PARTIAL DERIVATIVE PRACTICE

Problem 1. Consider the following function.

$$\pi(x_1, x_2) = 6(100x_1 + 50x_2 - x_1^2 + x_1x_2 - 3x_2^2) - 360x_1 - 90x_2$$

Find the following partial derivatives.

a.
$$\frac{\partial \pi(x_1, x_2)}{\partial x_1} =$$

b.
$$\frac{\partial \pi(x_1, x_2)}{\partial x_2} =$$

c.
$$\frac{\partial^2 \pi(x_1, x_2)}{\partial x_1^2} =$$

d.
$$\frac{\partial^2 \pi(x_1, x_2)}{\partial x_1 \partial x_2} =$$

e.
$$\frac{\partial^2 \pi(x_1, x_2)}{\partial x_2^2} =$$

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Problem 2. Consider the following function.

$$\pi(x_1, x_2) \;=\; 15x_1^{2/3}x_2^{1/5} \;-\; 5x_1 \;-\; 3x_2$$

Find the following partial derivatives.

a. $\dfrac{\partial \pi(x_1, x_2)}{\partial x_1} \;=$

b. $\dfrac{\partial \pi(x_1, x_2)}{\partial x_2} \;=$

c. $\dfrac{\partial^2 \pi(x_1, x_2)}{\partial x_1^2} \;=$

d. $\dfrac{\partial^2 \pi(x_1, x_2)}{\partial x_1 \partial x_2} \;=$

e. $\dfrac{\partial^2 \pi(x_1, x_2)}{\partial x_2^2} \;=$

Problem 3. Consider the following function.

$$\pi(x_1, x_2) \;=\; 336x_1^{1/3}x_2^{1/2} \;-\; 49x_1 \;-\; 96x_2$$

Find the following partial derivatives.

a. $\dfrac{\partial \pi(x_1, x_2)}{\partial x_1} \;=$

b. $\dfrac{\partial \pi(x_1, x_2)}{\partial x_2} \;=$

c. $\dfrac{\partial^2 \pi(x_1, x_2)}{\partial x_1^2} \;=$

d. $\dfrac{\partial^2 \pi(x_1, x_2)}{\partial x_1 \partial x_2} \;=$

e. $\dfrac{\partial^2 \pi(x_1, x_2)}{\partial x_2^2} \;=$

Problem 4. Consider the following function.

$$\mathcal{L}(x_1, x_2, \lambda) = 100x_1 + 40x_2 - 2x_1^2 + 3x_1x_2 - 2x_2^2 - \lambda(82x_1 + 60x_2 - 4976)$$

Find the following partial derivatives.

a. $\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1} =$

b. $\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2} =$

c. $\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda} =$

d. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1^2} =$

e. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1 \partial x_2} =$

f. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1 \partial \lambda} =$

g. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2^2} =$

h. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2 \partial \lambda} =$

i. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda^2} =$

Problem 5. Consider the following function.

$$\mathcal{L}(x_1, x_2, \lambda) = 150x_1 + 200x_2 - \lambda\left(100x_1 + 40x_2 - 2x_1^2 + 3x_1x_2 - 2x_2^2 - 3800\right)$$

Find the following partial derivatives.

a. $\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1} =$

b. $\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2} =$

c. $\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda} =$

d. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1^2} =$

e. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1 \partial x_2} =$

f. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1 \partial \lambda} =$

g. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2^2} =$

h. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2 \partial \lambda} =$

i. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda^2} =$

Problem 6. Consider the following function.

$$\mathcal{L}(x_1, x_2, \lambda) \;=\; 27x_1 \;+\; 16x_2 \;-\; \lambda\left(x_1^{1/3}x_2^{1/4} \;-\; 12\right)$$

Find the following partial derivatives.

a. $\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1} \;=$

b. $\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2} \;=$

c. $\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda} \;=$

d. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1^2} \;=$

e. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1 \partial x_2} \;=$

f. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1 \partial \lambda} \;=$

g. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2^2} \;=$

h. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2 \partial \lambda} \;=$

i. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda^2} \;=$

Problem 7. Consider the following function.

$$\mathcal{L}(x_1, x_2, \lambda) \;=\; 32x_1 \;+\; 75x_2 \;-\; \lambda\left(x_1^{1/3}x_2^{1/5} \;-\; 10\right)$$

Find the following partial derivatives.

a. $\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1} \;=$

b. $\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2} \;=$

c. $\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda} \;=$

d. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1^2} \;=$

e. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1 \partial x_2} \;=$

f. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1 \partial \lambda} \;=$

g. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2^2} \;=$

h. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2 \partial \lambda} \;=$

i. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda^2} \;=$

Problem 8. Consider the following function.

$$\mathcal{L}(x_1, x_2, \lambda) = x_1^{1/4} x_2^{1/2} - \lambda(6x_1 + 4x_2 - 162)$$

Find the following partial derivatives.

a. $\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1} =$

b. $\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2} =$

c. $\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda} =$

d. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1^2} =$

e. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1 \partial x_2} =$

f. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1 \partial \lambda} =$

g. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2^2} =$

h. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2 \partial \lambda} =$

i. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda^2} =$

Problem 9. Consider the following function.

$$\pi(x_1, x_2) = 4(200x_1 + 300x_2 - x_1^2 + x_1x_2 - 2x_2^2) - 400x_1 - 252x_2$$

Find the following partial derivatives.

a. $\dfrac{\partial \pi(x_1, x_2)}{\partial x_1} =$

b. $\dfrac{\partial \pi(x_1, x_2)}{\partial x_2} =$

c. $\dfrac{\partial^2 \pi(x_1, x_2)}{\partial x_1^2} =$

d. $\dfrac{\partial^2 \pi(x_1, x_2)}{\partial x_1 \partial x_2} =$

e. $\dfrac{\partial^2 \pi(x_1, x_2)}{\partial x_2^2} =$

Problem 10. Consider the following function.

$$\pi(x_1, x_2) = 8640x_1^{2/5}x_2^{1/6} - 256x_1 - 405x_2$$

Find the following partial derivatives.

a. $\dfrac{\partial \pi(x_1, x_2)}{\partial x_1} =$

b. $\dfrac{\partial \pi(x_1, x_2)}{\partial x_2} =$

c. $\dfrac{\partial^2 \pi(x_1, x_2)}{\partial x_1^2} =$

d. $\dfrac{\partial^2 \pi(x_1, x_2)}{\partial x_1 \partial x_2} =$

e. $\dfrac{\partial^2 \pi(x_1, x_2)}{\partial x_2^2} =$

Problem 11. Consider the following function.

$$\mathcal{L}(x_1, x_2, \lambda) = 150x_1 + 200x_2 - \lambda\left(100x_1 + 80x_2 - 2x_1^2 + 2x_1x_2 - 2x_2^2 - 3450\right)$$

Find the following partial derivatives.

a. $\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1} =$

b. $\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2} =$

c. $\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda} =$

d. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1^2} =$

e. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1 \partial x_2} =$

f. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1 \partial \lambda} =$

g. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2^2} =$

h. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2 \partial \lambda} =$

i. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda^2} =$

Problem 12. Consider the following function.

$$\mathcal{L}(x_1, x_2, \lambda) = x_1^{1/4} x_2^{3/5} - \lambda(5x_1 + 4x_2 - 1377)$$

Find the following partial derivatives.

a. $\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1} =$

b. $\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2} =$

c. $\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda} =$

d. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1^2} =$

e. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1 \partial x_2} =$

f. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1 \partial \lambda} =$

g. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2^2} =$

h. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2 \partial \lambda} =$

i. $\dfrac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda^2} =$