ECONOMICS 207
SPRING 2006
LABORATORY EXERCISE 3 KEY

Problem 1. Solve the following equations for x.

a $\frac{2x+5}{x+3} = \frac{13}{7}$

$$\begin{aligned} \frac{2x+5}{x+3} &= \frac{13}{7} \\ &\Rightarrow 7(2x+5) = 13(x+3) \\ &\Rightarrow 14x + 35 = 13x + 39 \\ &\Rightarrow 14x - 13x = 39 - 35 \\ &\Rightarrow x = 4 \end{aligned}$$

b $\frac{2x-4}{8-2x} = \frac{-5}{7}$

$$\begin{aligned} \frac{2x-4}{8-2x} &= \frac{-5}{7} \\ &\Rightarrow 7(2x-4) = -5(8-2x) \\ &\Rightarrow 14x - 28 = -40 + 10x \\ &\Rightarrow 4x = -12 \\ &\Rightarrow x = -3 \end{aligned}$$

c $\frac{6x-2}{13} = \frac{8x-6}{19}$

$$\begin{aligned} \frac{6x-2}{13} &= \frac{8x-6}{19} \\ &\Rightarrow 19(6x-2) = 13(8x-6) \\ &\Rightarrow 114x - 38 = 104x - 78 \\ &\Rightarrow 10x = -40 \\ &\Rightarrow x = -4 \end{aligned}$$

Date: 28 December 2006.

d $\frac{5}{10-3x} = \frac{-25}{4x-6}$

$$\begin{aligned}\frac{5}{10-3x} &= \frac{-25}{4x-6}\\ &\Rightarrow 5(4x-6) = -25(10-3x)\\ &\Rightarrow 20x-30 = -250+75x\\ &\Rightarrow -55x = -220\\ &\Rightarrow x = 4\end{aligned}$$

e $\frac{\frac{22x+26}{x+3}}{2x+4} = \frac{2}{2}$

$$\begin{aligned}\frac{\frac{22x+26}{x+3}}{2x+4} &= \frac{2}{2} = \frac{1}{1}\\ &\Rightarrow \frac{22x+26}{x+3} = 2x+4\\ &\Rightarrow 22x+26 = (2x+4)(x+3) = 2x^2+10x+12\\ &\Rightarrow 2x^2-12x-14 = 0\\ &\Rightarrow 2(x-7)(x+1) = 0\\ &\Rightarrow x = 7 \quad \text{or} \quad x = -1.\end{aligned}$$

Problem 2. Solve the following equations for x.

a $18x^2 + 3x - 6 = 0$

$$\begin{aligned} 18x^2 + 3x - 6 &= 0 \\ &\Rightarrow 3(2x-1)(3x+2) = 0 \\ &\Rightarrow x = \frac{1}{2} \text{ or } x = \frac{-2}{3}. \end{aligned}$$

b $-8x^2 + 26x - 15 = 0$

$$\begin{aligned} -8x^2 + 26x - 15 &= 0 \\ &\Rightarrow -(2x-5)(4x-3) = 0 \\ &\Rightarrow x = \frac{5}{2} \text{ or } x = \frac{3}{4}. \end{aligned}$$

c $6x^2 + 71x - 50 = 0$

$$\begin{aligned} 6x^2 + 71x - 50 &= 0 \\ &\Rightarrow (3x-2)(2x+25) = 0 \\ &\Rightarrow x = \frac{2}{3} \text{ or } x = \frac{-25}{2}. \end{aligned}$$

d $x^2 + 4x + 13 = 0$

$$\begin{aligned} x^2 + 4x + 13 &= 0 \\ &\Rightarrow (x+2)^2 + 9 = 0 \\ &\Rightarrow (x+2)^2 = -9 \\ &\quad No\ solution. \end{aligned}$$

e $4x^2 - 9x - 9 = 0$

$$\begin{aligned} 4x^2 - 9x - 9 &= 0 \\ &\Rightarrow (x-3)(4x+3) = 0 \\ &\Rightarrow x = 3 \text{ or } x = \frac{-3}{4} \end{aligned}$$

Problem 3. Solve the following equations for x_1.

a $6x_1^{-1/2} - 12 = 0$

$$\begin{aligned} 6x_1^{-1/2} - 12 &= 0 \\ &\Rightarrow 6x_1^{-1/2} = 12 \\ &\Rightarrow x_1^{-1/2} = \frac{12}{6} = 2 \\ &\Rightarrow x_1 = 2^{-2} = \frac{1}{4} \end{aligned}$$

b $10x_1^{-1/2} - 5 = 0$

$$\begin{aligned} 10x_1^{-1/2} - 5 &= 0 \\ &\Rightarrow 10x_1^{-1/2} = 5 \\ &\Rightarrow x_1^{-1/2} = \frac{5}{10} = \frac{1}{2} \\ &\Rightarrow x_1 = \left(\frac{1}{2}\right)^{-2} = 4 \end{aligned}$$

c $6x_1^{-1/2} - 4 = 0$

$$\begin{aligned} 6x_1^{-1/2} - 4 &= 0 \\ &\Rightarrow 6x_1^{-1/2} = 4 \\ &\Rightarrow x_1^{-1/2} = \frac{4}{6} = \frac{2}{3} \\ &\Rightarrow x_1 = \left(\frac{2}{3}\right)^{-2} = \frac{9}{4} \end{aligned}$$

d $405x_1^{-4/5} - 5 = 0$

$$\begin{aligned} 405x_1^{-4/5} - 5 &= 0 \\ &\Rightarrow 405x_1^{-4/5} = 5 \\ &\Rightarrow x_1^{-4/5} = \frac{5}{405} = \frac{1}{81} = 3^{-4} \\ &\Rightarrow x_1 = (3^{-4})^{-5/4} = 3^5 = 243 \end{aligned}$$

e $343x_1^{-3/5} - 1 = 0$

$$
\begin{aligned}
343x_1^{-3/5} - 1 &= 0 \\
&\Rightarrow 343x_1^{-3/5} = 1 \\
&\Rightarrow x_1^{-3/5} = \frac{1}{343} = 7^{-3} \\
&\Rightarrow x_1 = (7^{-3})^{-5/3} = 7^5 = 16807
\end{aligned}
$$

Problem 4. Solve the following equations for x_1.

a $10x_1^{1/2} = x_1$

$$\begin{aligned}
\text{Obviously } x_1 &= 0 \text{ is a solution to this equation.}\\
\text{If } x_1 &\neq 0, \text{ then } 10x_1^{1/2} = x_1\\
&\Rightarrow 10 = \frac{x_1}{x_1^{1/2}} = x_1^{1/2}\\
&\Rightarrow x_1 = 10^2 = 100\\
\text{So, } x_1 &= 0 \text{ or } x_1 = 100
\end{aligned}$$

b $10x_1^{1/2} = 2x_1$

$$\begin{aligned}
\text{Obviously } x_1 &= 0 \text{ is a solution to this equation.}\\
\text{If } x_1 &\neq 0, \text{ then } 10x_1^{1/2} = 2x_1\\
&\Rightarrow \frac{10}{2} = \frac{x_1}{x_1^{1/2}} = x_1^{1/2}\\
&\Rightarrow x_1^{1/2} = 5\\
&\Rightarrow x_1 = 5^2 = 25\\
\text{So, } x_1 &= 0 \text{ or } x_1 = 25
\end{aligned}$$

c $32x_1^{-1/3} = 2x_1$

$$\begin{aligned}
32x_1^{-1/3} &= 2x_1\\
&\Rightarrow \frac{32}{2} = \frac{x_1}{x_1^{-1/3}} = x_1^{4/3}\\
&\Rightarrow x_1^{4/3} = 16 = 2^4\\
&\Rightarrow x_1 = (2^4)^{3/4} = 2^3 = 8
\end{aligned}$$

d $405x_1^{-2/5} = 5x_1^{2/5}$

$$\begin{aligned} 405x_1^{-2/5} &= 5x_1^{2/5} \\ &\Rightarrow \frac{405}{5} = \frac{x_1^{2/5}}{x_1^{-2/5}} = x_1^{4/5} \\ &\Rightarrow x_1^{4/5} = \frac{405}{5} = 81 = 3^4 \\ &\Rightarrow x_1 = (3^4)^{5/4} = 3^5 = 243 \end{aligned}$$

e $6x_1^{2/3} = 12x_1^{5/3}$

$$\begin{aligned} \text{Obviously } x_1 &= 0 \text{ is a solution to this equation.} \\ \text{If } x_1 &\neq 0, \text{ then } 6x_1^{2/3} = 12x_1^{5/3} \\ &\Rightarrow \frac{6}{12} = \frac{x_1^{5/3}}{x_1^{2/3}} = x_1 \\ &\Rightarrow x_1 = \frac{1}{2} \\ \text{So, } x_1 &= 0 \text{ or } x_1 = \frac{1}{2} \end{aligned}$$

Problem 5. Solve the following systems of equations for x_1 and x_2 using the method of substitution

a

$$\begin{aligned} 5x_1 &+ 2x_2 &= 8 \\ 7x_1 &+ 3x_2 &= 11 \end{aligned}$$

$$\begin{aligned}
5x_1 + 2x_2 &= 8 \\
&\Rightarrow 2x_2 = 8 - 5x_1 \Rightarrow x_2 = \frac{1}{2}(8 - 5x_1) \\
&\quad \text{Substitute } x_2 = \frac{1}{2}(8 - 5x_1) \text{ into equation 2.} \\
7x_1 + 3 \times \frac{1}{2}(8 - 5x_1) &= 11 \Rightarrow 14x_1 + 3(8 - 5x_1) = 22 \\
&\Rightarrow 14x_1 + 24 - 15x_1 = 22 \\
&\Rightarrow -x_1 = 22 - 24 = -2 \\
&\Rightarrow x_1 = 2 \\
\text{Substitute } x_1 &= 2 \text{ into equation 1.} \\
5 \times 2 + 2x_2 &= 8 \\
&\Rightarrow 2x_2 = 8 - 10 = -2 \\
&\Rightarrow x_2 = -1 \\
\text{So, } x_1 &= 2 \quad x_2 = -1.
\end{aligned}$$

b

$$\begin{aligned} 8x_1 &+ 4x_2 &= 20 \\ 4x_1 &+ 3x_2 &= 11 \end{aligned}$$

$$\begin{aligned}
8x_1 + 4x_2 &= 20 \\
&\Rightarrow 4x_2 = 20 - 8x_1 \\
&\Rightarrow x_2 = 5 - 2x_1 \\
&\quad \text{Substitute above equation into equation 2.} \\
4x_1 + 3 \times (5 - 2x_1) &= 11 \\
&\Rightarrow 4x_1 + 15 - 6x_1 = 11 \\
&\Rightarrow -2x_1 = -4 \Rightarrow x_1 = 2 \\
\text{Substitute } x_1 &= 2 \text{ into equation 1.} \\
8 \times 2 + 4x_2 &= 20 \\
&\Rightarrow 4x_2 = 20 - 16 = 4 \\
&\Rightarrow x_2 = 1 \\
\text{So, } x_1 &= 2 \quad x_2 = 1.
\end{aligned}$$

c

$$\begin{aligned} x_1 \ + \ 3x_2 \ &= 12 \\ 4x_1 \ + \ 12x_2 \ &= 48 \end{aligned}$$

$$\begin{aligned} x_1 + 3x_2 &= 12 \\ &\Rightarrow \quad x_1 = 12 - 3x_2 \\ &\quad \text{Substitute above equation into equation 2.} \\ 4 \times (12 - 3x_2) + 12x_2 &= 48 \\ &\Rightarrow \quad 48 - 12x_2 + 12x_2 = 48 \Rightarrow 0 = 0 \end{aligned}$$

This is a true statement, but provides no information about the values of x_1 and x_2 that solve the system. So, this system has infinite solutions. Any x_1. x_2 which satisfies $x_1 = 12 - 3x_2$ is a solution to this system.

d

$$\begin{aligned} 2x_1 \ + \ 3x_2 \ &= 6 \\ 4x_1 \ + \ 6x_2 \ &= 7 \end{aligned}$$

$$\begin{aligned} 2x_1 + 3x_2 &= 6 \\ &\Rightarrow \quad x_1 = \frac{1}{2}(6 - 3x_2) \\ \text{Substitute } x_1 &= \frac{1}{2}(6 - 3x_2) \text{ into equation 2.} \\ 4 \times \frac{1}{2}(6 - 3x_2) + 6x_2 &= 7 \\ &\Rightarrow \quad 12 - 6x_2 + 6x_2 = 7 \Rightarrow 12 = 7 \end{aligned}$$

Obviously, this is an inconsistent system. There is no solution for this system.

e

$$\begin{aligned} x_1 &+ 2x_2 &= 8 \\ 3x_1 &+ 4x_2 &= 20 \end{aligned}$$

$$\begin{aligned}
x_1 + 2x_2 &= 8 \\
&\Rightarrow x_1 = 8 - 2x_2 \\
\text{Substitute } x_1 &= 8 - 2x_2 \text{ into equation 2.} \\
3(8 - 2x_2) + 4x_2 &= 20 \\
&\Rightarrow 24 - 6x_2 + 4x_2 = 20 \\
&\Rightarrow -2x_2 = -4 \Rightarrow x_2 = 2 \\
\text{Substitute } x_2 &= 2 \text{ into equation 1.} \\
x_1 + 2 \times 2 &= 8 \\
&\Rightarrow x_1 = 8 - 4 = 4 \\
\text{So, } x_1 &= 4 \quad x_2 = 2.
\end{aligned}$$

f

$$\begin{aligned} 2x_1 &+ 3x_2 &= 6 \\ 12x_1 &+ 18x_2 &= 36 \end{aligned}$$

$$\begin{aligned}
2x_1 + 3x_2 &= 6 \\
&\Rightarrow x_1 = \frac{1}{2}(6 - 3x_2) \\
&\text{Substitute } x_1 = \frac{1}{2}(6 - 3x_2) \text{ into equation 2.} \\
12 \times \frac{1}{2}(6 - 3x_2) + 18x_2 &= 36 \\
&\Rightarrow 36 - 18x_2 + 18x_2 = 36 \Rightarrow 36 = 36
\end{aligned}$$

This is a true statement, but provides no information about the values of x_1 and x_2 that solve the system. So, this system has infinite solutions. any x_1. x_2 which satisfies $x_1 = \frac{1}{2}(6 - 3x_2)$ is a solution to this system.

g

$$\begin{aligned} 2x_1 \ + \ 3x_2 \ &= 6 \\ 8x_1 \ + \ 12x_2 \ &= 21 \end{aligned}$$

$$\begin{aligned} 2x_1 + 3x_2 &= 6 \\ &\Rightarrow x_1 = \frac{1}{2}(6 - 3x_2) \\ \text{Substitute } x_1 &= \frac{1}{2}(6 - 3x_2) \text{ into equation 2.} \\ 8 \times \frac{1}{2}(6 - 3x_2) + 12x_2 &= 21 \\ &\Rightarrow 24 - 12x_2 + 12x_2 = 21 \Rightarrow 24 = 21 \end{aligned}$$

Obviously, this is an inconsistent system. There is no solution for this system.

Problem 6. Solve the following systems of equations for x_1, x_2, and x_3 using the method of substitution

a

$$\begin{aligned} x_1 + 2x_2 + 4x_3 &= 1 \\ 3x_1 + 7x_2 + 10x_3 &= 7 \\ 2x_1 + 3x_2 + 11x_3 &= -3 \end{aligned}$$

$$\begin{aligned}
x_1 + 2x_2 + 4x_3 &= 1 \\
&\Rightarrow x_1 = 1 - 2x_2 - 4x_3 \\
\text{Substitute } x_1 &= 1 - 2x_2 - 4x_3 \text{ into equation 2 and equation 3.} \\
3(1 - 2x_2 - 4x_3). + 7x_2 + 10x_3 &= 7 \\
2(1 - 2x_2 - 4x_3) + 3x_2 + 11x_3 &= -3 \\
\text{Simplify these two equations we will have} &: \\
x_2 - 2x_3 &= 4 \Rightarrow x_2 = 4 + 2x_3 \\
-x_2 + 3x_3 &= -5 \\
\text{Substitute } x_2 &= 4 + 2x_3 \text{ into the second equation .} \\
-(4 + 2x_3) + 3x_3 &= -5 \\
&\Rightarrow x_3 = -5 + 4 = -1 \\
\text{Substitute } x_3 &= -1 \text{ into the equation } x_2 = 4 + 2x_3\text{, we obtain,} \\
x_2 &= 4 + 2 \times (-1) = 4 - 2 = 2 \\
\text{Substitute } x_2 &= 2,\ x_3 = -1 \text{ into equation 1.} \\
x_1 + 2 \times 2 + 4 \times (-1) &= 1 \\
&\Rightarrow x_1 = 1 + 4 - 4 = 1 \\
\text{So, } x_1 &= 1 \quad x_2 = 2 \;\; x_3 = -1
\end{aligned}$$

b

$$
\begin{aligned}
-2x_1 - 2x_2 + 2x_3 &= 2 \\
4x_1 - 3x_2 + 2x_3 &= 16 \\
2x_1 - 2x_2 - 3x_3 &= 5
\end{aligned}
$$

$$
\begin{aligned}
-2x_1 - 2x_2 + 2x_3 &= 2 \\
&\Rightarrow x_1 = -1 - x_2 + x_3 \\
\text{Substitute } x_1 &= -1 - x_2 + x_3 \text{ into equation 2 and equation 3.} \\
4 \times (-1 - x_2 + x_3) - 3x_2 + 2x_3 &= 16 \\
2 \times (-1 - x_2 + x_3) - 2x_2 - 3x_3 &= 5 \\
\text{Simplify these two equations we will have } &: \\
-7x_2 + 6x_3 &= 20 \\
-4x_2 - x_3 &= 7 \Rightarrow x_3 = -4x_2 - 7 \\
\text{Substitute } x_3 &= -4x_2 - 7 \text{ into the first equation .} \\
-7x_2 + 6 \times (-4x_2 - 7) &= 20 \\
&\Rightarrow -31x_2 = 62 \Rightarrow x_2 = -2 \\
\text{Substitute } x_2 &= -2 \text{ into the equation } x_3 = -4x_2 - 7 \text{ . We obtain} \\
x_3 &= -4 \times (-2) - 7 = 1 \\
\text{Substitute } x_2 &= -2,\ x_3 = 1 \text{ into equation 1} \\
-2x_1 - 2 \times (-2) + 2 \times 1 &= 2 \\
&\Rightarrow 2x_1 = 4 + 2 - 2 = 4 \Rightarrow x_1 = 2 \\
\text{So, } x_1 &= 2 \quad x_2 = -2 \ \ x_3 = 1
\end{aligned}
$$

c

$$\{x_1 = -\tfrac{1}{2},\ x_2 = -1,\ x_3 = \tfrac{3}{2}\}$$

$$x_1 - 5x_2 + 3x_3 = 9$$
$$2x_1 - x_2 + 4x_3 = 6$$
$$3x_1 - 2x_2 + x_3 = 2$$

$$x_1 - 5x_2 + 3x_3 = 9$$
$$\Rightarrow x_1 = 9 + 5x_2 - 3x_3$$

Substitute $x_1 = 9 + 5x_2 - 3x_3$ into equation 2 and equation 3

$$2(9 + 5x_2 - 3x_3) - x_2. + 4x_3 = 6$$
$$3(9 + 5x_2 - 3x_3) - 2x_2 + x_3 = 2$$

Simplify these two equations we will have :

$$9x_2 - 2x_3 = -12 \Rightarrow x_3 = \frac{1}{2}(9x_2 + 12)$$
$$13x_2 - 8x_3 = -25$$

Substitute $x_3 = \frac{1}{2}(9x_2 + 12)$ into the second equation .

$$13x_2 - 8(\frac{1}{2}(9x_2 + 12)) = -25$$
$$\Rightarrow -23x_2 = 23 \Rightarrow x_2 = -1$$

Substitute $x_2 = -1$ into the equation $x_3 = \frac{1}{2}(9x_2 + 12)$. We obtain

$$x_3 = \frac{1}{2}(9 \times -1 + 12) = \frac{3}{2}$$

Substitute $x_2 = -1,\ x_3 = \frac{3}{2}$ into equation 1.

$$x_1 - 5 \times -1 + 3 \times \frac{3}{2} = 9 \Rightarrow x_1 = 9 - 5 - \frac{9}{2} = -\frac{1}{2}$$

So, $x_1 = -\frac{1}{2},\ x_2 = -1,\ x_3 = \frac{3}{2}$

d

$$x_1 - x_2 - x_3 = 1$$
$$-x_1 + 2x_2 - 3x_3 = -4$$
$$3x_1 - 2x_2 - 7x_3 = 0$$

$$\begin{aligned}
x_1 - x_2 - x_3 &= 1 \\
&\Rightarrow x_1 = 1 + x_2 + x_3 \\
\text{Substitute } x_1 &= 1 + x_2 + x_3 \text{ into equation 2 and equation 3} \\
-(1 + x_2 + x_3) + 2x_2 - 3x_3 &= -4 \\
3(1 + x_2 + x_3) - 2x_2 - 7x_3 &= 0 \\
\text{Simplify these two equations we will have } &: \\
x_2 - 4x_3 &= -3 \Rightarrow x_2 = -3 + 4x_3 \\
x_2 - 4x_3 &= -3 \\
\text{Substitute } x_2 &= -3 + 4x_3 \text{ into the second equation .} \\
-3 + 4x_3 - 4x_3 &= -3 \Rightarrow -3 = -3
\end{aligned}$$

This is a true statement, but provides no information about the values of x_1 x_2 and x_3 that solve the system. So, this system has infinite solutions. For any value of x_3, $x_2 = -3 + 4x_3$ *and* $x_1 = -2 + 5x_3$ will be a solution to this system.

Problem 7. Solve the following systems of equations for x_1 and x_2 using the method of substitution.

a

$$40x_1^{-3/5}x_2^{1/5} - 10 = 0$$
$$20x_1^{2/5}x_2^{-4/5} - 5 = 0$$

$$\begin{aligned}
40x_1^{-3/5}x_2^{1/5} - 10 &= 0 \\
&\Rightarrow 40x_2^{1/5} = 10x_1^{3/5} \\
&\Rightarrow x_2^{1/5} = \frac{1}{4}x_1^{3/5} \Rightarrow x_2 = (\frac{1}{4}x_1^{3/5})^5 = 4^{-5}x_1^3 \\
\text{Substitute } x_2 &= 4^{-5}x_1^3 \text{ into the second equation. We obtain,} \\
20x_1^{2/5}(4^{-5}x_1^3)^{-4/5} - 5 &= 0 \\
&\Rightarrow 20 \times 4^4 \times x_1^{-2} = 5 \\
&\Rightarrow x_1^{-2} = \frac{5}{20 \times 4^4} = 4^{-5} \\
&\Rightarrow x_1 = 4^{5/2} = 2^5 = 32 \\
\text{Substitute } x_1 &= 32 \text{ into the equation } x_2 = 4^{-5}x_1^3 \\
x_2 &= 4^{-5} \times 2^{15} = 2^{-10} \times 2^{15} = 2^5 = 32 \\
\text{So, } x_1 &= 32,\ x_2 = 32
\end{aligned}$$

b

$$6x_1^{-2/5}x_2^{1/5} - 3 = 0$$
$$2x_1^{3/5}x_2^{-4/5} - 2 = 0$$

$$\begin{aligned}
6x_1^{-2/5}x_2^{1/5} - 3 &= 0 \\
&\Rightarrow 6x_2^{1/5} = 3x_1^{2/5} \Rightarrow x_2^{1/5} = \frac{1}{2}x_1^{2/5} \\
&\Rightarrow x_2 = 2^{-5}x_1^2 \\
\text{Substitute } x_2 &= 2^{-5}x_1^2 \text{ into the second equation. We obtain,} \\
2x_1^{3/5} \times (2^{-5}x_1^2)^{-4/5} - 2 &= 0 \\
&\Rightarrow x_1^{-1} = 2^{-4} \Rightarrow x_1 = 2^4 = 16 \\
\text{Substitute } x_1 &= 16 \text{ into the equation } x_2 = 2^{-5}x_1^2 \\
x_2 &= 2^{-5} \times (2^4)^2 = 2^3 = 8 \\
\text{So, } x_1 &= 16,\ x_2 = 8
\end{aligned}$$