ECONOMICS 207
SPRING 2006
LABORATORY EXERCISE 4 KEY

Problem 1. Solve the following equations for x.

a $3x \; + \; 5 \; = \; 14$

$$\begin{aligned} 3x + 5 &= 14 \\ &\Rightarrow \; 3x = 14 - 5 = 9 \\ &\Rightarrow \; x = \frac{9}{3} \\ &\Rightarrow \; x = 3 \end{aligned}$$

b $3x \; - \; 5 \; = \; 20 \; - \; 2x$

$$\begin{aligned} 3x - 5 &= 20 - 2x \\ &\Rightarrow \; 3x + 2x = 20 + 5 \\ &\Rightarrow \; 5x = 25 \\ &\Rightarrow \; x = 5 \end{aligned}$$

c $\frac{2x \; + \; 4}{3x + 8} \; = \; \frac{5}{8}$

$$\begin{aligned} \frac{2x \; + \; 4}{3x + 8} &= \frac{5}{8} \\ &\Rightarrow \; 8(2x \; + \; 4) = 5(3x + 8) \\ &\Rightarrow \; 16x + 32 = 15x + 40 \\ &\Rightarrow \; 16x - 15x = 40 - 32 \\ &\Rightarrow \; x = 8 \end{aligned}$$

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d $\frac{2x - 5}{3} = x - 1$

$$\begin{aligned} \frac{2x - 5}{3} &= x - 1 \\ &\Rightarrow 2x - 5 = 3(x - 1) \\ &\Rightarrow 2x - 5 = 3x - 3 \\ &\Rightarrow 3 - 5 = 3x - 2x \\ &\Rightarrow x = -2 \end{aligned}$$

Problem 2. Solve the following equations for x.

a $10x^2 + 41x + 21 = 0$

$$\begin{aligned} 10x^2 + 41x + 21 &= (5x+3)(2x+7) = 0 \\ &\Rightarrow 5x+3=0 \text{ or } 2x+7=0 \\ &\Rightarrow x = \frac{-3}{5} \text{ or } x = \frac{-7}{2} \end{aligned}$$

b $3x^2 + 13x - 10 = 0$

$$\begin{aligned} 3x^2 + 13x - 10 &= (x+5)(3x-2) = 0 \\ &\Rightarrow x+5=0 \text{ or } 3x-2=0 \\ &\Rightarrow x = -5 \text{ or } x = \frac{2}{3} \end{aligned}$$

c $6x^2 - 45x = 81$

$$\begin{aligned} 6x^2 - 45x - 81 &= (3x-27)(2x+3) = 0 \\ &\Rightarrow 3x-27=0 \text{ or } 2x+3=0 \\ &\Rightarrow x = 9 \text{ or } x = \frac{-3}{2} \end{aligned}$$

Problem 3. Solve the following equations for x_1.

a $343x_1^{-3/4} - 27 = 0$

$$\begin{aligned} 343x_1^{-3/4} - 27 &= 0 \\ &\Rightarrow 343x_1^{-3/4} = 27 \\ &\Rightarrow x_1^{-3/4} = \frac{27}{343} = \frac{3^3}{7^3} \\ &\Rightarrow x_1 = (\frac{3}{7})^{3\times\frac{-4}{3}} = (\frac{3}{7})^{-4} \\ &\Rightarrow x_1 = (\frac{7}{3})^4 = \frac{2401}{81} \end{aligned}$$

b $256x_1^{-4/5} - 81 = 0$

$$\begin{aligned} 256x_1^{-4/5} - 81 &= 0 \\ &\Rightarrow 256x_1^{-4/5} = 81 \\ &\Rightarrow x_1^{-4/5} = \frac{81}{256} = \frac{3^4}{4^4} \\ &\Rightarrow x_1 = \left(\frac{3}{4}\right)^{4\times\frac{-5}{4}} = (\frac{3}{4})^{-5} \\ &\Rightarrow x_1 = \left(\frac{4}{3}\right)^5 = \frac{1024}{243} \end{aligned}$$

Problem 4. Solve the following equations for x_1.

a $686x_1^{-1/4} = 54x_1^{1/2}$

$$
\begin{aligned}
686x_1^{-1/4} &= 54x_1^{1/2} \\
&\Rightarrow \frac{686}{54} = \frac{x_1^{1/2}}{x_1^{-1/4}} \\
&\Rightarrow x_1^{3/4} = \frac{343}{27} = \frac{7^3}{3^3} \\
&\Rightarrow x_1 = \left(\frac{7}{3}\right)^{3\times\frac{4}{3}} = (\frac{7}{3})^4 \\
&\Rightarrow x_1 = \frac{2401}{81}
\end{aligned}
$$

b $768x_1^{-3/5} = 243x_1^{1/5}$

$$
\begin{aligned}
768x_1^{-3/5} &= 243x_1^{1/5} \\
&\Rightarrow \frac{768}{243} = \frac{x_1^{1/5}}{x_1^{-3/5}} \\
&\Rightarrow x_1^{4/5} = \frac{768}{243} = \frac{256}{81} = (\frac{4}{3})^4 \\
&\Rightarrow x_1 = \left(\frac{4}{3}\right)^{4\times\frac{5}{4}} = (\frac{4}{3})^5 \\
&\Rightarrow x_1 = \frac{1024}{243}
\end{aligned}
$$

c $121x_1^{-1/3} = 64x_1^{1/3}$

$$
\begin{aligned}
121x_1^{-1/3} &= 64x_1^{1/3} \\
&\Rightarrow \frac{121}{64} = \frac{x_1^{1/3}}{x_1^{-1/3}} \\
&\Rightarrow x_1^{2/3} = \left(\frac{11}{8}\right)^2 \\
&\Rightarrow x_1 = \left(\frac{11}{8}\right)^{2\times\frac{3}{2}} = (\frac{11}{8})^3 \\
&\Rightarrow x_1 = \frac{1331}{512}
\end{aligned}
$$

Problem 5. Solve the following systems of equations for x_1 and x_2 using the method of elimination.

a

$$\begin{aligned} 5x_1 \ + \ 2x_2 \ &= 8 \\ 7x_1 \ + \ 3x_2 \ &= 11 \end{aligned}$$

Multiply the first equation by -1.5 and add it to the second equation. We have,

$$\begin{aligned} -7.5x_1 - 3x_2 &= -12 \\ 7x_1 + 3x_2 &= 11 \\ &\Rightarrow -0.5x_1 = -1 \Rightarrow x_1 = \frac{-1}{-0.5} \\ &\Rightarrow x_1 = 2 \end{aligned}$$

Multiply the above equation by-5 and add it to the first equation.We have,

$$\begin{aligned} -5x_1 &= -10 \\ 5x_1 + 2x_2 &= 8 \\ &\Rightarrow 2x_2 = -2 \Rightarrow x_2 = -1 \\ \text{So, } x_1 &= 2, \ \ x_2 = -1. \end{aligned}$$

b

$$\begin{aligned} 8x_1 \ + \ 4x_2 \ &= 20 \\ 4x_1 \ + \ 3x_2 \ &= 11 \end{aligned}$$

Multiply the first equation by -0.5 and add it to the second equation. We have,

$$\begin{aligned} -4x_1 - 2x_2 &= -10 \\ 4x_1 + 3x_2 &= 11 \\ &\Rightarrow x_2 = 1 \end{aligned}$$

Multiply the above equation by -4 and add it to the first equation. We have,

$$\begin{aligned} -4x_2 &= -4 \\ 8x_1 + 4x_2 &= 20 \\ &\Rightarrow 8x_1 = 16 \Rightarrow x_1 = 2 \\ \text{So, } x_1 &= 2, \ \ x_2 = 1. \end{aligned}$$

c

$$\begin{aligned} x_1 \; + \; 3x_2 \; &= 12 \\ 4x_1 \; + \; 12x_2 \; &= 48 \end{aligned}$$

Multiply the first equation by -4 and add it to the second equation. We have

$$\begin{aligned} -4x_1 - 12x_2 &= -48 \\ 4x_1 + 12x_2 &= 48 \\ 0x_1 + 0x_2 &= 0 \end{aligned}$$

We now have an equation stating that $0x_1 + 0x_2 = 0$. This is a true statement, but provides no information about the values of x_1 and x_2 that solve the system. Solve the second equation for x_1 to obtain

$$\begin{aligned} 4x_1 + 12x_2 &= 48 \\ &\Rightarrow \; 4x_1 = 48 - 12x_2 \\ &\Rightarrow \; x_1 = 12 - 3x_2 \end{aligned}$$

Any x_1 satisfying the above equation will satisfy both equations. There are infinite solutions to this system, one for every possible value of x_2 among the real numbers.

d

$$\begin{aligned} 2x_1 \; + \; 3x_2 \; &= 6 \\ 4x_1 \; + \; 6x_2 \; &= 7 \end{aligned}$$

Multiply the first equation by -2 and add it to the second equation. We have,

$$\begin{aligned} -4x_1 - 6x_2 &= -12 \\ 4x_1 + 6x_2 &= 7 \\ 0x_1 + 0x_2 &= -5 \end{aligned}$$

It is clear that we have an inconsistent system.

e

$$\begin{aligned} x_1 &+ 2x_2 &= 8 \\ 3x_1 &+ 4x_2 &= 20 \end{aligned}$$

Multiply the first equation by -3 and add it to the second equation. We have,

$$\begin{aligned} -3x_1 - 6x_2 &= -24 \\ 3x_1 + 4x_2 &= 20 \\ 0x_1 - 2x_2 &= -4 \Rightarrow x_2 = 2 \end{aligned}$$

Multiply the above equation by -2 and add it to the first equation. We have,

$$\begin{aligned} -2x_2 &= -4 \\ x_1 + 2x_2 &= 8 \\ x_1 + 0x_2 &= 4 \Rightarrow x_1 = 4 \\ \text{So, } x_1 &= 4, \;\; x_2 = 2. \end{aligned}$$

f

$$\begin{aligned} 2x_1 &+ 3x_2 &= 6 \\ 12x_1 &+ 18x_2 &= 36 \end{aligned}$$

Multiply the first equation by -6 and add it to the second equation. We have,

$$\begin{aligned} -12x_1 - 18x_2 &= -36 \\ 12x_1 + 18x_2 &= 36 \\ 0x_1 + 0x_2 &= 0 \end{aligned}$$

We now have an equation stating that $0x_1 + 0x_2 = 0$. This is a true statement, but provides no information about the values of x_1 and x_2 that solve the system. Solve the second equation for x_1 to obtain

$$\begin{aligned} 12x_1 + 18x_2 &= 36 \\ &\Rightarrow 12x_1 = 36 - 18x_2 \\ &\Rightarrow x_1 = 3 - \frac{3}{2}x_2 \end{aligned}$$

Any x_1 satisfying the above equation will satisfy both equations. There are infinite solutions to this system, one for every possible value of x_2 among the real numbers.

g

$$\begin{aligned} 2x_1 \ + \ 3x_2 \ &= 6 \\ 8x_1 \ + \ 12x_2 \ &= 21 \end{aligned}$$

Multiply the first equation by -4 and add it to the second equation. We have,

$$\begin{aligned} -8x_1 - 12x_2 &= -24 \\ 8x_1 + 12x_2 &= 21 \\ 0x_1 + 0x_2 &= -3 \end{aligned}$$

It is clear that we have an inconsistent system.

Problem 6. Solve the following systems of equations for x_1, x_2, and x_3 using the method of elimination.

a

$$\{x_1 = 1, x_2 = 2, x_3 = -1\}$$

$$\begin{aligned} x_1 + 2x_2 + 4x_3 &= 1 \\ 3x_1 + 7x_2 + 10x_3 &= 7 \\ 2x_1 + 3x_2 + 11x_3 &= -3 \end{aligned}$$

Multiply the first equation by -3 and add it to the second equation. And, Multiply the first equation by -2 and add it to the third equation. The system now is given by,

$$\begin{aligned} x_1 + 2x_2 + 4x_3 &= 1 \\ 0x_1 + x_2 - 2x_3 &= 4 \\ 0x_1 - x_2 + 3x_3 &= -5 \end{aligned}$$

Third equation plus the second equation, the system will be,

$$\begin{aligned} x_1 + 2x_2 + 4x_3 &= 1 \\ 0x_1 + x_2 - 2x_3 &= 4 \\ 0x_1 + 0x_2 + x_3 &= -1 \end{aligned}$$

Multiply the third equation by 2 and add it to the second equation.

$$\begin{aligned} x_1 + 2x_2 + 4x_3 &= 1 \\ 0x_1 + x_2 + 0x_3 &= 2 \\ 0x_1 + 0x_2 + x_3 &= -1 \end{aligned}$$

Multiply the third equation by -4 and multiply the second equation by -2 and add them to the first equation.

$$\begin{aligned} x_1 + 0x_2 + 0x_3 &= 1 \\ 0x_1 + x_2 + 0x_3 &= 2 \\ 0x_1 + 0x_2 + x_3 &= -1 \\ \text{So, } x_1 &= 1,\; x_2 = 2,\; x_3 = -1.. \end{aligned}$$

b

$\{x_1 = 2, x_2 = -2, x_3 = 1\}$

$$-2x_1 - 2x_2 + 2x_3 = 2$$
$$4x_1 - 3x_2 + 2x_3 = 16$$
$$2x_1 - 2x_2 - 3x_3 = 5$$

Multiply the first equation by 2 and add it to the second equation. And, add the first equation to the third equation. The system now is given by,

$$\begin{aligned} -2x_1 - 2x_2 + 2x_3 &= 2 \\ 0x_1 - 7x_2 + 6x_3 &= 20 \\ 0x_1 - 4x_2 - 1x_3 &= 7 \end{aligned}$$

Multiply the third equation by 6 and add it to the second equation. Then divide the second equation by -31. The system will be,

$$\begin{aligned} -2x_1 - 2x_2 + 2x_3 &= 2 \\ 0x_1 + 1x_2 + 0x_3 &= -2 \\ 0x_1 - 4x_2 - 1x_3 &= 7 \end{aligned}$$

Multiply the second equation by 4 and add it to the third equation. Then divide the second equation by -1. The system will be,

$$\begin{aligned} -2x_1 - 2x_2 + 2x_3 &= 2 \\ 0x_1 + 1x_2 + 0x_3 &= -2 \\ 0x_1 + 0x_2 + 1x_3 &= 1 \end{aligned}$$

Multiply the third equation by -2 and multiply the second equation by 2 and add them to the first equation.

$$\begin{aligned} -2x_1 + 0x_2 + 0x_3 &= -4 \\ 0x_1 + 1x_2 + 0x_3 &= -2 \\ 0x_1 + 0x_2 + 1x_3 &= 1 \\ \text{So, } x_1 &= 2, \;\; x_2 = -2, \;\; x_3 = 1. \end{aligned}$$

c

$\{x_1 = -\frac{1}{2},\ x_2 = -1,\ x_3 = \frac{3}{2}\}$

$$x_1 - 5x_2 + 3x_3 = 9$$
$$2x_1 - x_2 + 4x_3 = 6$$
$$3x_1 - 2x_2 + x_3 = 2$$

Multiply the first equation by -2 and add it to the second equation. And, multiply the first equation by -3 and add it to the third equation. The system now is given by,

$$\begin{aligned} x_1 - 5x_2 + 3x_3 &= 9 \\ 0x_1 + 9x_2 - 2x_3 &= -12 \\ 0x_1 + 13x_2 - 8x_3 &= -25 \end{aligned}$$

Multiply the second equation by -4 and add it to the third equation.

$$\begin{aligned} x_1 - 5x_2 + 3x_3 &= 9 \\ 0x_1 + 9x_2 - 2x_3 &= -12 \\ 0x_1 - 23x_2 + 0x_3 &= 23 \end{aligned}$$

Divide the third equation by -23.

$$\begin{aligned} x_1 - 5x_2 + 3x_3 &= 9 \\ 0x_1 + 9x_2 - 2x_3 &= -12 \\ 0x_1 + x_2 + 0x_3 &= -1 \end{aligned}$$

Multiply the third equation by -9 and add it to the second equation. And, multiply the third equation by 5 and add it to the first equation. The system now is given by,

$$\begin{aligned} x_1 + 0x_2 + 3x_3 &= 4 \\ 0x_1 + 0x_2 - 2x_3 &= -3 \\ 0x_1 + x_2 + 0x_3 &= -1 \end{aligned}$$

Divide the second equation by -2: $0x_1 + 0x_2 + 1x_3 = \frac{3}{2}$ Multiply this equation by -3 and add it to the first equation.

$$\begin{aligned} x_1 + 0x_2 + 0x_3 &= \frac{-1}{2} \\ 0x_1 + 0x_2 + 1x_3 &= \frac{3}{2} \\ 0x_1 + x_2 + 0x_3 &= -1 \\ \text{So, } x_1 &= -\frac{1}{2},\ x_2 = -1,\ x_3 = \frac{3}{2}. \end{aligned}$$

d

$$\begin{aligned} x_1 - x_2 - x_3 &= 1 \\ -x_1 + 2x_2 - 3x_3 &= -4 \\ 3x_1 - 2x_2 - 7x_3 &= 0 \end{aligned}$$

You will end up with

$$\begin{aligned} x_1 - x_2 - x_3 &= 1 \\ 0x_1 + 1x_2 - 4x_3 &= -3 \\ 0x_1 + 0x_2 + 0x_3 &= 0 \end{aligned}$$

Add the first equation to the second equation. And, multiply the first equation by -3 and add it to the third equation. The system now is given by,

$$\begin{aligned} x_1 - x_2 - x_3 &= 1 \\ 0x_1 + x_2 - 4x_3 &= -3 \\ 0x_1 + x_2 - 4x_3 &= -3 \end{aligned}$$

Multiply the second equation by -1 and add it to the third equation.

$$\begin{aligned} x_1 - x_2 - x_3 &= 1 \\ 0x_1 + x_2 - 4x_3 &= -3 \\ 0x_1 + 0x_2 + 0x_3 &= 0 \end{aligned}$$

This is a true statement ,but provides no information about the values of x_1 x_2 and x_3 that solve the system. There are infinite solutions to this system.

Problem 7. Solve the following systems of equations for x_1 and x_2 using the method of substitution.

a

$$5120x_1^{-3/4}x_2^{2/5} - 1280 = 0$$
$$8192x_1^{1/4}x_2^{-3/5} - 512 = 0$$

$$\begin{aligned}
5120x_1^{-3/4}x_2^{2/5} - 1280 &= 0 \\
&\Rightarrow x_1^{-3/4} = \frac{1280}{5120}x_2^{-2/5} \\
&\Rightarrow x_1 = \left(\frac{1280}{5120}x_2^{-2/5}\right)^{-4/3} = 4^{4/3}x_2^{8/15} \\
\text{Substitute } x_1 &= 4^{4/3}x_2^{8/15} \text{ into the second equation,} \\
8192(4^{4/3}x_2^{8/15})^{1/4}x_2^{-3/5} - 512 &= 0 \\
&\Rightarrow 8192 \times 4^{1/3} \times x_2^{-1/15} = 512 \\
x_2^{-7/15} &= \frac{512}{8192 \times 4^{1/3}} = \frac{1}{16 \times 4^{1/3}} = 4^{-7/3} \\
&\Rightarrow x_2 = (4^{-7/3})^{-15/7} = 4^5 \\
x_1 &= 4^{4/3}x_2^{8/15} = 4^{4/3} \times 4^{8/3} = 4^4 \\
So,\ x_1 &= 4^4 = 256, x_2 = 4^5 = 1024
\end{aligned}$$

b

$\{x_1 \ = \ 125, \ x_2 \ = \ 256\}$

$$800x_1^{-2/3}x_2^{1/2} \ - \ 512 \ = 0$$
$$1200x_1^{1/3}x_2^{-1/2} \ - \ 375 \ = 0$$

$$\begin{aligned}
800x_1^{-2/3}x_2^{1/2} - 512 &= 0 \\
&\Rightarrow \ x_1^{-2/3} = \frac{512}{800}x_2^{-1/2} \\
&\Rightarrow \ x_1 = \left(\frac{512}{800}x_2^{-1/2}\right)^{-3/2} = (\frac{4}{5})^{-3}x_2^{3/4} \\
\text{Substitute } x_1 &= \left(\frac{4}{5}\right)^{-3} x_2^{3/4} \text{ into the second equation,} \\
1200\left(\left(\frac{4}{5}\right)^{-3} x_2^{3/4}\right)^{1/3} x_2^{-1/2} - 375 &= 0 \\
&\Rightarrow \ 1200 \times \frac{5}{4} \times x_2^{-1/4} = 375 \\
x_2^{-1/4} &= \frac{375}{1200 \times \frac{5}{4}} = \frac{1}{4} \\
&\Rightarrow \ x_2 = \left(\frac{1}{4}\right)^{-4} = 4^4 = 256 \\
x_1 &= \left(\frac{4}{5}\right)^{-3} x_2^{3/4} = (\frac{4}{5})^{-3} \times 4^3 = 5^3 = 125 \\
So, \ x_1 &= 125, \ x_2 = 256.
\end{aligned}$$