & 1 \\ -3 & 7 \end{vmatrix} - \begin{vmatrix} -5 & 1 \\ 2 & 4 \end{vmatrix} \\ &= -2\,(-26) + (-1)\,(-32) - (-22) \\ &= 2 \end{aligned}$$

Problem 4. (17 Points) Use Cremer's Rule to solve the following system of equations:

$$\begin{bmatrix} 3 & 0 & 4 \\ 2 & -1 & 7 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$x_1 = \frac{\begin{vmatrix} 1 & 0 & 4 \\ 0 & -1 & 7 \\ 1 & 0 & -1 \end{vmatrix}}{\begin{vmatrix} 3 & 0 & 4 \\ 2 & -1 & 7 \\ 1 & 0 & -1 \end{vmatrix}} = \frac{5}{7} \quad x_2 = \frac{\begin{vmatrix} 3 & 1 & 4 \\ 2 & 0 & 7 \\ 1 & 1 & -1 \end{vmatrix}}{\begin{vmatrix} 3 & 0 & 4 \\ 2 & -1 & 7 \\ 1 & 0 & -1 \end{vmatrix}} = \frac{-4}{7} \quad x_3 = \frac{\begin{vmatrix} 3 & 0 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & 0 & 4 \\ 2 & -1 & 7 \\ 1 & 0 & -1 \end{vmatrix}} = \frac{-2}{7}$$

$$\left\{ \begin{array}{c} \text{Determinant of} \\ \text{the coeficient} \\ \text{matrix} \end{array} \right\} \begin{vmatrix} 3 & 0 & 4 \\ 2 & -1 & 7 \\ 1 & 0 & -1 \end{vmatrix} = -0 \begin{vmatrix} 2 & 7 \\ 1 & -1 \end{vmatrix} + (-1) \begin{vmatrix} 3 & 4 \\ 1 & -1 \end{vmatrix} - 0 \begin{vmatrix} 3 & 4 \\ 2 & 7 \end{vmatrix} = 0 + 7 - 0 = 7$$

$$\left\{ \begin{array}{c} \text{Numerator of} \\ \text{the } x_1 \text{ ratio.} \end{array} \right\} \begin{vmatrix} 1 & 0 & 4 \\ 0 & -1 & 7 \\ 1 & 0 & -1 \end{vmatrix} = -0 + (-1) \begin{vmatrix} 1 & 4 \\ 1 & -1 \end{vmatrix} - 0 = 5$$

$$\left\{ \begin{array}{c} \text{Numerator of} \\ \text{the } x_2 \text{ ratio.} \end{array} \right\} \begin{vmatrix} 3 & 1 & 4 \\ 2 & 0 & 7 \\ 1 & 1 & -1 \end{vmatrix} = -1 \begin{vmatrix} 2 & 7 \\ 1 & -1 \end{vmatrix} + 0 - 1 \begin{vmatrix} 3 & 4 \\ 2 & 7 \end{vmatrix} = 9 - 13 = -4$$

$$\left\{ \begin{array}{c} \text{Numerator of} \\ \text{the } x_3 \text{ ratio.} \end{array} \right\} \begin{vmatrix} 3 & 0 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = -0 + (-1) \begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} - 0 = -2$$

Problem 5. (17 Points) Find all the first and second order partial derivatives of $f(x, y, z) = 30x^{\frac{1}{2}}y^{\frac{1}{3}}z^{\frac{1}{5}}$

$f_x = 15x^{-\frac{1}{2}}y^{\frac{1}{3}}z^{\frac{1}{5}}$	$f_y = 10x^{\frac{1}{2}}y^{-\frac{2}{3}}z^{\frac{1}{5}}$	$f_z = 6x^{\frac{1}{2}}y^{\frac{1}{3}}z^{-\frac{4}{5}}$
$f_{xx} = -\frac{15}{2}x^{-\frac{3}{2}}y^{\frac{1}{3}}z^{\frac{1}{5}}$	$f_{xy} = 5x^{-\frac{1}{2}}y^{-\frac{2}{3}}z^{\frac{1}{5}}$	$f_{xz} = 3x^{-\frac{1}{2}}y^{\frac{1}{3}}z^{-\frac{4}{3}}$
$f_{yx} = 5x^{-\frac{1}{2}}y^{-\frac{2}{3}}z^{\frac{1}{5}}$	$f_{yy} = -\frac{20}{3}x^{\frac{1}{2}}y^{-\frac{5}{3}}z^{\frac{1}{3}}$	$f_{yz} = 2x^{\frac{1}{2}}y^{-\frac{2}{3}}z^{-\frac{4}{3}}$
$f_{zx} = 3x^{-\frac{1}{2}}y^{\frac{1}{3}}z^{-\frac{4}{3}}$	$f_{zy} = 2x^{\frac{1}{2}}y^{-\frac{2}{3}}z^{-\frac{4}{3}}$	$f_{zz} = -\frac{24}{5}x^{\frac{1}{2}}y^{\frac{1}{3}}z^{-\frac{9}{5}}$

Problem 6. (15 Points) Find all the first and second order partial derivatives of $f(x, y) = e^{x^2 y}$

$$f_x = 2xye^{x^2 y} \qquad f_y = x^2 e^{x^2 y}$$

$$f_{xx} = 2xy\left(2xye^{x^2 y}\right) + e^{x^2 y}(2y) \qquad f_{xy} = 2xy\left(x^2 e^{x^2 y}\right) + e^{x^2 y}(2x)$$

$$f_{yx} = 2x^3 ye^{x^2 y} + 2xe^{x^2 y} \qquad f_{yy} = x^4 e^{x^2 y}$$