Problem 1. For each of the following systems of equations, find the solution vector \( \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \) by appending the right hand side vector to the coefficient matrix and performing row reduction.

a. 
\[
\begin{pmatrix} 2 & 3 \\ 4 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 13 \\ 2 \end{pmatrix}
\]
b. 

\[
\begin{pmatrix}
3 & 1 \\
6 & -3
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2
\end{pmatrix}
=
\begin{pmatrix}
5 \\
0
\end{pmatrix}
\]
Problem 2. Solve the following system of equations.

\[
\begin{align*}
8x_1^{-1/2}x_2^{1/4} - 4 &= 0 \\
4x_1^{1/2}x_2^{-3/4} - 1 &= 0
\end{align*}
\]
Problem 3. For the following competitive firm, find producer surplus at the given price.

\[ \text{price} = p = $589 \]
\[ \text{cost} = c(y) = 300 + 400y - 30y^2 + 3y^3 \]
Problem 4. For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise. Check to see if there are points of inflection at points other than critical points.

a. \( f(x) = 3x^4 - 16x^3 + 18x^2 \)

b. \( y = x^3 - 3x^2 + 15 \)
c. \( g(x) = x^3 + 3x^2 - 24x \)

d. \( f(x) = \frac{x}{x^2 + 1} \)
Problem 5. Consider the following matrices.

\[
A_2 = \begin{bmatrix} 4 & 2 \\ 3 & 2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -1 & 1 \\ 2 & -3 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}
\]

\[
D = \begin{bmatrix} 2 & -1 & 4 \\ 1 & 0 & 2 \\ 4 & -1 & 7 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & -3 & 2 \\ -2 & 5 & -2 \\ 4 & -11 & 7 \end{bmatrix}, \quad F = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 5 & 7 \end{bmatrix}
\]

a. Find the determinant of the matrix \(A_2\).

b. Find the inverse of the matrix \(A_2\) using the adjoint method.

c. Find the inverse of the matrix \(A_2\) using row reduction.
d. Find the determinant of the matrix B2.

e. Find the inverse of the matrix B2 using the adjoint method.

f. Find the inverse of the matrix B2 using row reduction.
g. Find the determinant of the matrix C2.

h. Find the inverse of the matrix C2 using the adjoint method.

i. Find the inverse of the matrix C2 using row reduction.
j. Find the determinant of the matrix F.

k. Find the inverse of the matrix F using the adjoint method.
1. Find the inverse of the matrix F using row reduction.
Problem 6. Consider the following matrices.

\[
A = \begin{bmatrix}
1 & 2 & 1 \\
2 & 5 & 2 \\
-3 & -4 & -2
\end{bmatrix}, \quad
B = \begin{bmatrix}
1 & 2 & 3 \\
0 & 5 & 2 \\
1 & 0 & 4
\end{bmatrix}, \quad
C = \begin{bmatrix}
1 & 2 & 2 \\
0 & 4 & 1 \\
1 & 0 & 2
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
2 & -1 & 4 \\
1 & 0 & 2 \\
4 & -1 & 7
\end{bmatrix}, \quad
E = \begin{bmatrix}
1 & -3 & 2 \\
-2 & 5 & -2 \\
4 & -11 & 7
\end{bmatrix}, \quad
F = \begin{bmatrix}
1 & 2 & 3 \\
2 & 3 & 5 \\
3 & 5 & 7
\end{bmatrix}
\]

a. Find the determinant of the matrix C, then find its inverse using the adjoint method. Multiply the matrix C by its inverse to verify that your answer is correct.
b. Find inverse of the matrix D using the row reduction. Multiply the matrix D by its inverse to verify that your answer is correct.
c. Find inverse of the matrix $E$ using the row reduction. Multiply the matrix $E$ by its inverse to verify that your answer is correct.