ECONOMICS 207
SPRING 2005
FINAL EXAMINATION

Problem 1 (30 points)**.** Consider the following matrices.

$$J = \begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$$

a. Find the determinant of the matrix J.

The determinant of a 2 x 2 matrix is defined as, $\mid J \mid = a_{11}a_{22} - a_{12}a_{21}$. Substituting the values in, we get, |J| $= (4)(2) - (3)(2) = 2.$

b. Find the inverse of the matrix J using the adjoint method.

The adjoint matrix of J is the $n \times n$ matrix whose (i, j) entry is the (j, i) cofactor of J. So by finding the cofactor of each element in J, we get the adjoint matrix:

$$\begin{bmatrix} 2 & -3 \\ -2 & 4 \end{bmatrix}$$

Then to find the inverse of J by the adjoint matrix, take the adjoint matrix times $\frac{1}{|J|}$. This gets us,

$$\begin{aligned} J^{-1} &= \frac{1}{\mid J \mid} \times adjoint\ matrix \\ &= \frac{1}{2} \begin{bmatrix} 2 & -3 \\ -2 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 1 & \frac{-3}{2} \\ -1 & 2 \end{bmatrix} \end{aligned}$$

c. Using the inverse from part b, solve the system of equations

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$$\begin{aligned} J \times \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} &= \begin{pmatrix} 8 \\ 4 \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = J^{-1} \times \begin{pmatrix} 8 \\ 4 \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} 1 & \frac{-3}{2} \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 8 \\ 4 \end{bmatrix} \\ &\Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \end{aligned}$$

[2,0] is the solution vector.

d. Using row reduction, find the inverse of the matrix J and the solution to the system of equations $J\,x \;=\; b$.

$$\begin{pmatrix} 4 & 3 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$$

To find the inverse of the matrix and the solution vector, set up a matrix that has J on the left, the identity matrix in the center, and the vector b on the right. Then row reduce to get the solution.

$$\begin{bmatrix} 4 & 3 & 1 & 0 & 8 \\ 2 & 2 & 0 & 1 & 4 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & \frac{3}{4} & \frac{1}{4} & 0 & 2 \\ 0 & \frac{1}{2} & \frac{-1}{2} & 1 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 1 & \frac{-3}{2} & 2 \\ 0 & 1 & -1 & 2 & 0 \end{bmatrix}$$

The inverse of J is

$$\begin{bmatrix} 1 & \frac{-3}{2} \\ -1 & 2 \end{bmatrix}$$

And the solution vector is [2,0].

Problem 2 (30 points)**.** Consider a competitive firm with the following production function where y represents the level of output of the firm and x represents the level of the single variable input.

$$y \;=\; f(x) \;=\; 400x \;+\; 60x^2 \;-\; 3x^3$$

The firm faces an output price of p = 10 and an input price of w = 5440.

a. Write a function representing the profit of the firm as a function of the given output and input prices and the input level.

To find profit, take the revenue of the firm and subtract the firm's total cost.

$$\begin{aligned}\pi &= p \times y - TC \\ &= pf(x) - wx \\ &= (10)(400x + 60x^2 - 3x^3) - wx \\ &= 4000x + 600x^2 - 30x^3 - 5440x \\ &= -1440x + 600x^2 - 30x^3\end{aligned}$$

b. What is the profit maximizing level of input for this firm? Verify that the input level you choose is the profit maximizing point.

To find the profit maximizing level of input for the firm, take the derivative of the profit function with respect to input x ,set it equal to zero, and then solve for x.

$$\begin{aligned}\pi(x) &= -1440x + 600x^2 - 30x^3 \\ \frac{d\pi(x)}{dx} &= -1440 + 1200x - 90x^2 = 0 \\ &\Rightarrow -90(x - \frac{4}{3})(x - 12) = 0 \\ &\Rightarrow x = \frac{4}{3} \; or \; 12\end{aligned}$$

To determine which of these points is a maximum, take the second derivative of the profit function which is $\pi''(x) = 1200 - 180x$. At $x = \frac{4}{3}$, $\pi''(\frac{4}{3}) = 1200 - 180(\frac{4}{3}) = 960 > 0$, so it is a minimum. At $x = 12$, $\pi''(12) = 10(120 - 18(12)) = -960 < 0$, so $x = 12$ is a maximum and the optimal input level.

To find the optimal profit, substitute $x = 12$ into the profit function.

$$\begin{aligned}\pi &= -1440x + 600x^2 - 30x^3 \\ &= -1440(12) + 600(12)^2 - 30(12)^3 \\ &= 17280\end{aligned}$$

The profit at the optimal input level is 17,280.

c. What is the level of output of the firm at the profit maximizing input level?

To find the optimal level of output given $x = 12$, we substitute$x = 12$ into the output function.

$$\begin{aligned}y &= f(x) = 400x + 60x^2 - 3x^3 \\ f(12) &= 400(12) + 60(12)^2 - 3(12)^3 \\ &= 8256\end{aligned}$$

8256 is the optimal output level for the firm.

d. What is the marginal product (MP_x) of the variable input at the profit maximizing input level?

Method 1: The partial derivative of $f(x)$ with respect to x is the marginal product of x. So we obtain

$$\begin{aligned} f^{'}(x) &= 400 + 120x - 9x^2 \\ f^{'}(12) &= 400 + 120 \times 12 - 9 \times 12^2 \\ &= 544 \end{aligned}$$

Method 2: The optimality conditions for the profit maximization problem can be interpreted as

$$\frac{\partial \pi}{\partial x} = p \frac{\partial f(x)}{\partial x} - w = 0$$

And since $\frac{\partial f(x)}{\partial x}$ is equal to MP$_x$ we get,

$$\begin{aligned} \frac{\partial f(x)}{\partial x} &= \frac{w}{p} = \frac{5440}{10} \\ &\Rightarrow MP_x = 544 \end{aligned}$$

e. Verify that pMP$_x$ = w at the profit maximizing input level?

To verify that pMP$_x$ = w, we plug in the respective values.

$$pMP_x = (10)(544) = 5440 = w.$$

Problem 3 (40 points)**.** For the following problem, write an equation that represents profit as a function of the two inputs x_1 and x_2. Output price is represented by p, output by y, the price of the first input by w_1 and the price of the second input by w_2. The production function is given by y = f(x_1,x_2). Then find critical values of x_1 and x_2. Verify that profit is maximized at these critical values.

$$\begin{aligned} f(x_1, x_2) &= x_1^{2/3} x_2^{1/5} \\ p &= 15 \\ w_1 &= 5, \qquad w_2 = 3 \end{aligned}$$

The optimal level of x_2 is 32 and $\dfrac{\partial^2 \pi(x_1, x_2)}{\partial x_2 \partial x_2}$ = $\frac{-3}{40}$.

$$\begin{aligned} \pi &= pf(x_1, x_2) - w_1 x_1 - w_2 x_2 \\ &= 15 x_1^{\frac{2}{3}} x_2^{\frac{1}{5}} - 5x_1 - 3x_2 \end{aligned}$$

First, we need to solve the cortical points for x_1 and x_2. To do this, take the partial derivatives of x_1 and x_2, set them equal to zero, and solve for x_1 and x_2.

$$\begin{aligned} \frac{\partial \pi}{\partial x_1} &= 10 x_1^{\frac{-1}{3}} x_2^{\frac{1}{5}} - 5 = 0 \\ \frac{\partial \pi}{\partial x_2} &= 3 x_1^{\frac{2}{3}} x_2^{\frac{-4}{5}} - 3 = 0 \end{aligned} \qquad (1)$$

Since we know $x_2 = 32$, we plug $x_2 = 32$ into the equation,

$$\begin{aligned}
\frac{\partial \pi}{\partial x_1} &= 10x_1^{\frac{-1}{3}}(32)^{\frac{1}{5}} - 5 = 0 \\
\Rightarrow\ & 20x_1^{\frac{-1}{3}} = 5 \\
\Rightarrow\ & x_1^{\frac{-1}{3}} = \frac{1}{4} \\
\Rightarrow\ & x_1 = \frac{1}{4}^{-3} = 4^3 \\
\Rightarrow\ & x_1 = 64
\end{aligned}$$

The critical points are $x_1 = 64$ and $x_2 = 32$.

We can verify that (x_1^*, x_2^*) is a maximum if we set up a Hessian Matrix such as:

$$\begin{vmatrix} \frac{\partial^2 \pi(x_1,x_2)}{\partial x_1^2} & \frac{\partial^2 \pi(x_1,x_2)}{\partial x_1\, \partial x_2} \\ \frac{\partial^2 \pi(x_1,x_2)}{\partial x_2\, \partial x_1} & \frac{\partial^2 \pi(x_1,x_2)}{\partial x_2^2} \end{vmatrix}$$

If the determinant of the Hessian Matrix (discriminant) is greater than 0 and the (1,1) entry is negative, then f has a local maximum. The first entry in the Hessian Matrix is found by taking the derivative of f with respect to x_1 two times, the (1,2) entry is determined by taking the derivative of f with respect to x_2 and with respect to x_1. The (2,1) entry is found by taking the derivative of f with respect to x_2 and then with respect to x_1, and the (2,2) entry is found by taking the derivative of f with the respect to x_2 twice.

The Hessian Matrix of the profit equation is:

$$\begin{vmatrix} \frac{\partial^2 \pi(x_1,x_2)}{\partial x_1^2} & \frac{\partial^2 \pi(x_1,x_2)}{\partial x_1\, \partial x_2} \\ \frac{\partial^2 \pi(x_1,x_2)}{\partial x_2\, \partial x_1} & \frac{\partial^2 \pi(x_1,x_2)}{\partial x_2^2} \end{vmatrix} = \begin{vmatrix} -\frac{10x_2^{\frac{1}{5}}}{3x_1^{\frac{4}{3}}} & \frac{2}{x_1^{\frac{1}{3}} x_2^{\frac{4}{5}}} \\ \frac{2}{x_1^{\frac{1}{3}} x_2^{\frac{4}{5}}} & \frac{-12x_1^{\frac{2}{3}}}{5x_2^{\frac{9}{5}}} \end{vmatrix}$$

The Hessian Matrix of the profit equation at the critical input levels $x_1 = 64$ and $x_2 = 32$ is:

$$\begin{vmatrix} \frac{-5}{192} & \frac{1}{32} \\ \frac{1}{32} & \frac{-3}{40} \end{vmatrix} = \frac{1}{1024} > 0$$

Since the discriminant of the Hessian Matrix is greater than zero, there is a local maximum with $x_1 = 64$ and $x_2 = 32$.

To find the profit at the input levels, substitute $x_1 = 64$ and $x_2 = 32$ into the profit equation.

$$\begin{aligned}
\pi &= 15x_1^{\frac{2}{3}}x_2^{\frac{1}{5}} - 5x_1 - 3x_2 \\
&= 15(64)^{\frac{2}{3}}(32)^{\frac{1}{5}} - 5(64) - 3(32) \\
&= 64
\end{aligned}$$

The maximized profit for the firm is 64.

For your information, the bordered Hessian in the constrained optimization problem written as

$$\mathcal{L}(x_1,\, x_2,\, \lambda) \;=\; f(x_1,\, x_2) \;-\; \lambda\, g(x_1,\, x_2)$$

is given by

$$H_B = \begin{bmatrix} \frac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1 \partial x_1} & \frac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1 \partial x_2} & \frac{\partial g(x_1, x_2)}{\partial x_1} \\ \frac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2 \partial x_1} & \frac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2 \partial x_2} & \frac{\partial g(x_1, x_2)}{\partial x_2} \\ \frac{\partial g(x_1, x_2)}{\partial x_1} & \frac{\partial g(x_1, x_2)}{\partial x_2} & 0 \end{bmatrix}$$

Problem 4 (50 points)**.** Consider a firm with a production function given by

$$y = f(x_1, x_2) = 80x_1 + 40x_2 - 2x_1^2 + 3x_1x_2 - 2x_2^2$$

The firm faces prices and a cost constraint given by

$$\begin{aligned} w_1 &= 6 \\ w_2 &= 3 \\ c_0 &= 96 \end{aligned}$$

Find the maximum output the firm can produce given the cost constraint and the stated prices. Verify that this output is a maximum.

First, we must find the equation of the budget constraint. The budget constraint can be written as $w_1x_1 + w_2x_2 = 96$.

$$\begin{aligned} Budget\ Constraint &= w_1x_1 + w_2x_2 = c_0 \\ &\Rightarrow 6x_1 + 3x_2 = 96 \\ &\Rightarrow 6x_1 + 3x_2 - 96 = 0 \end{aligned}$$

Then to maximize output with a cost of \$96 given these prices we set up the following Lagrangian

$$\begin{aligned} L &= 80x_1 + 40x_2 - 2x_1^2 + 3x_1x_2 - 2x_2^2 - \lambda(6x_1 + 3x_2 - 96) \\ \frac{\partial L}{\partial x_1} &= 80 - 4x_1 + 3x_2 - 6\lambda = 0 \\ \frac{\partial L}{\partial x_2} &= 40 + 3x_1 - 4x_2 - 3\lambda = 0 \\ \frac{\partial L}{\partial \lambda} &= -6x_1 - 3x_2 + 96 = 0 \end{aligned}$$

If we take the ratio of the first two first order conditions we obtain:

$$\begin{aligned} \frac{80 - 4x_1 + 3x_2}{40 + 3x_1 - 4x_2} &= \frac{6\lambda}{3\lambda} = 2 \\ &\Rightarrow 80 - 4x_1 + 3x_2 = 2(40 + 3x_1 - 4x_2) \\ &\Rightarrow 10x_1 = 11x_2 \\ &\Rightarrow x_1 = 1.1x_2 \end{aligned}$$

Now plug this value for x_1 into the last first order condition to obtain.

$$\begin{aligned} -6x_1 - 3x_2 + 96 &= 0 \\ &\Rightarrow -6(1.1x_2) - 3x_2 + 96 = 0 \\ &\Rightarrow 9.6x_2 = 96 \\ &\Rightarrow x_2 = 10 \\ &\Rightarrow x_1 = 1.1x_2 = 1.1 \times 10 = 11 \end{aligned}$$

We can also find the maximum y by substituting in for x_1 and x_2.

$$\begin{aligned} y &= f(x_1,\, x_2) \;=\; 80x_1 \;+\; 40x_2 \;-\; 2x_1^2 \;+\; 3x_1x_2 \;-\; 2x_2^2 \\ f(11,10) &= 80 \times 11 \;+\; 40 \times 10 \;-\; 2 \times 11^2 \;+\; 3 \times 11 \times 10 \;-\; 2 \times 10^2 \\ &= 880 + 400 - 242 + 330 - 200 \\ &= 1168 \end{aligned}$$

The Lagrangian multiplier can be obtained by solving the first equation that was obtained by differentiating L with respect to x_1.

$$\begin{aligned} 80 - 4x_1 + 3x_2 - 6\lambda &= 0 \\ &\Rightarrow 80 - 4 \times 11 + 3 \times 10 - 6\lambda = 0 \\ &\Rightarrow 6\lambda = 66 \\ &\Rightarrow \lambda = 11 \end{aligned}$$

To check for a maximum or minimum we need to set up the bordered Hessian

$$H_B = \begin{bmatrix} \frac{\partial^2 \mathfrak{L}(x_1,x_2,\lambda)}{\partial x_1 \partial x_1} & \frac{\partial^2 \mathfrak{L}(x_1,x_2,\lambda)}{\partial x_1 \partial x_2} & \frac{\partial g(x_1,x_2)}{\partial x_1} \\ \frac{\partial^2 \mathfrak{L}(x_1,x_2,\lambda)}{\partial x_2 \partial x_1} & \frac{\partial^2 \mathfrak{L}(x_1,x_2,\lambda)}{\partial x_2 \partial x_2} & \frac{\partial g(x_1,x_2)}{\partial x_2} \\ \frac{\partial g(x_1,x_2)}{\partial x_1} & \frac{\partial g(x_1,x_2)}{\partial x_2} & 0 \end{bmatrix}$$

We compute the various elements of the bordered Hessian as follows

$$\begin{aligned} L &= \;80x_1 \;+\; 40x_2 \;-\; 2x_1^2 \;+\; 3x_1x_2 \;-\; 2x_2^2 - \lambda(6x_1 \;+\; 3x_2 \;-\; 96) \\ \frac{\partial L}{\partial x_1} &= 80 - 4x_1 + 3x_2 - 6\lambda \\ \frac{\partial L}{\partial x_2} &= 40 + 3x_1 - 4x_2 - 3\lambda \\ \frac{\partial^2 \mathfrak{L}(x_1,x_2,\lambda)}{\partial x_1 \partial x_1} &= -4\,, \quad \frac{\partial^2 \mathfrak{L}(x_1,x_2,\lambda)}{\partial x_2 \partial x_2} = -4 \\ \frac{\partial^2 \mathfrak{L}(x_1,x_2,\lambda)}{\partial x_2 \partial x_1} &= \frac{\partial^2 \mathfrak{L}(x_1,x_2,\lambda)}{\partial x_1 \partial x_2} = 3 \\ \frac{\partial g(x_1,x_2)}{\partial x_1} &= 6, \quad \frac{\partial g(x_1,x_2)}{\partial x_2} = 3 \end{aligned}$$

The derivatives are all constants. The bordered Hessian is given by

$$HB = \begin{bmatrix} -4 & 3 & 6 \\ 3 & -4 & 3 \\ 6 & 3 & 0 \end{bmatrix}$$

The determinant of the bordered Hessian is

$$\begin{aligned} &(-1)^2 \times (-4) \begin{vmatrix} -4 & 3 \\ 3 & 0 \end{vmatrix} + (-1)^3 \times 3 \begin{vmatrix} 3 & 3 \\ 6 & 0 \end{vmatrix} + (-1)^4 \times 6 \begin{vmatrix} 3 & -4 \\ 6 & 3 \end{vmatrix} \\ &= -4 \times (-9) - 3 \times (-18) + 6 \times 33 \\ &= 288 > 0 \end{aligned}$$

The condition for a maximum is that $(-1)^2|HB| > 0$, so this point $x_1 = 11,\ x_2 = 10$ is a relative maximum. 1168 is the maximum output level within the given budget constraint.

Problem 5 (50 points). Consider a consumer with a utility function given by

$$u = u(x_1, x_2) = x_1^{1/3} x_2^{1/5}$$

The consumer faces prices and a utility target given by

$$\begin{aligned} p_1 &= 32 \\ p_2 &= 75 \\ u_0 &= 10 \end{aligned}$$

Find the minimum cost way to obtain the given level of utility. Verify that this cost is a minimum. Some of the optimal values are as follows

$$\begin{aligned} x_2 &= 32 \\ \frac{\partial^2 \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2 \partial x_1} &= \frac{-1}{5}, \qquad \frac{\partial g(x_1, x_2)}{\partial x_2} = \frac{1}{16} \\ |H_B| &= \frac{-1}{375} \end{aligned}$$

First, we must find the equation of the constraint

$$\begin{aligned} x_1^{1/3} x_2^{1/5} &= U_0 = 10 \\ &\Rightarrow x_1^{1/3} x_2^{1/5} - 10 = 0 \end{aligned}$$

Then to minimize the cost with utility of 10 given these prices we set up the following Lagrangian

$$\begin{aligned} L &= P_1 x_1 + P_2 x_2 - \lambda(x_1^{1/3} x_2^{1/5} - 10) \\ L &= 32 x_1 + 75 x_2 - \lambda(x_1^{1/3} x_2^{1/5} - 10) \\ \frac{\partial L}{\partial x_1} &= 32 - \frac{1}{3}\lambda x_1^{-2/3} x_2^{1/5} = 0 \\ \frac{\partial L}{\partial x_2} &= 75 - \frac{1}{5}\lambda x_1^{1/3} x_2^{-4/5} = 0 \\ \frac{\partial L}{\partial \lambda} &= -x_1^{1/3} x_2^{1/5} + 10 = 0 \end{aligned}$$

Since we have already been given the optimal value of x_2 is 32, substituting in the last equation of the first order conditions, we obtain:

$$\begin{aligned} -x_1^{1/3}(32)^{1/5} + 10 &= 0 \\ &\Rightarrow x_1^{1/3} = 5 \\ &\Rightarrow x_1 = 125 \end{aligned}$$

Plug $x_1 = 125,\ x_2 = 32$ in the first equation of the first order conditions, we get

$$\begin{aligned} 32 - \frac{1}{3}\lambda x_1^{-2/3} x_2^{1/5} &= 0 \\ 32 - \frac{1}{3}\lambda (125)^{-2/3}(32)^{1/5} &= 0 \\ &\Rightarrow \lambda \times \frac{1}{25} \times 2 = 32 \times 3 \\ &\Rightarrow \lambda = 1200 \end{aligned}$$

We can also find the minimum cost by substituting in for x_1 and x_2.

$$\begin{aligned} Cost &= P_1 x_1 + P_2 x_2 \\ &= 32 \times 125 + 75 \times 32 \\ &= 6400 \end{aligned}$$

To verify that $x_1 = 125$ and $x_2 = 32$ are minimums, set up a bordered Hessian matrix of the Lagrangian.

$$det \begin{bmatrix} \frac{\partial^2 \mathcal{L}(x_1,x_2,\lambda)}{\partial x_1 \partial x_1} & \frac{\partial^2 \mathcal{L}(x_1,x_2,\lambda)}{\partial x_1 \partial x_2} & \frac{\partial g(x_1,x_2)}{\partial x_1} \\ \frac{\partial^2 \mathcal{L}(x_1,x_2,\lambda)}{\partial x_2 \partial x_1} & \frac{\partial^2 \mathcal{L}(x_1,x_2,\lambda)}{\partial x_2 \partial x_2} & \frac{\partial g(x_1,x_2)}{\partial x_2} \\ \frac{\partial g(x_1,x_2)}{\partial x_1} & \frac{\partial g(x_1,x_2)}{\partial x_2} & 0 \end{bmatrix}$$

We compute the various elements of the bordered Hessian as follows

$$\begin{aligned} L &= 32x_1 \ + \ 75x_2 \ - \ \lambda(x_1^{1/3}x_2^{1/5} - 10) \\ \frac{\partial L}{\partial x_1} &= 32 - \frac{1}{3}\lambda x_1^{-2/3} x_2^{1/5} \\ \frac{\partial L}{\partial x_2} &= 75 - \frac{1}{5}\lambda x_1^{1/3} x_2^{-4/5} \\ \frac{\partial^2 \mathcal{L}(x_1,x_2,\lambda)}{\partial x_1 \partial x_1} &= \frac{64}{375} \quad , \frac{\partial^2 \mathcal{L}(x_1,x_2,\lambda)}{\partial x_2 \partial x_2} = \frac{15}{8} \\ \frac{\partial^2 \mathcal{L}(x_1,x_2,\lambda)}{\partial x_2 \partial x_1} &= \frac{\partial^2 \mathcal{L}(x_1,x_2,\lambda)}{\partial x_1 \partial x_2} = \frac{-1}{5} \\ \frac{\partial g(x_1,x_2)}{\partial x_1} &= \frac{2}{75}, \qquad \frac{\partial g(x_1,x_2)}{\partial x_2} = \frac{1}{16} \end{aligned}$$

The bordered Hessian is given by

$$\begin{aligned} HB &= \begin{bmatrix} \frac{64}{375} & \frac{-1}{5} & \frac{2}{75} \\ \frac{-1}{5} & \frac{15}{8} & \frac{1}{16} \\ \frac{2}{75} & \frac{1}{16} & 0 \end{bmatrix} \\ Det(HB) &= \frac{-1}{375} < 0 \end{aligned}$$

The condition for a minimum is that $(-1)^2|HB| < 0$, so this point $x_1 = 125,\ x_2 = 32$ is a relative minimum. 6400 is the minimum cost to get the utility level of 10.