

ECONOMICS 207
SPRING 2006
LABORATORY EXERCISE 1 KEY

Problem 1. Consider the following six sets.

$$A = \{1, 2, 4\}$$

$$B = \{2, 4, 5\}$$

$$C = \{1, 3, 5\}$$

$$D = \{1, 2, 3, 4, 5, 6, 8\}$$

$$E = \{1, 2, 4, 6, 8, 10\}$$

$$F = \{0, 2, 3, 4, 6, 7, 8\}$$

$$X = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

a A is a subset of which other sets?

A is a subset of D, E, X.

b What is $A \cap B$?

$$A \cap B = \{2, 4\}$$

c What is $C \cap F$?

$$C \cap F = \{3\}$$

d What is $A \cap B \cap C$?

$$A \cap B \cap C = \emptyset$$

e What is $A \cup (B \cap D)$?

$$\begin{aligned} B \cap D &= \{2, 4, 5\} \\ A \cup (B \cap D) &= \{1, 2, 4, 5\} \end{aligned}$$

f What is $(A \cup B) \cap D$?

$$\begin{aligned} A \cup B &= \{1, 2, 4, 5\} \\ (A \cup B) \cap D &= \{1, 2, 4, 5\} \end{aligned}$$

g What is $(A \cup B) \cap (A \cup D)$?

$$\begin{aligned} A \cup B &= \{1, 2, 4, 5\} \\ A \cup D &= \{1, 2, 3, 4, 5, 6, 8\} \\ (A \cup B) \cap (A \cup D) &= \{1, 2, 4, 5\} \end{aligned}$$

h What is $A \cap (B \cup D)$?

$$\begin{aligned} B \cup D &= \{1, 2, 3, 4, 5, 6, 8\} \\ A \cap (B \cup D) &= \{1, 2, 4\} \end{aligned}$$

i What is $(A \cap B) \cup (A \cap D)$?

$$\begin{aligned} A \cap B &= \{2, 4\} \\ A \cap D &= \{1, 2, 4\} \\ (A \cap B) \cup (A \cap D) &= \{1, 2, 4\} \end{aligned}$$

j What is $(A \cap B) \cup D$?

$$\begin{aligned}A \cap B &= \{2, 4\} \\(A \cap B) \cup D &= \{1, 2, 3, 4, 5, 6, 8\}\end{aligned}$$

k What is $(A \cup D) \cap (B \cup D)$?

$$\begin{aligned}A \cup D &= \{1, 2, 3, 4, 5, 6, 8\} \\B \cup D &= \{1, 2, 3, 4, 5, 6, 8\} \\(A \cup D) \cap (B \cup D) &= \{1, 2, 3, 4, 5, 6, 8\}\end{aligned}$$

l Given X , what is A^C ?

$$A^C = \{0, 3, 5, 6, 7, 8, 9, 10\}$$

m Given X , what is $(A \cup B)^C$?

$$\begin{aligned}A \cup B &= \{1, 2, 4, 5\} \\(A \cup B)^C &= \{0, 3, 6, 7, 8, 9, 10\}\end{aligned}$$

n Given X , what is $(A \cap B)^C$?

$$\begin{aligned}A \cap B &= \{2, 4\} \\(A \cap B)^C &= \{0, 1, 3, 5, 6, 7, 8, 9, 10\}\end{aligned}$$

o What is $D \setminus E$?

$$D \setminus E = \{3, 5\}$$

p What is $F \setminus A$?

$$F \setminus A = \{0, 3, 6, 7, 8\}$$

Problem 2. Consider the following sets.

$$A = \left\{ \frac{a}{b} : a \in \{0, 1, 2, 3, 4\}, -1 \leq b \leq 3 \text{ and } b \in \text{integers}, b \neq 0 \right\}$$

$$B = \{ \{x, y\} : x + y = 5, x < 3 \text{ and } x \in \text{natural numbers}, y \leq 7 \}$$

$$C = \{ \{x, y\} : x + y = 5, x < 10 \text{ and } x \in \text{natural numbers}, y \leq 7 \text{ and } y \in \text{integers} \}$$

$$D = \{ \{x, y\} : x + 2y = 12, x < 10 \text{ and } x \in \text{natural numbers}, y \leq 7 \text{ and } y \in \text{integers} \}$$

$$E = \{ \{x, y\} : x + 2y = 12, x < 8 \text{ and } x \in \text{integers}, y \leq 7 \}$$

$$F = \{ \{x, y\} : 4x + y = -1, x < 2 \text{ and } x \in \text{integers}, y \leq 7 \}$$

$$G = \{ \{x, y\} : x + y = 5, x < 2 \text{ and } x \in \text{integers}, y \leq 7 \}$$

$$X = \{ \{x, y\} : |x| < 10, |y| < 5 \}$$

a. List the elements of each of the sets: A, B, C, D, E, F, G, and X.

$$A = \left\{ 0, -1, 1, \frac{1}{2}, \frac{1}{3}, -2, 2, \frac{2}{3}, -3, 3, \frac{3}{2}, -4, 4, \frac{4}{3} \right\}$$

$$B = \{ \{1, 4\}, \{2, 3\} \}$$

$$C = \{ \{1, 4\}, \{2, 3\}, \{3, 2\}, \{4, 1\}, \{5, 0\}, \{6, -1\}, \{7, -2\}, \{8, -3\}, \{9, -4\} \}$$

$$D = \{ \{2, 5\}, \{4, 4\}, \{6, 3\}, \{8, 2\} \}$$

$$E = \left\{ \{-2, 7\}, \{-1, \frac{13}{2}\}, \{0, 6\}, \{1, \frac{11}{2}\}, \{2, 5\}, \{3, \frac{9}{2}\}, \{4, 4\}, \{5, \frac{7}{2}\}, \{6, 3\}, \{7, \frac{5}{2}\} \right\}$$

$$F = \{ \{-2, 7\}, \{-1, 3\}, \{0, -1\}, \{1, -5\} \}$$

$$G = \{ \{-2, 7\}, \{-1, 6\}, \{0, 5\}, \{1, 4\} \}$$

There are infinite elements in X, hence can not be listed.

b. What is $A \cap B$?

$$A \cap B = \emptyset$$

c. What is $B \cap C$?

$$B \cap C = \{\{1, 4\}, \{2, 3\}\}$$

d. What is $B \cap D$?

$$B \cap D = \emptyset$$

e. What is $B \cap E$?

$$B \cap E = \emptyset$$

f. What is $E \cap F$?

$$E \cap F = \{\{-2, 7\}\}$$

g. What is $E \cap G$?

$$E \cap G = \{\{-2, 7\}\}$$

h. What is $E \cap F \cap G$?

$$E \cap F \cap G = \{\{-2, 7\}\}$$

Problem 3. Simplify the following fractions.

a. $\frac{14}{16}$

$$\frac{14}{16} = \frac{2 \times 7}{2 \times 8} = \frac{7}{8}$$

b. $\frac{105}{77}$

$$\frac{105}{77} = \frac{15 \times 7}{11 \times 7} = \frac{15}{11}$$

c. $\frac{252}{462}$

$$\frac{252}{462} = \frac{6 \times 42}{11 \times 42} = \frac{6}{11}$$

d. $\frac{693}{2520}$

$$\frac{693}{2520} = \frac{7 \times 9 \times 11}{7 \times 9 \times 40} = \frac{11}{40}$$

e. $\frac{79002}{149625}$

$$\frac{79002}{149625} = \frac{7 \times 9 \times 79 \times 66}{7 \times 9 \times 79 \times 125} = \frac{66}{125}$$

f. $\frac{15015}{35343}$

$$\frac{15015}{35343} = \frac{3 \times 7 \times 11 \times 65}{3 \times 7 \times 11 \times 153} = \frac{65}{153}$$

Problem 4. Complete the following operations.

a. $\frac{14}{16} + \frac{3}{4}$

$$\frac{14}{16} + \frac{3}{4} = \frac{7}{8} + \frac{6}{8} = \frac{13}{8}$$

b. $\frac{\frac{105}{77}}{\frac{5}{7}}$

$$\frac{\frac{105}{77}}{\frac{5}{7}} = \frac{105 \times 7}{5 \times 77} = \frac{21 \times 5 \times 7}{5 \times 7 \times 11} = \frac{21}{11}$$

c. $\frac{16}{21} + \frac{3}{7}$

$$\frac{16}{21} + \frac{3}{7} = \frac{16}{21} + \frac{3 \times 3}{7 \times 3} = \frac{16}{21} + \frac{9}{21} = \frac{25}{21}$$

d. $\frac{17}{30} + \frac{7}{8}$

$$\frac{17}{30} + \frac{7}{8} = \frac{17 \times 4}{30 \times 4} + \frac{7 \times 15}{8 \times 15} = \frac{68}{120} + \frac{105}{120} = \frac{173}{120}$$

e. $\frac{6}{7} + \frac{29}{42} + \frac{5}{6}$

$$\frac{6}{7} + \frac{29}{42} + \frac{5}{6} = \frac{6 \times 6}{7 \times 6} + \frac{29}{42} + \frac{5 \times 7}{6 \times 7} = \frac{36}{42} + \frac{29}{42} + \frac{35}{42} = \frac{100}{42} = \frac{50}{21}$$

f. $\frac{7}{8} + \frac{5}{24} + \frac{1}{6} + \frac{15}{36}$

$$\begin{aligned} \frac{7}{8} + \frac{5}{24} + \frac{1}{6} + \frac{15}{36} &= \frac{7 \times 9}{8 \times 9} + \frac{5 \times 3}{24 \times 3} + \frac{1 \times 12}{6 \times 12} + \frac{15 \times 2}{36 \times 2} \\ &= \frac{63}{72} + \frac{15}{72} + \frac{12}{72} + \frac{30}{72} = \frac{120}{72} = \frac{12 \times 2 \times 5}{12 \times 2 \times 3} \\ &= \frac{5}{3} \end{aligned}$$