ECONOMICS 207
FALL 2006
PROBLEM SET 10

Problem 1. Consider the following matrix and vectors.

$$A = \begin{bmatrix} 1 & -\frac{1}{3} & 2 \\ 3 & 0 & 6 \\ 5 & -\frac{5}{3} & 11 \end{bmatrix}$$

$$c = \begin{bmatrix} 2 \\ 9 \\ 11 \end{bmatrix}$$

(i) Find the inverse of the matrix A.

$$\begin{bmatrix} 1 & -\frac{1}{3} & 2 & 1 & 0 & 0 \\ 3 & 0 & 6 & 0 & 1 & 0 \\ 5 & -\frac{5}{3} & 11 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} R2 \to \widetilde{-3R1} + R2 \\ R3 \to -5R1 + R3 \end{matrix}$$

$$\begin{bmatrix} 1 & -\frac{1}{3} & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -3 & 1 & 0 \\ 0 & 0 & 1 & -5 & 0 & 1 \end{bmatrix} R1 \to \widetilde{\frac{1}{3}R2} + R1$$

$$\begin{bmatrix} 1 & 0 & 2 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & 0 & -3 & 1 & 0 \\ 0 & 0 & 1 & -5 & 0 & 1 \end{bmatrix} R1 \to \widetilde{R1} - 2R3$$

$$\begin{bmatrix} 1 & 0 & 0 & 10 & \frac{1}{3} & -2 \\ 0 & 1 & 0 & -3 & 1 & 0 \\ 0 & 0 & 1 & -5 & 0 & 1 \end{bmatrix}$$

So, $A^{(-1)} = \begin{bmatrix} 10 & \frac{1}{3} & -2 \\ -3 & 1 & 0 \\ -5 & 0 & 1 \end{bmatrix}$

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(ii) Find the solution vector $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ by using Cramer's Rule.

$$x_1 = \frac{\begin{vmatrix} 2 & -\frac{1}{3} & 2 \\ 9 & 0 & 6 \\ 11 & 6 & 11 \end{vmatrix}}{\begin{vmatrix} 1 & -\frac{1}{3} & 2 \\ 3 & 0 & 6 \\ 5 & 6 & 11 \end{vmatrix}} = 1 \qquad x_2 = \frac{\begin{vmatrix} 1 & 2 & 2 \\ 3 & 9 & 6 \\ 5 & 11 & 11 \end{vmatrix}}{\begin{vmatrix} 1 & -\frac{1}{3} & 2 \\ 3 & 0 & 6 \\ 5 & 6 & 11 \end{vmatrix}} = 3 \qquad x_3 = \frac{\begin{vmatrix} 1 & -\frac{1}{3} & 2 \\ 3 & 0 & 9 \\ 5 & 6 & 11 \end{vmatrix}}{\begin{vmatrix} 1 & -\frac{1}{3} & 2 \\ 3 & 0 & 6 \\ 5 & 6 & 11 \end{vmatrix}} = 1$$

Problem 2. For the following problem, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise. Check to see if there are points of inflection **at points other than** critical points.

(i) [a.]
(ii) $f(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 28x + 5$

$$\begin{aligned} f'(x) &= x^2 + 3x - 28 = 0 \\ 0 &= (x-4)(x+7) \\ x &= 4 \text{ or } x = -7 \end{aligned}$$

Now lets check to see if these are local maximum or minima or inflection points.

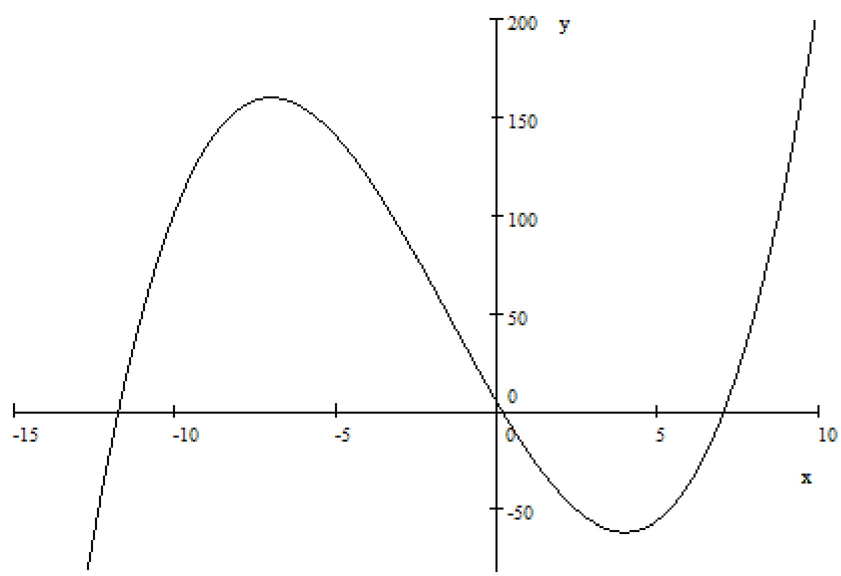
$$\begin{aligned} f''(x) &= 2x + 3 \\ f''(4) &= 11 > 0 \\ f''(-7) &= -11 < 0 \end{aligned}$$

So, $x = 4$ is a local minima, and $x = -7$ is a local maxima. Now we need to check if there are any inflection points.

x

$$\begin{aligned} f''(x) &= 2x + 3 \\ 0 &= 2x + 3 \\ x &= -\frac{3}{2} \end{aligned}$$

So , $x = -\frac{3}{2}$ is an inflection point. Here is the graph of $f(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 28x + 5$



Problem 3. Find all first partial derivatives of each of the following

(i) $y = -x_1^2 e^{5x_2} + 3x_1$

$$\begin{aligned} \frac{\partial y}{\partial x_1} &= -2x_1 e^{5x_2} + 3 \\ \frac{\partial y}{\partial x_2} &= -5x_1^2 e^{5x_2} \end{aligned}$$

(ii) $y = x_1 e^{-x_1^2 x_2^2} \backslash$

$$\begin{aligned} \frac{\partial y}{\partial x_1} &= x_1 \left(-2x_1 e^{-x_1^2 x_2^2} \right) + \left(e^{-x_1^2 x_2^2} \right) \\ \frac{\partial y}{\partial x_2} &= 2x_1 x_2 e^{-x_1^2 x_2^2} \end{aligned}$$

(iii) $y = x_1^3 x_2^3 \ln(x_2)$

$$\begin{aligned} \frac{\partial y}{\partial x_1} &= 3x_1^2 x_2^3 \ln(x_2) \\ \frac{\partial y}{\partial x_2} &= \frac{x_1^3 x_2^3}{x_2} + 3x_1^3 x_2^2 \ln(x_2) = x_1^3 x_2^2 + 3x_1 x_2 \ln(x_2) \end{aligned}$$

(iv) $y = x_1^7 \cdot \ln(x_2) + \frac{9}{x_1^3} - \sqrt{x_2}$

$$\begin{aligned} \frac{\partial y}{\partial x_1} &= 7x_1^6 \ln(x_2) - 27x_1^{-4} \\ \frac{\partial y}{\partial x_2} &= \frac{x_1^7}{x_2} - \frac{1}{2} x_2^{-\frac{1}{2}} \end{aligned}$$