

**ECONOMICS 207**  
**SPRING 2007**  
**EXAM 3**

**Problem 1** (21 points).

Consider the following matrices.

$$A = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}, \quad E = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}, \quad F = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -4 \\ -4 & -5 & 0 \end{bmatrix}$$

$$x = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}, \quad b = \begin{bmatrix} -1 \\ -3 \\ 6 \end{bmatrix}, \quad c = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Compute the following

a.  $A + B$

b.  $AB$

c.  $c'B$

d. b'F

e. FA

f. EA

g. Fx

**Problem 2** (14 Points).

Solve the following system of equations for  $x_1$ ,  $x_2$ , and  $x_3$  using the method of elimination. The answer for  $x_2$  is  $x_2 = -2$ .

$$x_1 + 2x_2 - x_3 = -1$$

$$3x_1 + 7x_2 - 4x_3 = -3$$

$$-4x_1 - 5x_2 + 0x_3 = 6$$

**Problem 3** (24 Points).

For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise. Also find the points of inflection for each function.

a.  $f(x) = x^3 - 6x^2 - 15x$

b.  $f(x) = \frac{1}{2}x^6 + \frac{3}{5}x^5 - 9x^4$

Mathematics Helps:

$$15 \times 256 = 3840,$$

$$15 \times 81 = 1215$$

$$12 \times 64 = 768,$$

$$12 \times 27 = 324$$

$$108 \times 16 = 1728,$$

$$108 \times 9 = 972$$

$$16 \times 46 = 736$$

c.  $f(x) = -4x^3 + 6x^4$

**Problem 4** (17 points).

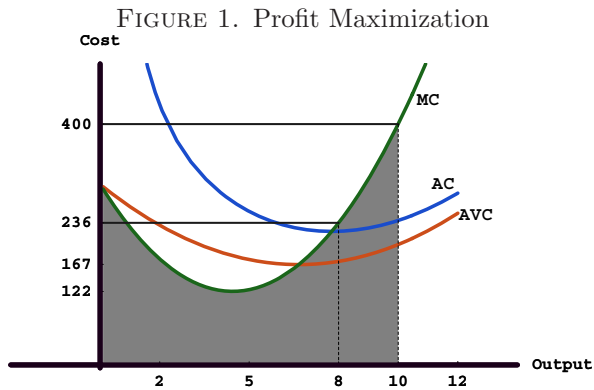
Solve the following system of equations.

$$9x_1^{-3/4}x_2^{1/3} - 1 = 0$$

$$12x_1^{1/4}x_2^{-2/3} - 4 = 0$$

**Problem 5** (24 points).

The cost function for a firm is a rule or mapping that tells the total cost of production of any output level produced by the firm. If the variable  $y$  represents the output of the firm, then the cost function is given by  $c(y)$ . Marginal cost represents the change in the cost of production for the firm as output changes and is given by the derivative of the cost function with respect to output, i.e., Marginal Cost (MC) =  $\frac{dc(y)}{dy}$ . A competitive firm facing a fixed output price maximizes profit at the output level where marginal cost is equal to price as in the diagram below.



Consider a firm with marginal cost function given by

$$MC = 9y^2 - 80y + 300$$

a. What is the variable cost function for this firm?

b. What is this firm's variable cost if it produces 10 units of output?



c. Assuming this firm has fixed costs of \$400, write the cost equation for this firm.

$$c(y) =$$

d. If the output price faced by this firm is \$400, write the profit equation for this firm.

$$\pi(y) =$$

e. Find potential profit maximizing output levels for the firm.

f. Show which of the potential levels in part e maximizes profit.

g. What is producer surplus for this firm when the price is \$400?

h. Crosshatch producer surplus in figure 1.