Problem 1. The cost function for a firm is a rule or mapping that tells the total cost of production of any output level produced by the firm. If \( y \) represents the output of the firm, then the cost function is given by \( c(y) \). Consider the following competitive firm.

\[
\begin{align*}
\text{price} & = p = 600 \\
\text{cost} & = c(y) = 200 + 400y - 20y^2 + 2y^3
\end{align*}
\]

a. What is the level of fixed cost for this firm?

b. Write a function representing the revenue of the firm as a function of the given output price and output level.

c. Write an equation that gives the variable cost of the firm as a function of the level of output.

d. Write a function representing the profit of the firm as a function of the given output price and output level.
e. What is the profit maximizing level of output for this firm? Verify that the output level you choose is the profit maximizing point.

h. What is the marginal cost of the firm at this output level?
Problem 2. Find the critical values for each of the following functions. Check the second order conditions to determine whether the critical values indicate a maximum, a minimum, or something else.

a. 
\[ y = 40x_1 + 30x_2 + 2x_1^2 + 2x_1x_2 + x_2^2 \]
b. 

\[ y = 46x_1 - x_1^2 + 40x_2 + 4x_1x_2 - x_2^2 \]
c. \[ y = -20x_1 - 10x_2 + x_1^2 + 2x_1x_2 + x_2^2 \]
d. 

\[ y = 40x_1^{3/5}x_2^{1/5} - 12x_1 - 8x_2 \]
Problem 3. For each of the following problems, write an equation that represents profit as a function of the two inputs $x_1$ and $x_2$. Output price is represented by $p$, the price of the first input by $w_1$ and the price of the second input as $w_2$. Write it in the form $\pi = pf(x_1,x_2) - w_1x_1 - w_2x_2$ and then simplify the expression. Then find critical values of $x_1$ and $x_2$. Verify that profit is maximized at these critical values.

a. 
\[ f(x_1, x_2) = 60x_1 + 42x_2 - x_1^2 + x_1x_2 - x_2^2 \]
\[ p = 4 \]
\[ w_1 = 60, \quad w_2 = 12 \]
b. 

\[ f(x_1, x_2) = 20x_1 + 14x_2 - 2x_1^2 + x_1x_2 - x_2^2 \]

\[ p = 4 \]

\[ w_1 = 60, \quad w_2 = 12 \]
c. 

\[ f(x_1, x_2) = 20x_1 + 14x_2 - 2x_1^2 + 2x_1x_2 - x_2^2 \]

\[ p = 5 \]

\[ w_1 = 10, \quad w_2 = 20 \]
d. 

\[ f(x_1, x_2) = x_1^{1/3} x_2^{1/6} \]

\[ p = 60 \]

\[ w_1 = 4, \quad w_2 = 2 \]
e.

\[ f(x_1, x_2) = x_1^{1/2} x_2^{1/6} \]

\[ p = 48 \]

\[ w_1 = 4, \quad w_2 = 3 \]
f. 

\[ f(x_1, x_2) = x_1^{2/5} x_2^{1/5} \]

\[ p = 20 \]

\[ w_1 = 2, \quad w_2 = 4 \]