ECONOMICS 207
SPRING 2006
LABORATORY EXERCISE 12

Consider the following matrices and vectors.

$$R = \begin{bmatrix} 4 & -1 \\ 6 & -2 \end{bmatrix}, \quad S = \begin{bmatrix} -3 & 2 \\ 2 & -2 \end{bmatrix}, \quad T = \begin{bmatrix} 1 & -2 & -3 \\ -4 & 9 & 6 \\ 2 & -6 & 5 \end{bmatrix},$$

$$r = \begin{bmatrix} 6 \\ 10 \end{bmatrix}, \quad s = \begin{bmatrix} 18 \\ -14 \end{bmatrix}, \quad t = \begin{bmatrix} -14 \\ 40 \\ 1 \end{bmatrix},$$

Problem 1. a Find the determinant of the matrix R.

$$\begin{aligned} Det(R) &= 4 \times (-2) - (-1) \times 6 \\ &= -8 + 6 = -2 \end{aligned}$$

b Find the inverse of the matrix R using the adjoint method.

$$\begin{aligned} Adj(R) &= \begin{bmatrix} -2 & 1 \\ -6 & 4 \end{bmatrix} \\ Inv(R) &= \frac{1}{Det(R)} Adj(R) \\ &= -\frac{1}{2} \begin{bmatrix} -2 & 1 \\ -6 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -\frac{1}{2} \\ 3 & -2 \end{bmatrix} \end{aligned}$$

Date: 11 September 2006.

c Using the inverse from part b, solve the system of equations

$$R\begin{pmatrix}x_1\\x_2\end{pmatrix} = r$$

$$\begin{aligned}
R\begin{pmatrix}x_1\\x_2\end{pmatrix} &= r\\
&\Rightarrow \begin{pmatrix}x_1\\x_2\end{pmatrix} = R^{-1}\times r\\
&\Rightarrow \begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{bmatrix}1 & -\frac{1}{2}\\ 3 & -2\end{bmatrix}\times\begin{pmatrix}6\\10\end{pmatrix}\\
&\Rightarrow \begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}1\\-2\end{pmatrix}
\end{aligned}$$

d Using row reduction, find the inverse of the matrix R and the solution to the system of equations

$$R\begin{pmatrix}x_1\\x_2\end{pmatrix} = r$$

$$\begin{aligned}
\begin{pmatrix}4 & -1 & 1 & 0 & 6\\ 6 & -2 & 0 & 1 & 10\end{pmatrix} &\rightarrow \begin{pmatrix}1 & 0 & 1 & -\frac{1}{2} & 1\\ 6 & -2 & 0 & 1 & 10\end{pmatrix}\rightarrow\begin{pmatrix}1 & 0 & 1 & -\frac{1}{2} & 1\\ 0 & -2 & -6 & 4 & 4\end{pmatrix}\\
&\rightarrow \begin{pmatrix}1 & 0 & 1 & -\frac{1}{2} & 1\\ 0 & 1 & 3 & -2 & -2\end{pmatrix}\Rightarrow\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}1\\-2\end{pmatrix}\\
Inv(R) &= \begin{bmatrix}1 & -\frac{1}{2}\\ 3 & -2\end{bmatrix}
\end{aligned}$$

Problem 2. a Find the determinant of the matrix S.

$$\begin{aligned} Det(S) &= -3 \times (-2) - 2 \times 2 \\ &= 6 - 4 = 2 \end{aligned}$$

b Find the inverse of the matrix S using the adjoint method.

$$\begin{aligned} Adj(S) &= \begin{bmatrix} -2 & -2 \\ -2 & -3 \end{bmatrix} \\ Inv(S) &= \frac{1}{Det(S)} Adj(S) \\ &= \frac{1}{2} \begin{bmatrix} -2 & -2 \\ -2 & -3 \end{bmatrix} \\ &= \begin{bmatrix} -1 & -1 \\ -1 & -\frac{3}{2} \end{bmatrix} \end{aligned}$$

c Using the inverse from part b, solve the system of equations

$$S\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = s$$

$$\begin{aligned} S\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} &= s \\ \Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} &= S^{-1} \times s \\ \Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} &= \begin{bmatrix} -1 & -1 \\ -1 & -\frac{3}{2} \end{bmatrix} \times \begin{pmatrix} 18 \\ -14 \end{pmatrix} \\ \Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} &= \begin{pmatrix} -4 \\ 3 \end{pmatrix} \end{aligned}$$

d Using row reduction, find the inverse of the matrix S and the solution to the system of equations

$$S\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = s$$

$$\begin{aligned} \begin{pmatrix} -3 & 2 & 1 & 0 & 18 \\ 2 & -2 & 0 & 1 & -14 \end{pmatrix} &\rightarrow \begin{pmatrix} -1 & 0 & 1 & 1 & 4 \\ 2 & -2 & 0 & 1 & -14 \end{pmatrix} \rightarrow \begin{pmatrix} -1 & 0 & 1 & 1 & 4 \\ 0 & -2 & 2 & 3 & -6 \end{pmatrix} \\ &\rightarrow \begin{pmatrix} 1 & 0 & -1 & -1 & -4 \\ 0 & 1 & -1 & -\frac{3}{2} & 3 \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} \\ Inv(S) &= \begin{bmatrix} -1 & -1 \\ -1 & -\frac{3}{2} \end{bmatrix} \end{aligned}$$

Problem 3. a Find the determinant of the matrix T.

$$\begin{aligned} Det(T) &= (-1)^2 \begin{vmatrix} 9 & 6 \\ -6 & 5 \end{vmatrix} + (-1)^3 \times (-2) \begin{vmatrix} -4 & 6 \\ 2 & 5 \end{vmatrix} + (-1)^4 (-3) \begin{vmatrix} -4 & 9 \\ 2 & -6 \end{vmatrix} \\ &= (45 + 36) + 2(-20 - 12) - 3(24 - 18) \\ &= 81 - 64 - 18 = 81 - 82 = -1 \end{aligned}$$

b Find the inverse of the matrix T using the adjoint method.

$$\begin{aligned} Adj(T) &= \begin{bmatrix} \begin{vmatrix} 9 & 6 \\ -6 & 5 \end{vmatrix} & -\begin{vmatrix} -2 & -3 \\ -6 & 5 \end{vmatrix} & \begin{vmatrix} -2 & -3 \\ 9 & 6 \end{vmatrix} \\ -\begin{vmatrix} -4 & 6 \\ 2 & 5 \end{vmatrix} & \begin{vmatrix} 1 & -3 \\ 2 & 5 \end{vmatrix} & -\begin{vmatrix} 1 & -3 \\ -4 & 6 \end{vmatrix} \\ \begin{vmatrix} -4 & 9 \\ 2 & -6 \end{vmatrix} & -\begin{vmatrix} 1 & -2 \\ 2 & -6 \end{vmatrix} & \begin{vmatrix} 1 & -2 \\ -4 & 9 \end{vmatrix} \end{bmatrix} \\ &= \begin{bmatrix} 81 & 28 & 15 \\ 32 & 11 & 6 \\ 6 & 2 & 1 \end{bmatrix} \\ Inv(T) &= \frac{1}{Det(T)} Adj(T) \\ &= \frac{1}{-1} \begin{bmatrix} 81 & 28 & 15 \\ 32 & 11 & 6 \\ 6 & 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -81 & -28 & -15 \\ -32 & -11 & -6 \\ -6 & -2 & -1 \end{bmatrix} \end{aligned}$$

c Using the inverse from part b, solve the system of equations

$$T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = t$$

$$\begin{aligned}
T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} &= t \\
\Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} &= T^{-1} \times t \\
\Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} &= \begin{bmatrix} -81 & -28 & -15 \\ -32 & -11 & -6 \\ -6 & -2 & -1 \end{bmatrix} \times \begin{pmatrix} -14 \\ 40 \\ 1 \end{pmatrix} \\
\Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} &= \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}
\end{aligned}$$

d Using row reduction, find the inverse of the matrix T and the solution to the system of equations

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = t$$

$$\left(\begin{array}{ccccccc} 1 & -2 & -3 & 1 & 0 & 0 & -14 \\ -4 & 9 & 6 & 0 & 1 & 0 & 40 \\ 2 & -6 & 5 & 0 & 0 & 1 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccccccc} 1 & -2 & -3 & 1 & 0 & 0 & -14 \\ 0 & 1 & -6 & 4 & 1 & 0 & -16 \\ 2 & -6 & 5 & 0 & 0 & 1 & 1 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccccccc} 1 & -2 & -3 & 1 & 0 & 0 & -14 \\ 0 & 1 & -6 & 4 & 1 & 0 & -16 \\ 0 & -2 & 11 & -2 & 0 & 1 & 29 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccccccc} 1 & -2 & -3 & 1 & 0 & 0 & -14 \\ 0 & 1 & -6 & 4 & 1 & 0 & -16 \\ 0 & 0 & 1 & -6 & -2 & -1 & 3 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccccccc} 1 & -2 & -3 & 1 & 0 & 0 & -14 \\ 0 & 1 & 0 & -32 & -11 & -6 & 2 \\ 0 & 0 & 1 & -6 & -2 & -1 & 3 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccccccc} 1 & 0 & 0 & -81 & -28 & -15 & -1 \\ 0 & 1 & 0 & -32 & -11 & -6 & 2 \\ 0 & 0 & 1 & -6 & -2 & -1 & 3 \end{array}\right) \Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$$

$$Inv(T) = \begin{bmatrix} -81 & -28 & -15 \\ -32 & -11 & -6 \\ -6 & -2 & -1 \end{bmatrix}$$

Problem 4. Solve the following system of equations.

$$\frac{10}{3}x_1^{-2/3}x_2^{3/5} - 10 = 0$$
$$6x_1^{1/3}x_2^{-2/5} - 2 = 0$$

First equation divided by the second equation, we have:

$$\frac{10}{2} = \frac{\frac{10}{3}x_1^{-2/3}x_2^{3/5}}{6x_1^{1/3}x_2^{-2/5}} \Rightarrow 5 = \frac{5}{9}\frac{x_2}{x_1}$$
$$\Rightarrow x_2 = 9x_1$$

Substituting $x_2 = 9x_1$ into the second equation, we obtain:

$$\begin{aligned} 6x_1^{1/3}(9x_1)^{-2/5} &= 2 \\ &\Rightarrow 3^{-4/5}x_1^{-1/15} = 3^{-1} \\ &\Rightarrow x_1^{-1/15} = 3^{-1/5} \\ &\Rightarrow x_1 = (3^{-1/5})^{-15} = 3^3 = 27 \\ x_2 &= 9x_1 = 9 \times 27 = 243 \end{aligned}$$

So, $x_1 = 27, \quad x_2 = 243$.

Problem 5. Find all first and second partial derivatives of each of the following

a $f(x_1, x_2) = -2x_1^2 - x_2^2 + 20x_1 + 30x_2$

$\frac{\partial f}{\partial x_1} = -4x_1 + 20$	$\frac{\partial f}{\partial x_2} = -2x_2 + 30$
$\frac{\partial^2 f}{\partial x_1 \partial x_1} = -4$	$\frac{\partial^2 f}{\partial x_1 \partial x_2} = 0$
$\frac{\partial^2 f}{\partial x_2 \partial x_1} = 0$	$\frac{\partial^2 f}{\partial x_2 \partial x_2} = -2$

b $f(x_1, x_2) = -x_1^2 + 4x_1x_2 - 2x_2^2 + 25x_1 + 40x_2$

$\frac{\partial f}{\partial x_1} = -2x_1 + 4_x2 + 25$	$\frac{\partial f}{\partial x_2} = 4x_1 - 4x_2 + 40$
$\frac{\partial^2 f}{\partial x_1 \partial x_1} = -2$	$\frac{\partial^2 f}{\partial x_1 \partial x_2} = 4$
$\frac{\partial^2 f}{\partial x_2 \partial x_1} = 4$	$\frac{\partial^2 f}{\partial x_2 \partial x_2} = -4$

c $f(x_1, x_2, x_3) = 10x_1^{1/2}x_2^{1/5}x_3^{1/3} - 4x_1 - 3x_2 - 5x_3$

$\frac{\partial f}{\partial x_1} = 5x_1^{-1/2}x_2^{1/5}x_3^{1/3} - 4$	$\frac{\partial f}{\partial x_2} = 2x_1^{1/2}x_2^{-4/5}x_3^{1/3} - 3$	$\frac{\partial f}{\partial x_3} = 10/3x_1^{1/2}x_2^{1/5}x_3^{-2/3} - 5$
$\frac{\partial^2 f}{\partial x_1 \partial x_1} = -5/2x_1^{-3/2}x_2^{1/5}x_3^{1/3}$	$\frac{\partial^2 f}{\partial x_1 \partial x_2} = x_1^{-1/2}x_2^{-4/5}x_3^{1/3}$	$\frac{\partial^2 f}{\partial x_1 \partial x_3} = -5/3x_1^{-1/2}x_2^{1/5}x_3^{-2/3}$
$\frac{\partial^2 f}{\partial x_2 \partial x_1} = x_1^{-1/2}x_2^{-4/5}x_3^{1/3}$	$\frac{\partial^2 f}{\partial x_2 \partial x_2} = -8/5x_1^{1/2}x_2^{-9/5}x_3^{1/3}$	$\frac{\partial^2 f}{\partial x_2 \partial x_3} = 2/3x_1^{1/2}x_2^{-4/5}x_3^{-2/3}$
$\frac{\partial^2 f}{\partial x_3 \partial x_1} = 5/3x_1^{-1/2}x_2^{1/5}x_3^{-2/3}$	$\frac{\partial^2 f}{\partial x_3 \partial x_2} = 2/3x_1^{1/2}x_2^{-4/5}x_3^{-2/3}$	$\frac{\partial^2 f}{\partial x_3 \partial x_3} = -20/9x_1^{1/2}x_2^{1/5}x_3^{-5/3}$

d $f(x_1, x_2, x_3) = 10x_1^{1/3}x_2^{1/2} + 2x_1x_2^2 - 3x_2x_1^3 + 4x_1x_2 - 5x_3x_1^2 + 3x_3$

$\frac{\partial f}{\partial x_1} = 10/3x_1^{-2/3}x_2^{1/2} + 2x_2^2 - 9x_2x_1^2 + 4x_2 - 10x_3x_1$	$\frac{\partial f}{\partial x_2} = 5x_1^{1/3}x_2^{-1/2} + 4x_1x_2 - 3x_1^3 + 4x_1$	$\frac{\partial f}{\partial x_3} = -5x_1^2 + 3$
$\frac{\partial^2 f}{\partial x_1 \partial x_1} = -20/9x_1^{-5/3}x_2^{1/2} - 18x_2x_1 - 10x_3$	$\frac{\partial^2 f}{\partial x_1 \partial x_2} = 5/3x_1^{-2/3}x_2^{-1/2} + 4x_2 - 9x_1^2 + 4$	$\frac{\partial^2 f}{\partial x_1 \partial x_3} = -10x_1$
$\frac{\partial^2 f}{\partial x_2 \partial x_1} = 5/3x_1^{-2/3}x_2^{-1/2} + 4x_2 - 9x_1^2 + 4$	$\frac{\partial^2 f}{\partial x_2 \partial x_2} = -5/2x_1^{1/3}x_2^{-3/2} + 4x_1$	$\frac{\partial^2 f}{\partial x_2 \partial x_3} = 0$
$\frac{\partial^2 f}{\partial x_3 \partial x_1} = -10x_1$	$\frac{\partial^2 f}{\partial x_3 \partial x_2} = 0$	$\frac{\partial^2 f}{\partial x_3 \partial x_3} = 0$

Problem 6. For each of the following problems, write an equation that represents profit as a function of the two inputs x_1 and x_2. Write it in the form $\pi = pf(x_1,x_2) - w_1x_1 - w_2x_2$ and then simplify the expression. Then find all first and second partial derivatives of the function at the specified point.

a.

$$f(x_1,x_2) = 60x_1 + 42x_2 - x_1^2 + x_1x_2 - x_2^2$$
$$p = 4$$
$$w_1 = 60, \quad w_2 = 12$$
$$x_1 = 43, \quad x_2 = 41$$

Profit function:

$$\begin{aligned}\pi &= pf(x_1,x_2) - w_1x_1 - w_2x_2\\ &= 4(60x_1 + 42x_2 - x_1^2 + x_1x_2 - x_2^2) - 60x_1 - 12x_2\\ &= -4x_1^2 + 4x_1x_2 - 4x_2^2 + 180x_1 + 156x_2\end{aligned}$$

$\frac{\partial \pi}{\partial x_1} = -8x_1 + 4x_2 + 180 = -8 \times 43 + 4 \times 41 + 180 = 0$	$\frac{\partial \pi}{\partial x_2} = 4x_1 - 8x_2 + 156 = 4 \times 43 - 8 \times 41 + 156 = 0$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1} = -8$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2} = 4$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1} = 4$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2} = -8$

b.

$$\begin{aligned} f(x_1, x_2) &= 20x_1 + 14x_2 - 2x_1^2 + x_1x_2 - x_2^2 \\ p &= 4 \\ w_1 &= 60, \qquad w_2 = 12 \\ x_1 &= 3, \qquad x_2 = 7 \end{aligned}$$

Profit function:

$$\begin{aligned} \pi &= pf(x_1, x_2) - w_1x_1 - w_2x_2 \\ &= 4(20x_1 + 14x_2 - 2x_1^2 + x_1x_2 - x_2^2) - 60x_1 - 12x_2 \\ &= -8x_1^2 + 4x_1x_2 - 4x_2^2 + 20x_1 + 44x_2 \end{aligned}$$

$\frac{\partial \pi}{\partial x_1} = -16x_1 + 4x_2 + 20 = -16 \times 3 + 4 \times 7 + 20 = 0$	$\frac{\partial \pi}{\partial x_2} = 4x_1 - 8x_2 + 44 = 4 \times 3 - 8 \times 7 + 44 = 0$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1} = -16$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2} = 4$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1} = 4$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2} = -8$

c.

$$\begin{aligned} f(x_1, x_2) &= 20x_1 + 14x_2 - 2x_1^2 + 2x_1x_2 - x_2^2 \\ p &= 5 \\ w_1 &= 10, \qquad w_2 = 20 \\ x_1 &= 14, \qquad x_2 = 19 \end{aligned}$$

Profit function:

$$\begin{aligned} \pi &= pf(x_1, x_2) - w_1x_1 - w_2x_2 \\ &= 5(20x_1 + 14x_2 - 2x_1^2 + 2x_1x_2 - x_2^2) - 10x_1 - 20x_2 \\ &= -10x_1^2 + 10x_1x_2 - 5x_2^2 + 90x_1 + 50x_2 \end{aligned}$$

$\frac{\partial \pi}{\partial x_1} = -20x_1 + 10x_2 + 90 = -20 \times 14 + 10 \times 19 + 90 = 0$	$\frac{\partial \pi}{\partial x_2} = 10x_1 - 10x_2 + 50 = 10 \times 14 - 10 \times 19 + 50 = 0$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1} = -20$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2} = 10$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1} = 10$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2} = -10$

d.

$$\begin{aligned} f(x_1,x_2) &= x_1^{1/3}x_2^{3/5} \\ p &= 10 \\ w_1 &= 10, \qquad w_2 = 2 \\ x_1 &= 27, \qquad x_2 = 243 \end{aligned}$$

Profit function:

$$\begin{aligned} \pi &= pf(x_1,x_2) - w_1x_1 - w_2x_2 \\ &= 10(x_1^{1/3}x_2^{3/5}) - 10x_1 - 2x_2 \\ &= 10x_1^{1/3}x_2^{3/5} - 10x_1 - 2x_2 \end{aligned}$$

$\frac{\partial\pi}{\partial x_1} = 10/3x_1^{-2/3}x_2^{3/5} - 10 = 10/3(3^3)^{-2/3}(3^5)^{3/5} - 10 = \frac{10}{3} \times \frac{1}{9} \times 3^3 - 10 = 0$	$\frac{\partial\pi}{\partial x_2} = 6x_1^{1/3}x_2^{-2/5} - 2 = 6(3^3)^{1/3}(3^5)^{-2/5} - 2 = 6 \times 3 \times \frac{1}{9} - 2 = 0$
$\frac{\partial^2\pi}{\partial x_1\partial x_1} = -20/9x_1^{-5/3}x_2^{3/5} = -20/9(3^3)^{-5/3}(3^5)^{3/5} = \frac{-20}{9} \times 3^{-5} \times 3^3 = \frac{-20}{81}$	$\frac{\partial^2\pi}{\partial x_1\partial x_2} = 2x_1^{-2/3}x_2^{-2/5} = 2(3^3)^{-2/3}(3^5)^{-2/5} = 2 \times \frac{1}{9} \times \frac{1}{9} = \frac{2}{81}$
$\frac{\partial^2\pi}{\partial x_2\partial x_1} = 2x_1^{-2/3}x_2^{-2/5} = 2(3^3)^{-2/3}(3^5)^{-2/5} = 2 \times \frac{1}{9} \times \frac{1}{9} = \frac{2}{81}$	$\frac{\partial^2\pi}{\partial x_2\partial x_2} = -12/5x_1^{1/3}x_2^{-7/5} = -12/5(3^3)^{1/3}(3^5)^{-7/5} = \frac{-12}{5} \times 3 \times 3^{-7} = \frac{-4}{1215}$

e.

$$f(x_1, x_2) = x_1^{1/4} x_2^{3/7}$$
$$p = 3024$$
$$w_1 = 224, \qquad w_2 = 243$$
$$x_1 = 81, \qquad x_2 = 128$$

Profit function:

$$\begin{aligned}\pi &= pf(x_1, x_2) - w_1 x_1 - w_2 x_2 \\ &= 3024(x_1^{1/4} x_2^{3/7}) - 224x_1 - 243x_2 \\ &= 3024x_1^{1/4} x_2^{3/7} - 224x_1 - 243x_2\end{aligned}$$

$\frac{\partial \pi}{\partial x_1} = 756x_1^{-3/4} x_2^{3/7} - 224 = 756(3^4)^-3/4(2^7)^3/7 - 224 = 756 \times 3^{-3} \times 2^3 - 224 = 0$	$\frac{\partial \pi}{\partial x_2} = 1296x_1^{1/4} x_2^{-4/7} - 243 = 1296 \times (3^4)^{1/4} \times (2^7)^{-4/7} - 243 = 1296 \times 3 \times 2^{-4} - 243 = 0$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1} = -567x_1^{-7/4} x_2^{3/7} = -567(3^4)^{-7/4}(2^7)^{3/7} = -567 \times 3^{-7} \times 2^3 = -2.07$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2} = 324x_1^{-3/4} x_2^{-4/7} = 324(3^4)^{-3/4}(2^7)^{-4/7} = 324 \times 3^{-3} \times 2^{-4} = 0.75$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1} = 324x_1^{-3/4} x_2^{-4/7} = 324(3^4)^{-3/4}(2^7)^{-4/7} = 324 \times 3^{-3} \times 2^{-4} = 0.75$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2} = \frac{-5184}{7} x_1^{1/4} x_2^{-11/7} = \frac{-5184}{7}(3^4)^{1/4}(2^7)^{-11/7} = \frac{-5184}{7} \times 3 \times 2^{-11} = -1.085$

Problem 7. For the following problem, write an equation that represents profit as a function of the input x. Write it in the form π = pf(x)-wx and then simplify the expression. Then find the first and second derivatives of the function. Then find the critical points. For each critical point state whether profit is at a relative maximum, relative minimum, or otherwise. Check to see if there are points of inflection **at points other than** critical points.

$$f(x) = 200x + 40x^2 - 2x^3$$
$$p = 10$$
$$w = 2260$$

Profit function:

$$\begin{aligned} \pi &= pf(x) - wx \\ &= 10(200x + 40x^2 - 2x^3) - 2260x \\ &= -20x^3 + 400x^2 - 260x \end{aligned}$$

The first and second derivatives of the function:

$$\begin{aligned} \frac{\partial \pi}{\partial x} &= -60x^2 + 800x - 260 \\ \frac{\partial^2 \pi}{\partial x^2} &= -120x + 800 \end{aligned}$$

Critical points:

$$\begin{aligned} \frac{\partial \pi}{\partial x} &= -60x^2 + 800x - 260 = 0 \\ &\Rightarrow -20(x - 13)(3x - 1) \\ &\Rightarrow x = 13 \quad or \quad x = 1/3 \end{aligned}$$

Substituting these values into the second derivatives of the fropit function:

$$\begin{aligned} If \quad x &= 13, \quad \frac{\partial^2 \pi}{\partial x^2} = -120x + 800 = -120 \times 13 + 800 = -760 < 0 \\ So, \quad x &= 13 \quad \text{is a relative maximum point.} \\ If \quad x &= 1/3, \quad \frac{\partial^2 \pi}{\partial x^2} = -120x + 800 = -120 \times 1/3 + 800 = 760 > 0 \\ So, \quad x &= 1/3 \quad \text{is a relative minimum point.} \end{aligned}$$

Since there are no other points that satisfy the first derivative function of the profit function equals 0, so there are no points of inflection at points other than critical points.