ECONOMICS 207
SPRING 2005
EXAM 2 ANSWER KEY

Problem 1 (18 Points).

Find the derivatives of each of the following functions with respect to x.

a. $y = 4x^3 - 3x^2 + 2x + 15$

$\frac{dy}{dx} = 12x^2 - 6x + 2$

b. $y = (5x - 2)(4x^2 + 3)$

$\frac{dy}{dx} = 5(4x^2 + 3) + 8x(5x - 2) = 20x^2 + 15 + 40x^2 - 16x = 60x^2 - 16x + 15$

c. $y = \frac{3x^2 + 2x}{x^3 + 2x}$

$\frac{dy}{dx} = \frac{(6x + 2)(x^3 + 2x) - (3x^2 + 2)(3x^2 + 2x)}{(x^3 + 2x)^2} = \frac{(6x^4 + 12x^2 + 2x^3 + 4x) - (9x^4 + 6x^3 + 6x^2 + 4x)}{(x^3 + 2x)^2} = \frac{-3x^4 - 4x^3 + 6x^2}{(x^3 + 2x)^2}$

d. $y = 10e^{4x^2 - 2x + 5}$

$\frac{dy}{dx} = 10e^{4x^2 - 2x + 5}(8x - 2) = 80xe^{4x^2 - 2x + 5} - 20e^{4x^2 - 2x + 5}$

e. $y = pAx^{3/8} - 20x$

$\frac{dy}{dx} = \frac{3}{8}\, pAx^{-5/8} - 20$

f. $y = ln\,[(2x^2 - 3x + 4)^3 + 3x^2]$

$\frac{dy}{dx} = \frac{1}{(2x^2 - 3x + 4)^3 + 3x^2}[3(2x^2 - 3x + 4)^2(4x - 3) + 6x]$

Problem 2 (18 Points).

Solve the following system of equations using any method you choose.

$$x + y - z = 4 \quad (1)$$
$$x + 2y + 3z = 10 \quad (2)$$
$$2x + 5y + 5z = 21 \quad (3)$$

$Solution:$ $\quad (1) + 2 \times (2) \Rightarrow$

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$$3x + 5y + 5z = 24 \tag{4}$$

$$(4) - (3) \Rightarrow$$

$$x = 3$$

$$Plug \quad x = 3 \quad into \quad (1) \quad and \quad (2) \Rightarrow$$

$$y - z = 1 \tag{5}$$

$$2y + 3z = 7 \tag{6}$$

$$(6) - 2 \times (5) \Rightarrow$$

$$5z = 5 \Rightarrow z = 1$$

$$Plug \quad x = 3 \quad and \quad z = 1 \quad into \quad (1) \Rightarrow$$

$$y = 2$$

$$So, \quad x = 3, \quad y = 2, \quad z = 1.$$

Problem 3 (15 points)**.**

Solve the following system of equations.

$$4x_1^{-1/2}x_2^{1/4} - 2 = 0 \tag{7}$$

$$2x_1^{1/2}x_2^{-3/4} - 4 = 0 \tag{8}$$

$$Solution(1): \qquad Rearrange\ (7) \quad and \quad (8) \Rightarrow$$

$$4x_1^{-1/2}x_2^{1/4} = 2 \tag{9}$$

$$2x_1^{1/2}x_2^{-3/4} = 4 \tag{10}$$

$$(10) \times (9) \Rightarrow$$

$$\begin{aligned} 8x_2^{-1/2} &= 8 \\ &\Rightarrow x_2^{-1/2} = 1 \\ &\Rightarrow x_2 = 1 \end{aligned}$$

$$Plug \quad x_2 = 1 \quad into \quad (10) \Rightarrow$$

$$\begin{aligned} 2x_1^{1/2} &= 4 \\ &\Rightarrow \sqrt{x_1} = 2 \\ &\Rightarrow x_1 = 4 \end{aligned}$$

$$So, \ x_1 = 4, \quad x_2 = 1.$$

$$Solution(2): \qquad Rearrange\ (7) \quad \Rightarrow$$

$$\begin{aligned} 4x_1^{-1/2}x_2^{1/4} &= 2 \\ &\Rightarrow 2x_2^{1/4} = x_1^{1/2} \\ &\Rightarrow x_1 = 4x_2^{1/2} \end{aligned}$$

$$Plug \quad x_1 = 4x_2^{1/2} \quad into (8) \Rightarrow$$

$$\begin{aligned} 2 \times 2x_2^{1/4} \times x_2^{-3/4} - 4 &= 0 \\ &\Rightarrow 4x_2^{1/4-3/4} = 4 \\ &\Rightarrow x_2^{-1/2} = 1 \\ &\Rightarrow x_2 = 1 \end{aligned}$$

$$plug \quad x_2 = 1 \quad into \quad (8) \Rightarrow$$

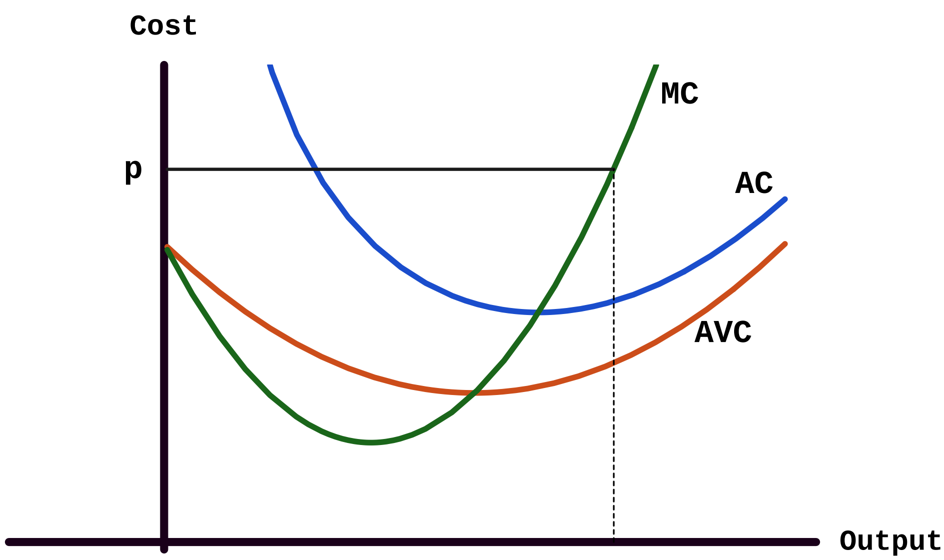
$$\begin{aligned} 2x_1^{1/2} - 4 &= 0 \\ &\Rightarrow x_1^{1/2} = 2 \\ &\Rightarrow x_1 = 4 \end{aligned}$$

$So, \;\; x_1 = 4, \quad x_2 = 1.$

Problem 4 (18 points)**.**

The cost function for a firm is a rule or mapping that tells the total cost of production of any output level produced by the firm. If the variable y represents the output of the firm, then the cost function is given by c(y). Marginal cost represents the change in the cost of production for the firm as output changes and is given by the derivative of the cost function with respect to output, i.e., Marginal Cost (MC) = $\frac{dc(y)}{dy}$. A competitive firm facing a fixed output price maximizes profit at the output level where marginal cost is equal to price as in the diagram below.

FIGURE 1. Profit Maximization



Find the profit maximizing level of output for the following firm.

$$price = p = 157 \tag{11}$$

$$cost = c(y) = 50 + 125y - 10y^2 + y^3 \tag{12}$$

$$MC = \frac{dc(y)}{dy} = 125 - 20y + 3y^2 = p = 157$$

$$\Rightarrow 3y^2 - 20y - 32 = 0$$

$$\Rightarrow (y - 8)(3y + 4) = 0$$

$$\Rightarrow y = 8 \quad or \quad y = -\tfrac{4}{3}(Dropped)$$

So, the profit maximizing level of output for the firm is $y = 8$.

Problem 5 (20 Points)**.**

In the following problem you are given a production function for a firm where y is the variable representing the level of output and x is the level of the variable input. You are given the price (p) of the output and the price (w) of the single variable input. The function representing output as a function of input is given by

$$output \ = \ y \ = \ f(x) \ = \ 50x \ + \ 20x^2 \ - x^3$$

a. Write a function representing the revenue of the firm as a function of price and output level.

   Revenue = py

b. Write a composite function representing the revenue of the firm as a function of price and input level.

   Revenue = $p(50x \ + \ 20x^2 \ - x^3)$

c. Write a function representing the cost of the firm as a function of input price and input level.

   Cost = w × x, This is the answer so you need do nothing here.

d. Write a function representing the profit of the firm as a function of input price, output price and input level.

   Profit = Revenue - Cost $= p(50x \ + \ 20x^2 \ - x^3) - w \times x$

e. Assume that the price of output for this firm is price = p = 20. Assume that the price of the input = w = 1960.

   Write an equation for the profit of this firm that depends on the input level.

   Profit = $p(50x \ + \ 20x^2 \ - x^3) - w \times x = 20(50x \ + \ 20x^2 \ - x^3) - 1960x = -20x^3 + 400x^2 - 960x$

f. Maximize profit by taking the derivative of the function in part e with respect to x, setting it equal to zero, and solving for the input level x. Hint: 960 is divisible by 20 and 48 is divisible by 12.

$$\frac{dProfit}{dx} = -60x^2 + 800x - 960 = 0 \Rightarrow 20(x-12)(-3x+4) = 0 \Rightarrow x = 12 \quad or \quad x = \frac{4}{3}$$

$$Profit(x = 12) = -20 \times 12^3 + 400 \times 12^2 - 960 \times 12 = 11520$$

$$Profit(x = 4/3) = -20 \times (\frac{4}{3})^3 + 400 \times (\frac{4}{3})^2 - 960 \times (\frac{4}{3}) = -616.3 < 0 < 11520$$

So, the optimal input level is12, and the corresponding maximized profit is11520.

Problem 6 (11 Points)**.**

What is a function?

A function is a relation, such that each element of a set (the domain) is associated with a unique element of another (possibly the same) set.