ECONOMICS 207
SPRING 2006
LABORATORY EXERCISE 2 KEY

Problem 1. Carry out the following long division operations.

a.

b. $14\overline{)935}$

c. $19\overline{)412}$

d. $45\overline{)5924}$

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e. $73 \overline{)6872}$

f. $241 \overline{)58472}$

g. $21 \overline{)38167}$

Problem 2. Complete the square in the following

a. $x^2 + 6x$

$$x^2 + 6x = x^2 + 6x + 9 - 9 = (x+3)^2 - 9$$

b. $x^2 - 12x$

$$x^2 - 12x = x^2 - 12x + 36 - 36 = (x-6)^2 - 36$$

c. $2x^2 - 12x$

$$\begin{aligned} 2x^2 - 12x &= 2(x^2 - 6x + 9 - 9) = 2[(x-3)^2 - 9] \\ &= 2(x-3)^2 - 18 \end{aligned}$$

d. $x^2 - 10x + 20$

$$\begin{aligned} x^2 - 10x + 20 &= x^2 - 10x + 20 + 5 - 5 \\ &= x^2 - 10x + 25 - 5 \\ &= (x-5)^2 - 5 \end{aligned}$$

e. $x^2 + \frac{1}{2}x - \frac{1}{4}$

$$\begin{aligned} x^2 + \frac{1}{2}x - \frac{1}{4} &= x^2 + \frac{1}{2}x + \frac{1}{16} - \frac{1}{16} - \frac{1}{4} \\ &= (x + \frac{1}{4})^2 - \frac{5}{16} \end{aligned}$$

f. $x^2 - 4x(a + bt)$

$$\begin{aligned} x^2 - 4x(a + bt) &= x^2 - 4x(a + bt) + (2(a + bt))^2 - (2(a + bt))^2 \\ &= (x - 2(a + bt))^2 - (2(a + bt))^2 \end{aligned}$$

Problem 3. Simplify, add, subtract, multiply or divide the following fractions. Express all answers in reduced form.

a. $\frac{63}{36} + \frac{7}{8}$

$$\begin{aligned}\frac{63}{36}+\frac{7}{8} &= \frac{63\times 2}{36\times 2}+\frac{7\times 9}{8\times 9}=\frac{126}{72}+\frac{63}{72}\\ &= \frac{126+63}{72}=\frac{189}{72}=\frac{21\times 9}{8\times 9}=\frac{21}{8}\end{aligned}$$

b. $\frac{17}{25} + \frac{7}{30}$

$$\begin{aligned}\frac{17}{25}+\frac{7}{30} &= \frac{17\times 6}{25\times 6}+\frac{7\times 5}{30\times 5}=\frac{102}{150}+\frac{35}{150}\\ &= \frac{102+35}{150}=\frac{137}{150}\end{aligned}$$

c. $\left(\frac{84}{448}\right)\left(\frac{7}{\frac{1}{4}}\right)$

$$\left(\frac{84}{448}\right)\left(\frac{7}{\frac{1}{4}}\right) = \left(\frac{84}{448}\right)\times 28=\frac{84\times 28}{16\times 28}=\frac{4\times 21}{4\times 4}=\frac{21}{4}$$

d. $\frac{1}{12}+\frac{2}{3} + \frac{7}{36} - \frac{5}{16}$

$$\begin{aligned}\frac{1}{12}+\frac{2}{3}+\frac{7}{36}-\frac{5}{16} &= \frac{12}{12\times 12}+\frac{2\times 48}{3\times 48}+\frac{7\times 4}{36\times 4}-\frac{5\times 9}{16\times 9}\\ &= \frac{12+96+28-45}{144}=\frac{91}{144}\end{aligned}$$

e. $\frac{1}{7}+\frac{12}{49} + \frac{6}{105}$

$$\begin{aligned}\frac{1}{7}+\frac{12}{49}+\frac{6}{105} &= \frac{105}{7\times 105}+\frac{12\times 15}{49\times 15}+\frac{6\times 7}{105\times 7}\\ &= \frac{105+180+42}{735}=\frac{327}{735}=\frac{109\times 3}{245\times 3}\\ &= \frac{109}{245}\end{aligned}$$

f. $\frac{7}{121}+\frac{5}{77} + \frac{6}{91}$

$$\begin{aligned}\frac{7}{121}+\frac{5}{77}+\frac{6}{91} &= \frac{7\times 91}{121\times 91}+\frac{5\times 143}{77\times 143}+\frac{6\times 121}{91\times 121}\\ &= \frac{637+715+726}{11011}=\frac{2078}{11011}\end{aligned}$$

Problem 4. Factor the following.

a. $x^2 - 6x + 9$

$$x^2 - 6x + 9 = (x-3)^2$$

b. $x^2 - 36$

$$x^2 - 36 = x^2 - 6^2 = (x-6)(x+6)$$

c. $4x^2 - 24x + 36$

$$\begin{aligned} 4x^2 - 24x + 36 &= 4(x^2 - 6x + 9) \\ &= 4(x-3)^2 \end{aligned}$$

d. $2x^2 - 4x - 30$

$$\begin{aligned} 2x^2 - 4x - 30 &= 2(x^2 - 2x - 15) \\ &= 2(x+3)(x-5) \end{aligned}$$

e. $6x^2 + 11x - 10$

$$6x^2 + 11x - 10 = (2x+5)(3x-2)$$

f. $21x^2 + 29x - 10$

$$21x^2 + 29x - 10 = (3x+5)(7x-2)$$

Problem 5. Solve the following equations for x.

a. $8x + 5 = 21$

$$\begin{aligned} 8x + 5 &= 21 \\ \Rightarrow \quad 8x &= 21 - 5 = 16 \\ \Rightarrow \quad x &= \frac{16}{8} = 2 \end{aligned}$$

b. $4x + 3 = 10 - 3x$

$$\begin{aligned} 4x + 3 &= 10 - 3x \\ \Rightarrow \quad 4x + 3x &= 10 - 3 \\ \Rightarrow \quad 7x &= 7 \\ \Rightarrow \quad x &= 1 \end{aligned}$$

c. $\frac{3x+2}{4x-4} = 2$

$$\begin{aligned} \frac{3x + 2}{4x - 4} &= 2 \\ \Rightarrow \quad 3x + 2 &= 2 \times (4x - 4) \\ \Rightarrow \quad 3x + 2 &= 8x - 8 \\ \Rightarrow \quad -5x &= -10 \\ \Rightarrow \quad x &= 2 \end{aligned}$$

d. $\frac{x+7}{6x+3} = \frac{2}{7}$

$$\begin{aligned} \frac{x + 7}{6x + 3} &= \frac{2}{7} \\ \Rightarrow \quad 7 \times (x + 7) &= 2 \times (6x + 3) \\ \Rightarrow \quad 7x + 49 &= 12x + 6 \\ \Rightarrow \quad -5x &= -43 \\ \Rightarrow \quad x &= \frac{43}{5} \end{aligned}$$

e. $\frac{x-5}{x+7} = \frac{3}{7}$

$$\begin{aligned} \frac{x-5}{x+7} &= \frac{3}{7} \\ &\Rightarrow 7(x-5) = 3(x+7) \\ &\Rightarrow 7x-35 = 3x+21 \\ &\Rightarrow 4x = 56 \\ &\Rightarrow x = \frac{56}{4} = 14 \end{aligned}$$

f. $6x^2 - x - 2 = 0$

$$\begin{aligned} 6x^2 - x - 2 &= 0 \\ &\Rightarrow (2x+1)(3x-2) = 0 \\ &\Rightarrow 2x+1 = 0 \text{ or } 3x-2 = 0 \\ &\Rightarrow x = \frac{-1}{2} \text{ or } x = \frac{2}{3} \end{aligned}$$

g. $x^2 + 2x - 3 = 0$

$$\begin{aligned} x^2 + 2x - 3 &= 0 \\ &\Rightarrow (x-1)(x+3) = 0 \\ &\Rightarrow x-1 = 0 \text{ or } x+3 = 0 \\ &\Rightarrow x = 1 \text{ or } x = -3 \end{aligned}$$

h. $x^2 + 4x - 21 = 0$

$$\begin{aligned} x^2 + 4x - 21 &= 0 \\ &\Rightarrow (x+7)(x-3) = 0 \\ &\Rightarrow x+7 = 0 \text{ or } x-3 = 0 \\ &\Rightarrow x = -7 \text{ or } x = 3 \end{aligned}$$

i. $2x^2 + 5x - 7 = 0$

$$\begin{aligned} 2x^2 + 5x - 7 &= 0 \\ &\Rightarrow (x-1)(2x+7) = 0 \\ &\Rightarrow x-1 = 0 \text{ or } 2x+7 = 0 \\ &\Rightarrow x = 1 \text{ or } x = \frac{-7}{2} \end{aligned}$$

j. $x^2 + 2x + 3 = 0$

$$\begin{aligned} x^2 + 2x + 3 &= 0 \\ &\Rightarrow (x+1)^2 + 2 = 0 \\ &\Rightarrow (x+1)^2 = -2 \\ &\quad \text{No solution.} \end{aligned}$$

Problem 6. Solve the following equations for x_1.

a. $16x_1^{-1/2} - 2 = 0$

$$\begin{aligned} 16x_1^{-1/2} - 2 &= 0 \\ &\Rightarrow \quad x_1^{-1/2} = \frac{2}{16} = 2^{-3} \\ &\Rightarrow \quad x_1 = (2^{-3})^{-2} = 2^6 = 64 \end{aligned}$$

b. $8x_1^{-1/2} - 2 = 0$

$$\begin{aligned} 8x_1^{-1/2} - 2 &= 0 \\ &\Rightarrow \quad x_1^{-1/2} = \frac{2}{8} = 2^{-2} \\ &\Rightarrow \quad x_1 = (2^{-2})^{-2} = 2^4 \\ &\Rightarrow \quad x_1 = 16 \end{aligned}$$

c. $32x_1^{-1/2} - 4 = 0$

$$\begin{aligned} 32x_1^{-1/2} - 4 &= 0 \\ &\Rightarrow \quad x_1^{-1/2} = \frac{4}{32} = 2^{-3} \\ &\Rightarrow \quad x_1 = (2^{-3})^{-2} = 2^6 \\ &\Rightarrow \quad x_1 = 64 \end{aligned}$$

d. $16x_1^{-3/4} - 2 = 0$

$$\begin{aligned} 16x_1^{-3/4} - 2 &= 0 \\ &\Rightarrow \quad x_1^{-3/4} = \frac{2}{16} = 2^{-3} \\ &\Rightarrow \quad x_1 = (2^{-3})^{-4/3} = 2^4 \\ &\Rightarrow \quad x_1 = 16 \end{aligned}$$

e. $64x_1^{-3/4} - 8 = 0$

$$\begin{aligned} 64x_1^{-3/4} - 8 &= 0 \\ &\Rightarrow \quad x_1^{-3/4} = \frac{8}{64} = \frac{1}{8} = 2^{-3} \\ &\Rightarrow \quad x_1 = (2^{-3})^{-4/3} = 2^4 \\ &\Rightarrow \quad x_1 = 16 \end{aligned}$$

f. $16x_1^{-3/5} - 2 = 0$

$$\begin{aligned} 16x_1^{-3/5} - 2 &= 0 \\ &\Rightarrow \quad x_1^{-3/5} = \frac{2}{16} = 2^{-3} \\ &\Rightarrow \quad x_1 = (2^{-3})^{-5/3} = 2^5 \\ &\Rightarrow \quad x_1 = 32 \end{aligned}$$

g. $12x_1^{-2/5} - 3 = 0$

$$\begin{aligned} 12x_1^{-2/5} - 3 &= 0 \\ &\Rightarrow \quad x_1^{-2/5} = \frac{3}{12} = \frac{1}{4} = 2^{-2} \\ &\Rightarrow \quad x_1 = (2^{-2})^{-5/2} = 2^5 \\ &\Rightarrow \quad x_1 = 32 \end{aligned}$$

h. $8x_1^{-2/3} - 2 = 0$

$$\begin{aligned} 8x_1^{-2/3} - 2 &= 0 \\ &\Rightarrow \quad x_1^{-2/3} = \frac{2}{8} = 2^{-2} \\ &\Rightarrow \quad x_1 = (2^{-2})^{-3/2} = 2^3 \\ &\Rightarrow \quad x_1 = 8 \end{aligned}$$

i. $18x_1^{-2/3} - 2 = 0$

$$\begin{aligned} 18x_1^{-2/3} - 2 &= 0 \\ &\Rightarrow \quad x_1^{-2/3} = \frac{2}{18} = \frac{1}{9} = 3^{-2} \\ &\Rightarrow \quad x_1 = (3^{-2})^{-3/2} = 3^3 \\ &\Rightarrow \quad x_1 = 27 \end{aligned}$$