ECONOMICS 207
SPRING 2007
LABORATORY EXERCISE 11

Problem 1. Consider the following matrix and vector.

$$P = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}, \quad p = \begin{bmatrix} 2 \\ 4 \end{bmatrix},$$

a. Use elementary row operations to find the inverse of P and solve the equation Px=p in one set of operations.

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b. Find the determinant of the matrix P.

$$P = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}, \quad p = \begin{bmatrix} 2 \\ 4 \end{bmatrix},$$

c. Find the inverse of the matrix P using the cofactor/adjoint method.

d. Solve the equation Px=p using the inverse you found in part 1c

e. Solve the equation Px=p using Cramer's rule.

$$P = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}, \quad p = \begin{bmatrix} 2 \\ 4 \end{bmatrix},$$

Problem 2. Consider the following matrix and vector.

$$Q = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}, \quad q = \begin{bmatrix} 5 \\ 5 \end{bmatrix},$$

a. Use elementary row operations to find the inverse of Q and solve the equation Qx=q in one set of operations.

b. Find the determinant of the matrix Q.

$$Q = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}, \quad q = \begin{bmatrix} 5 \\ 5 \end{bmatrix},$$

c. Find the inverse of the matrix Q using the cofactor/adjoint method.

d. Solve the equation Qx=q using the inverse you found in part 2c

e. Solve the equation Qx=q using Cramer's rule.

$$Q = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}, \quad q = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

Problem 3. Consider the following matrix and vector.

$$D = \begin{bmatrix} 1 & -2 & 1 \\ -3 & 5 & -2 \\ 4 & -6 & 3 \end{bmatrix}, \quad d = \begin{bmatrix} -2 \\ 5 \\ -5 \end{bmatrix}$$

a. Use elementary row operations to find the inverse of D and solve the equation Dx=d in one set of operations.

b. Find the determinant of the matrix D.

$$D = \begin{bmatrix} 1 & -2 & 1 \\ -3 & 5 & -2 \\ 4 & -6 & 3 \end{bmatrix}, \quad d = \begin{bmatrix} -2 \\ 5 \\ -5 \end{bmatrix}$$

c. Find the inverse of the matrix D using the cofactor/adjoint method.

$$D = \begin{bmatrix} 1 & -2 & 1 \\ -3 & 5 & -2 \\ 4 & -6 & 3 \end{bmatrix}, \quad d = \begin{bmatrix} -2 \\ 5 \\ -5 \end{bmatrix},$$

d. Solve the equation Dx=d using the inverse you found in part 3c

$$D = \begin{bmatrix} 1 & -2 & 1 \\ -3 & 5 & -2 \\ 4 & -6 & 3 \end{bmatrix}, \quad d = \begin{bmatrix} -2 \\ 5 \\ -5 \end{bmatrix}$$

e. Solve the equation Dx=d using Cramer's rule.

Problem 4. Consider the following matrix and vector.

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & -4 \\ 3 & -1 & 6 \end{bmatrix}, \quad a = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

a. Use elementary row operations to find the inverse of A and solve the equation Ax=a in one set of operations.

b. Find the determinant of the matrix A.

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & -4 \\ 3 & -1 & 6 \end{bmatrix}, \quad a = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

c. Find the inverse of the matrix A using the cofactor/adjoint method.

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & -4 \\ 3 & -1 & 6 \end{bmatrix}, \quad a = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

d. Solve the equation Ax=a using the inverse you found in part 4c

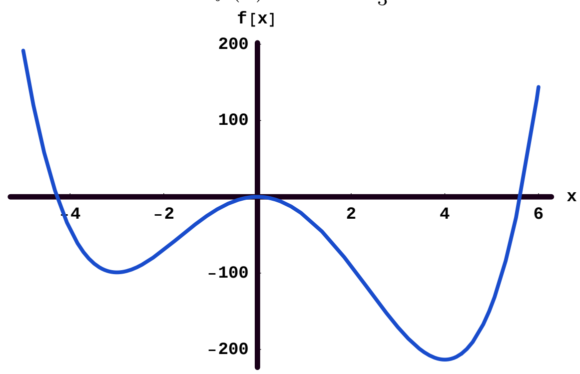
$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & -4 \\ 3 & -1 & 6 \end{bmatrix}, \quad a = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

e. Solve the equation Ax=a using Cramer's rule.

Problem 5. For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise. Check to see if there are potential points of inflection **at points other than** critical points.

a. $f(x) = x^4 - \frac{4}{3}x^3 - 24x^2$.

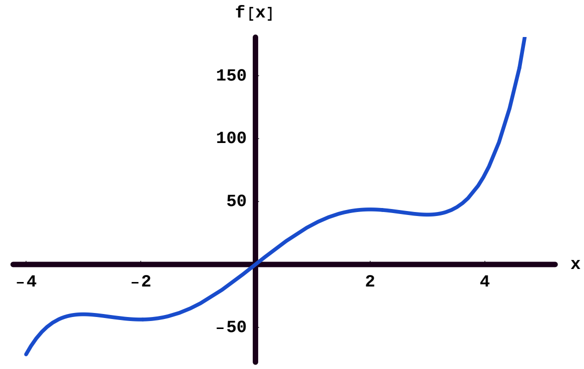
FIGURE 1. $f(x) = x^4 - \frac{4}{3}x^3 - 24x^2$



The inflection points are $\frac{1 \pm \sqrt{37}}{3}$.

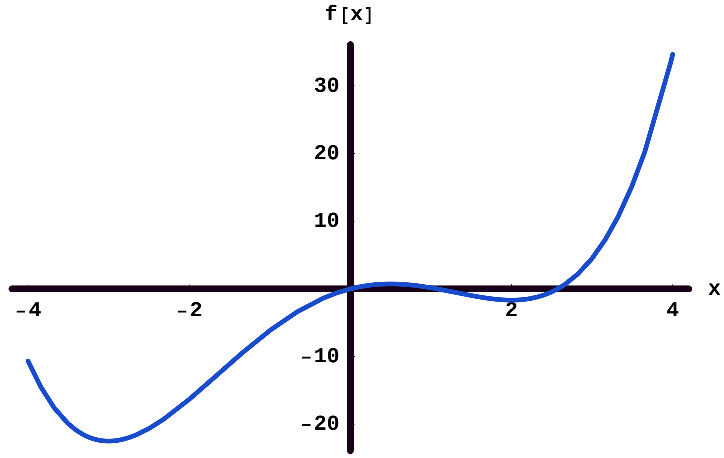
b. $f(x) \;=\; \dfrac{x^5}{5} - \dfrac{13}{3}x^3 + 36x$

FIGURE 2. $f(x) \;=\; \frac{x^5}{5} - \frac{13}{3}x^3 + 36x$



c. $f(x) = \frac{1}{4}x^4 + \frac{1}{6}x^3 - \frac{13}{4}x^2 + 3x$

FIGURE 3. $f(x) = \frac{1}{4}x^4 + \frac{1}{6}x^3 - \frac{13}{4}x^2 + 3x$



The inflection points are $\frac{-1\pm\sqrt{79}}{6}$

Problem 6. In the following problem you are given a production function for a firm where y is the level of output and x is the level of the variable input. You are given the price (p) of the output and the price (w) of the single variable input. Write down an equation that represents profit for the firm. Then maximize this function by taking its derivative with respect to the variable input x and set equal to zero. What is the optimal level of x? Show why this level of x maximizes profit.

$$output\ price \ = \ p \ = 5$$

$$input\ price \ = \ w \ = 970$$

$$y \ = \ output \ = \ f(x) \ = \ 40x + 40x^2 - 2x^3$$

Problem 7. Find all first partial derivatives of each of the following

a. $f(x) = 4x_1^3 - 6x_2^2 + 30$

b. $y = 3x_1^3 x_2^2$

c. $f(x) = 1080 x_1^{1/3} x_2^{2/5}$

d. $f(x) \;=\; 30x_1^{2/3}x_2^{3/5}x_3^{5/6}$

e. $f(x) \;=\; 30x_1^{1/3}x_2^{3/5}x_3^{1/2} - 4x_1 - 5x_2 - x_3$

f. $f(x) \;=\; 2x_1^{1/2} + 4x_2^{1/2} + 6x_3^{1/2} + 3x_1 + 2x_1^{1/2}x_2^{1/2} + 4x_1^{1/2}x_3^{1/2} + 5x_2 + 6x_2^{1/2}x_3^{1/2} + x_3$

Problem 8. Solve the following systems of equations for x_1 and x_2 using the method of substitution.
a.

$$360x_1^{-2/3}x_2^{2/5} - 160 = 0 \tag{8a.1}$$

$$432x_1^{1/3}x_2^{-3/5} - 162 = 0 \tag{8a.2}$$

Rearrange the first equation 8a.1 to obtain

$$\begin{aligned}
x_1^{-2/3}x_2^{2/5} &= \frac{160}{360} = \frac{4}{9} \\
\Rightarrow x_1^{2/3}x_1^{-2/3}x_2^{2/5} &= \frac{4}{9}x_1^{2/3} \\
\Rightarrow x_2^{2/5} &= \frac{4}{9}x_1^{2/3} \\
\Rightarrow x_2 &= \left(\frac{4}{9}\right)^{5/2}\left(x_1^{2/3}\right)^{5/2} \\
&= \left(\frac{4}{9}\right)^{5/2} x_1^{5/3}
\end{aligned} \tag{8a.1.a}$$

Rearrange the second equation 8a.2 slightly to obtain

$$x_1^{1/3}x_2^{-3/5} = \frac{162}{432} = \frac{3}{8} \tag{8a.2$'$}$$

Now substitute x_2 from equation 8a.1.a into equation 8a.2′ to obtain

$$\begin{aligned}
x_1^{1/3}\left(\left(\frac{4}{9}\right)^{5/2} x_1^{5/3}\right)^{-3/5} &= \frac{3}{8} \\
\Rightarrow x_1^{1/3}\left(\frac{4}{9}\right)^{-3/2} x_1^{-1} &= \frac{3}{8} \\
\Rightarrow x_1^{-2/3}\left(\frac{4}{9}\right)^{-3/2} &= \frac{3}{8} \\
\Rightarrow x_1^{-2/3} &= \frac{3}{8}\left(\frac{4}{9}\right)^{3/2} \\
\Rightarrow x_1 &= \left(\frac{3}{8}\left(\frac{4}{9}\right)^{3/2}\right)^{-3/2} = \left(\frac{3}{8}\right)^{-3/2}\left(\frac{4}{9}\right)^{-9/4} \\
&= 3^{-3/2}2^{9/2}2^{-18/4}3^{18/4} = 3^{6/2} = 3^3 = 27
\end{aligned} \tag{8a.2.a}$$

Now substitute x_1 from equation 8a.2.a into equation 8a.1.a to obtain

$$\begin{aligned}
x_2 &= \left(\frac{4}{9}\right)^{5/2} 27^{5/3} \\
&= 2^{10/2}\,3^{-10/2}\,3^5 = 2^5 = 32
\end{aligned}$$