Problem 1. For each of the following systems of equations, find the solution vector \( \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \) by appending the right hand side vector to the coefficient matrix and performing row reduction.

a. \[
\begin{pmatrix} 2 & -4 \\ 6 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 8 \\ 10 \end{pmatrix}
\]
b. 

\[
\begin{pmatrix}
4 & 1 \\
2 & 3 \\
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
\end{pmatrix}
=
\begin{pmatrix}
6 \\
-2 \\
\end{pmatrix}
\]
Problem 2. Solve the following system of equations.

\[ 6x_1^{-1/3}x_2^{1/6} - 3 = 0 \]
\[ \frac{3}{2}x_1^{2/3}x_2^{-5/6} - 2 = 0 \]
Problem 3 (32 points).

The cost function for a firm is a rule or mapping that tells the total cost of production of any output level produced by the firm. If the variable $y$ represents the output of the firm, then the cost function is given by $c(y)$. Then consider the following competitive firm.

\[
\text{price } = p = 566 \\
\text{cost } = c(y) = 300 + 500y - 30y^2 + 2y^3
\]

a. What is the level of fixed cost for this firm?

b. Write a function representing the revenue of the firm as a function of price and output level. Then substitute in the actual price the firm faces.

c. Write an equation that gives the variable cost of the firm as a function of the level of output.

d. Write a function representing the profit of the firm as a function of output price and output level. Then substitute in the actual price the firm faces.
e. What is the profit maximizing level of output for this firm? Verify that the output level you choose is the only profit maximizing point.
f. What is the total cost of the firm at this output level?


g. Show that the revenue of this profit maximizing firm is $6,226.


h. What is the marginal cost of the firm at this output level?


i. Using the marginal cost function from part h and revenue from part g, use integration to find producer surplus for this firm.


j. What is the profit level for this firm?
Problem 4. For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise. Check to see if there are points of inflection at points other than critical points.

a. \( f(x) = \frac{1}{2}x^4 - \frac{17}{4}x^3 - \frac{55}{2}x^2 \)

b. \( y = (x^2 - 3)e^x \)
Problem 5. Consider the following matrices.

\[ D_2 = \begin{bmatrix} 5 & 6 \\ 3 & 4 \end{bmatrix}, \quad G = \begin{bmatrix} 3 & -3 & 4 \\ 1 & -2 & 2 \\ -6 & 5 & -9 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 3 & 2 & 2 \\ -2 & 1 & -2 \end{bmatrix} \]

a. Find the determinant of the matrix \( D_2 \), then find its inverse using the adjoint method. Multiply the matrix \( D_2 \) by its inverse to verify that your answer is correct.
b. Find the determinant of the matrix $G$, then find its inverse using the adjoint method. Multiply the matrix $G$ by its inverse to verify that your answer is correct.
c. Find inverse of the matrix $H$ using the row reduction. Multiply the matrix $H$ by its inverse to verify that your answer is correct.