

ECONOMICS 207
SPRING 2007
LABORATORY EXERCISE 10

For this laboratory exercise, consider the following matrices and vectors. You may want to tear this page off so it is easy to view.

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & -4 \\ 3 & -1 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -2 & -2 \\ 3 & -5 & -5 \\ -2 & 2 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} -5 & 2 & 0 \\ 1 & -1 & -1 \\ -4 & 2 & 1 \end{bmatrix}$$
$$D = \begin{bmatrix} 1 & -2 & 1 \\ -3 & 5 & -2 \\ 4 & -6 & 3 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 4 & -4 \\ -4 & -5 & 3 \end{bmatrix}, \quad F = \begin{bmatrix} -8 & -13 & -12 \\ 7 & 11 & 10 \\ 1 & 1 & 1 \end{bmatrix}$$
$$a = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}, \quad c = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

Problem 1. Compute the following

a. $A + F$

b. BF

c. FC

d. Da

Problem 2. Compute the determinants of the following matrices.

a.

$$\begin{pmatrix} 2 & 3 \\ 3 & 1 \end{pmatrix}$$

b.

$$\begin{pmatrix} -2 & 3 \\ 3 & 4 \end{pmatrix}$$

c.

$$\begin{pmatrix} 3 & 6 \\ 2 & 4 \end{pmatrix}$$

d.

$$\begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix}$$

Problem 3. a. Use elementary row operations to solve the following system of equations.

$$\begin{pmatrix} 2 & 3 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 8 \\ 5 \end{pmatrix}$$

b.

$$\begin{pmatrix} 3 & 6 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 15 \\ 10 \end{pmatrix}$$

Problem 4. a. Use elementary row operations to find the inverse of the following matrices.

$$\begin{pmatrix} -2 & 3 \\ 3 & 4 \end{pmatrix}$$

The answer is $\begin{pmatrix} \frac{-4}{17} & \frac{3}{17} \\ \frac{3}{17} & \frac{2}{17} \end{pmatrix}$.

b.

$$\begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix}$$

Problem 5. a. Use the inverse you found in problem 4a to solve the following equation.

$$\begin{pmatrix} -2 & 3 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 11 \\ 9 \end{pmatrix}$$

b. Use the inverse you found in problem 4b to solve the following equation.

$$\begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

Problem 6. Compute the determinants of the following matrices using the cofactor factor method. Check using the general formula.

a. A

b. C

c. D

d. E

Problem 7. a. Use elementary row operations to solve the following system of equations. The answers are $x_1 = -9$, $x_2 = -4$, $x_3 = 3$.

$$Dx = b$$

$$\begin{bmatrix} 1 & -2 & 1 \\ -3 & 5 & -2 \\ 4 & -6 & 3 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$$

- b. Use elementary row operations to solve the following system of equations. Use elementary row operations to solve the following system of equations. The answers are $x_1 = 7$, $x_2 = -5$, $x_3 = 0$.

$$Ex = b$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 4 & -4 \\ -4 & -5 & 3 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$$

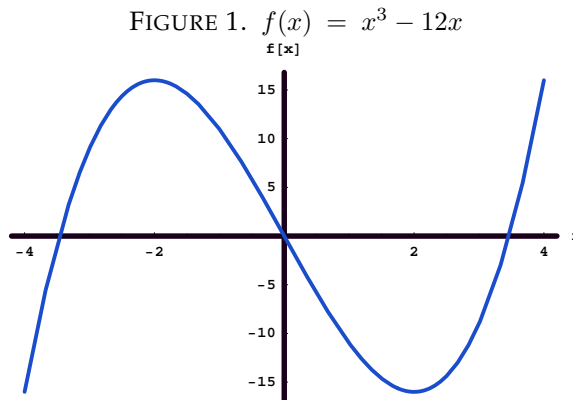
- c. Use elementary row operations to solve the following system of equations. The answers are $x_1 = 4$, $x_2 = 3$, $x_3 = -6$.

$$Fx = a$$

$$, \quad F = \begin{bmatrix} -8 & -13 & -12 \\ 7 & 11 & 10 \\ 1 & 1 & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

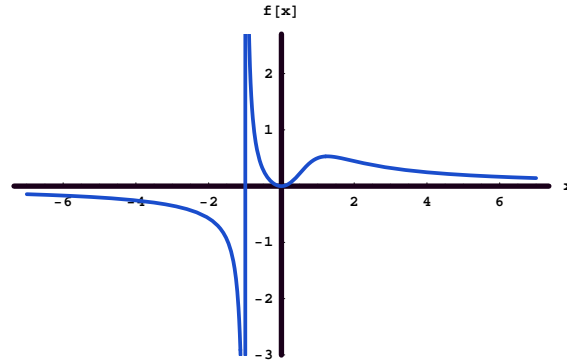
Problem 8. For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise. Check to see if there are potential points of inflection **at points other than** critical points.

a. $f(x) = x^3 - 12x$.



b. $f(x) = \frac{x^2}{x^3+1}$

FIGURE 2. $f(x) = \frac{x^2}{x^3+1}$



Hints: The second derivative of the function is $\frac{2(x^6-7x^3+1)}{(x^3+1)^3}$. To solve the following equation $x^6 - 7x^3 + 1 = 0$, use the substitution $z = x^3$. The real points of inflection (you still need to find them) are $\left(\frac{7 \pm 3\sqrt{5}}{2}\right)^{1/3}$.

c. $f(x) = \frac{1}{2}x^4 + 2x^3 - 9x^2 + 10x$

FIGURE 3. $f(x) = \frac{x^4}{2} + 2x^3 - 9x^2 + 10x$

