ECONOMICS 207
FALL 2006
PROBLEM SET 9

Problem 1. Consider a consumer with the following utility function:

$$U(x_1, x_2) = x_1^{1/2} x_2^{1/3}$$

and facing the following prices for x_1 and x_2, respectively. w is the consumer's income.

$$p_1 = 2$$
$$p_2 = 3$$
$$w = 1000$$

Form the Lagrangian for the constrained maximization problem; then, find the first order conditions and solve for the optimal consumption bundle. (i.e., find the optimal values for x_1 and x_2) Remember, the Lagrangian is of the form: $\mathcal{L} = h(x_1, x_2) - \lambda [g(x_1, x_2)]$ where $h(x_1, x_2)$ is the objective function (the thing to be maximized) and $g(x_1, x_2)$ is the constraint (the budget constraint)

Date: 28 November 2006.

Problem 2. Now check the second order condition on the bordered Hessian. Remember that the bordered Hessian in the constrained optimization problem written is given by:

$$H_B = \begin{bmatrix} \dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1 \partial x_1} & \dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1 \partial x_2} & \dfrac{\partial g(x_1, x_2)}{\partial x_1} \\ \dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2 \partial x_1} & \dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2 \partial x_2} & \dfrac{\partial g(x_1, x_2)}{\partial x_2} \\ \dfrac{\partial g(x_1, x_2)}{\partial x_1} & \dfrac{\partial g(x_1, x_2)}{\partial x_2} & 0 \end{bmatrix}$$

Problem 3. Consider a consumer with the following utility function:

$$U(x_1, x_2) = x_1^{1/4} x_2^{1/5}$$

and facing the following prices for x_1 and x_2, respectively. w is the consumer's income.

$$p_1 = 1$$
$$p_2 = 2$$
$$w = \bar{w}$$

Form the Lagrangian for the constrained maximization problem; then, find the first order conditions and solve for the optimal consumption bundle. (i.e., find the optimal values for x_1 and x_2)

Remember, the Lagrangian is of the form: $\mathcal{L} = h(x_1, x_2) - \lambda [g(x_1, x_2)]$ where $h(x_1, x_2)$ is the objective function (the thing to be maximized) and $g(x_1, x_2)$ is the constraint (the budget constraint)

Problem 4. Now for this problem, write out the bordered Hessian and plug in the optimal values you found above. State what the condition is that gaurantees we found a maximum. (**Note: This means you are to write out the bordered Hessian, but I'm not asking you to calculate the determinant. Just tell me what you would look for if you were to calculate it.)** Remember that the bordered Hessian in the constrained optimization problem written is given by:

$$H_B = \begin{bmatrix} \frac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1 \partial x_1} & \frac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1 \partial x_2} & \frac{\partial g(x_1, x_2)}{\partial x_1} \\ \frac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2 \partial x_1} & \frac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2 \partial x_2} & \frac{\partial g(x_1, x_2)}{\partial x_2} \\ \frac{\partial g(x_1, x_2)}{\partial x_1} & \frac{\partial g(x_1, x_2)}{\partial x_2} & 0 \end{bmatrix}$$

Problem 5. Consider a consumer with the following utility function:

$$U(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$$

and facing the following prices for x_1 and x_2, respectively. w is the consumer's income.

$$p_1 = 1$$
$$p_2 = 2$$
$$w = 100$$

Form the Lagrangian for the constrained maximization problem; then, find the first order conditions and solve for the optimal consumption bundle. (i.e., find the optimal values for x_1 and x_2)

Remember, the Lagrangian is of the form: $\mathcal{L} = h(x_1, x_2) - \lambda [g(x_1, x_2)]$ where $h(x_1, x_2)$ is the objective function (the thing to be maximized) and $g(x_1, x_2)$ is the constraint (the budget constraint)

Now for this problem, write out the bordered Hessian and plug in the optimal values you found above. State what the condition is that gaurantees we found a maximum. (**Note: This means you are to write out the bordered Hessian, but I'm not asking you to calculate the determinant. Just tell me what you would look for if you were to calculate it.)** Remember that the bordered Hessian in the constrained optimization problem written is given by:

$$H_B = \begin{bmatrix} \frac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1 \partial x_1} & \frac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1 \partial x_2} & \frac{\partial g(x_1, x_2)}{\partial x_1} \\ \frac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2 \partial x_1} & \frac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2 \partial x_2} & \frac{\partial g(x_1, x_2)}{\partial x_2} \\ \frac{\partial g(x_1, x_2)}{\partial x_1} & \frac{\partial g(x_1, x_2)}{\partial x_2} & 0 \end{bmatrix}$$

Problem 6. Consider a firm with production function $f(x_1, x_2)$, and who faces prices or $w_1,\ w_2$ for the inputs $x_1,\ x_2$ respectively. Find the profit maximizing quantities x_1 and x_2, and check the second order condition on the Hessian.

$$f(x_1, x_2) = 50x_1 + 30x_2 - 2x_1^2 + 2x_1x_2 - x_2^2$$
$$w_1 = 4$$
$$w_2 = 2$$
$$p = 1$$