Problem 1. Solve the following system of equations.

\[ 4x_1^{1/2}x_2^{3/8} - 1 = 0 \]
\[ 3x_1^{1/2}x_2^{-5/8} - 3 = 0 \]
Problem 2. Find the derivatives of each of the following functions with respect to $x$.

a. $y = \frac{\ln(x^2) - 3x}{2x^2 + e^x}$

b. $y = (2x^4 + 5x^3 - x^2 - 2)(2x^2 + 7x)$

c. $y = e^{2x^2} + 4x(3x^2 - 2x + 5)^3$
**Problem 3.** In the following problem you are given a production function for a firm where $y$ is the variable representing the level of output and $x$ is the level of the variable input. You are given the price ($p$) of the output and the price ($w$) of the single variable input. The function representing output as a function of input is given by

$$output = y = f(x) = 20x + 200x^2 - 3x^3$$

a. Write a function representing the revenue of the firm as a function of price and output level.

$$Revenue =$$

b. Write a composite function representing the revenue of the firm as a function of price and input level.

$$Revenue =$$

c. Write a function representing the cost of the firm as a function of input price and input level.

$$Cost =$$

d. Write a function representing the profit of the firm as a function of input price, output price and input level.

$$Profit =$$
e. Assume that the price of output for this firm is \( p = 3 \). Assume that the price of the input \( w = 8268 \).

Write an equation for the profit of this firm that depends on the input level.

\[
\text{Profit} = \]

f. Maximize profit by taking the derivative of the function in part e with respect to \( x \), setting it equal to zero, and solving for the input level \( x \).
Problem 4. Find the second derivative of each of the following functions with respect to the indicated variable

a. \( y = 60x + 600x^2 - 9x^3 - 8268, \ x \)

b. \( y = 1000x + 400x^2 - 20x^3 - 1960, \ x \)

c. \( y = 16x^{3/8}z^{1/4} - 3x - 2z, \ x \)
d. \( y = 16x^{3/8}z^{1/4} - 3x - 2z, \quad z \)

e. \( y = 8x_1^{1/2}x_2^{3/8} - x_1 - 3x_2, \quad x_1 \)

f. \( y = 8x_1^{1/2}x_2^{3/8} - x_1 - 3x_2, \quad x_2 \)
Problem 5. Find the indefinite integral of each of the following functions with respect to the variable indicated. Write in the form $F(x) + c$.

a. $f(x) = 6x^2 + 2x - 5$, $x$

b. $f(x) = -\frac{5}{7}x^{-2/7}$, $x$

c. $4x_1^{-1/2}x_2^{3/8} - 1$, $x_1$
Problem 6. Find the definite integral of each of the problems.

a. \( \int_{1}^{4} (5x + 4) \, dx \)

b. \( \int_{1}^{5} (3x^2 - 4x + 2) \, dx \)

c. \( \int_{0}^{10} (9x^2 - 60x + 20) \, dx \)