**ECONOMICS 207**
**FALL 2006**
**PROBLEM SET 5**

**Problem 1.** Find the second derivative of each of the following functions with respect to x

(i) $y \ = \ 160x^{1/4}z^{3/5} \ - \ 45x \ - \ 35z$

$$\begin{aligned} y' &= 40x^{\frac{-3}{4}}z^{\frac{3}{5}} - 45 \\ y'' &= -30x^{-\frac{7}{4}}z^{\frac{3}{5}} \end{aligned}$$

(ii) $f(x) \ = \ 9x^3e^{3x^2-x}$

$$f'(x) = 9x^3\left[e^{3x^2-x}(6x-1)\right] + 27x^2e^{3x^2-x}$$

$$f''(x) = \left[\left[54x^4 - 9x^3\right]e^{3x^2-x}(6x-1) + e^{3x^2-x}\left[216x^3 - 27x^2\right]\right] + \left[27x^2e^{3x^2-x}(6x-1) + e^{3x^2-x}(54x)\right]$$

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**Problem 2.** Find the definite integral of each of the following functions.

(i) $\int_0^9 \left(9x^2 \ - \ 12x \ + \ 100\right) \, dx$

$$\begin{aligned} &= 3x^3 - 6x^2 + 100x|_0^9 \\ &= 3*9^3 - 6*9^2 + 100*9 \\ &= 2601 \end{aligned}$$

(ii) $\int_{27}^{64} \left(50x^{-1/3}z^{1/5} \ - \ 16\right) \, dx, \quad z \ = \ 243$. (When you are done evaluating the integral, plug in $z = 243$ to get a number)

$$\begin{aligned} &= 75x^{\frac{2}{3}}z^{\frac{1}{5}} - 16x|_{27}^{64} \\ &= \left(75*64^{\frac{2}{3}}z^{\frac{1}{5}} - 16*64\right) - \left(75*27^{\frac{2}{3}}z^{\frac{1}{5}} - 16*27\right) \\ &= \left(1200z^{\frac{1}{5}} - 592\right) - \left(675z^{\frac{1}{5}} - 432\right) \\ &= 525z^{\frac{1}{5}} - 592 \\ &= 525*243^{\frac{1}{5}} - 592 \\ &= 983 \end{aligned}$$

**Problem 3.**

(i) Find the profit maximizing level of output for the following firm. Demonstrate that the level you choose maximizes profit.

$$price \;=\; p \;= \$392$$

$$cost \;=\; c(y) \;= 300 \;+\; 200y \;-\; 12y^2 \;+\; 2y^3$$

$$\begin{aligned}\pi &= 392y - \left(300 + 200y - 12y^2 + 2y^3\right)\\ \pi' &= 392 - 200 + 24y - 6y^2\\ 0 &= 192 + 24y - 6y^2\\ y &= 8 \text{ or } -4\end{aligned}$$

$$\begin{aligned}\pi'' &= 24 - 12y\\ \pi''(8) &= 24 - 12 * 8 < 0 \text{ Hence } y = 8 \text{ is a max.}\end{aligned}$$

(ii) What is revenue minus variable cost for this firm when price is \$392?

$$\begin{aligned}PS &= \text{Rev} - \text{ VC}\\ &= 392 * 8 - \left(200 * 8 - 12 * 8^2 + 2 * 8^3\right)\\ &= 1280\end{aligned}$$

(iii) Find producer surplus for this firm assuming you are only given the following marginal cost function: $MC(y) = 200 - 24y + 6y^2$ and a price of $392.

$$
\begin{aligned}
& \int_0^8 392 - \left(200 - 24y + 6y^2\right) dy \\
= \; & 392 * y - \left(200 * y - 12 * y^2 + 2 * y^3\right) |_0^8 \\
= \; & 392 * 8 - \left(200 * 8 - 12 * 8^2 + 2 * 8^3\right) - 0 \\
= \; & 1280
\end{aligned}
$$

**Problem 4.**

(i) Find the profit maximizing level of output for the following firm. Demonstrate that the level you choose maximizes profit.

$$price = p = \$769$$

$$cost = c(y) = 500 + 400y - 20y^2 + 3y^3$$

$$
\begin{aligned}
\pi &= 769y - \left(500 + 400y - 20y^2 + 3y^3\right) \\
\pi' &= 769 - \left(400 - 40y + 9y^2\right) \\
0 &= 369 + 40y - 9y^2 \\
y &= 9 \text{ or } -\frac{41}{9}
\end{aligned}
$$

$$
\begin{aligned}
\pi'' &= 40 - 18y \\
\pi''(9) &= 40 - 18 * 9 < 0 \quad \text{Hence } y = 9 \text{ is the profit maximising output level.}
\end{aligned}
$$

(ii) What is revenue minus variable cost for this firm when price is $769?

$$
\begin{aligned}
PS &= \text{Rev} - \text{VC} \\
&= 769 * 9 - \left(400 * 9 - 20 * 9^2 + 3 * 9^3\right) \\
&= 2754
\end{aligned}
$$

(iii) Find producer surplus for this firm assuming you are only given the following marginal cost function: $MC(y) = 400 - 40y + 9y^2$ and a price of \$769.

$$\begin{aligned} & \int_0^9 769 - \left(400 - 40y + 9y^2\right) dy \\ = & \quad 769 * y - \left(400 * y - 20 * y^2 + 3 * y^3\right) |_0^9 \\ = & \quad 2754 \end{aligned}$$

**Problem 5.** For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise.

(i) $y = 9x^3 - 27x^2$

$$\begin{aligned} y' &= 27x^2 - 54x = 0 \\ 0 &= x\,(27x - 54) \\ x &= 0 \text{ or } 27x - 54 = 0 \quad \Rightarrow x = 2 \\ \text{So, } x &= 0, 2 \text{ are the critical points of this function} \end{aligned}$$

$$\begin{aligned} y'' &= 54x - 54 \\ y''(2) &> 0 \quad \text{Hence } x = 2 \text{ is a relative minimum} \\ y''(0) &< 0 \quad \text{Hence } x = 0 \text{ is a relative maximum} \end{aligned}$$

(ii) $f(x) = x^2 + \frac{1}{x^2}$

$$\begin{aligned} f'(x) &= 2x - \frac{2}{x^3} = 0 \\ 0 &= 2x^4 - 2 \\ x^4 &= 1 \\ x &= \pm 1 \\ \text{Hence, } x &= \pm 1 \text{ are the critical points of this function} \end{aligned}$$

$$\begin{aligned} f''(x) &= 2 + \frac{6}{x^4} \\ f''(-1) &= 2 + 6 > 0 \text{ Hence } x = 1 \text{ is a relative minimum} \\ f''(1) &= 2 + 6 > 0 \text{ Hence } x = -1 \text{ is a relative minimum} \end{aligned}$$

(iii) $f(x) = -3x^5 + 5x^3$

$$\begin{aligned} f'(x) &= -15x^4 + 15x^2 = 0 \\ 0 &= x^2\left(-15x^2 + 15\right) \\ x &= 0 \text{ or } -15x^2 + 15 = 0 \\ \text{So, } x &= 0, \pm 1 \text{ are the critical points of this function} \end{aligned}$$

$$\begin{aligned} f''(x) &= -60x^3 + 30x \\ f''(0) &= 0 \text{ Hence } x = 0 \text{ is an inflection point} \\ f''(1) &= -60 + 30 < 0 \text{ Hence } x = 1 \text{ is a relative maximum} \\ f''(-1) &= 60 + 30 > 0 \text{ Hence } x = -1 \text{ is a relative minimum} \end{aligned}$$

**Problem 6.** Do the following problems from the book.

(i) Section 9.4

(a) 5

The equilibrium quantity is found by setting demand and supply equal to one another. Then the equilibrium price can be found by plugging in the equilibrium quantity into either the demand or supply equation.

$$\begin{aligned} 200 - .2Q &= 20 + .1Q \\ Q &= 600 \\ P &= 200 - .2 * 600 = 80 \end{aligned}$$

Consumer surplus is the area underneath the demand curve and above the price line.

$$\begin{aligned} CS &= \int_0^{600} (200 - .2Q) - 80 dQ \\ &= 200Q - \frac{1}{10}Q^2 - 80Q|_0^{600} \\ &= 36,000 \end{aligned}$$

Producer surplus is the area under the price line and above the supply curve.

$$\begin{aligned} PS &= \int_0^{600} 80 - (20 + .1Q)\, dQ \\ &= 80Q - \left(20Q + \frac{1}{20}Q^2\right)|_0^{600} \\ &= 18,000 \end{aligned}$$

(ii) Section 8.7

You will need to use Theorem 8.7.1 for these problems.

(a) 1a

$$\begin{aligned} f'(x) &= 3x^2 + 3x - 6 = 0 \\ x &= 1 \text{ and } -2 \text{ are the critical values of the function} \\ f'(-4) &= 30 > 0 \quad \text{Hence, } f(x) \text{ is increasing on } (-\infty, -2) \\ f'(0) &= -6 < 0 \quad \text{Hence } f(x) \text{ is decreasing on } (-2, 1) \\ f'(2) &= 12 > 0 \quad \text{Hence } f(x) \text{ is increasing on } (1, \infty) \end{aligned}$$

(b) 1b

$$\begin{aligned} f''(x) &= 6x + 3 = 0 \\ x &= -\frac{1}{2} \text{ is the inflection point of this function} \end{aligned}$$