ECONOMICS 207
SPRING 2007
LABORATORY EXERCISE 12

Problem 1. Consider the following matrix and vector.

$$P = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}, \quad p = \begin{bmatrix} 2 \\ 4 \end{bmatrix},$$

a. Use elementary row operations to find the inverse of P and solve the equation Px=p in one set of operations.

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b. Find the determinant of the matrix P.

$$P = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}, \quad p = \begin{bmatrix} 2 \\ 4 \end{bmatrix},$$

c. Find the inverse of the matrix P using the cofactor/adjoint method.

d. Solve the equation Px=p using the inverse you found in part 1c

e. Solve the equation Px=p using Cramer's rule.

$$P = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}, \quad p = \begin{bmatrix} 2 \\ 4 \end{bmatrix},$$

Problem 2. Consider the following matrix and vector.

$$D = \begin{bmatrix} 1 & 1 & -1 \\ -5 & -3 & 4 \\ 2 & -1 & 0 \end{bmatrix}, \quad d = \begin{bmatrix} -1 \\ 5 \\ 2 \end{bmatrix}$$

a. Use elementary row operations to find the inverse of D and solve the equation Dx=d in one set of operations.

b. Find the determinant of the matrix D.

$$D = \begin{bmatrix} 1 & 1 & -1 \\ -5 & -3 & 4 \\ 2 & -1 & 0 \end{bmatrix}, \quad d = \begin{bmatrix} -1 \\ 5 \\ 2 \end{bmatrix}$$

c. Find the inverse of the matrix D using the cofactor/adjoint method.

$$D = \begin{bmatrix} 1 & 1 & -1 \\ -5 & -3 & 4 \\ 2 & -1 & 0 \end{bmatrix}, \quad d = \begin{bmatrix} -1 \\ 5 \\ 2 \end{bmatrix}$$

d. Solve the equation Dx=d using the inverse you found in part 2c

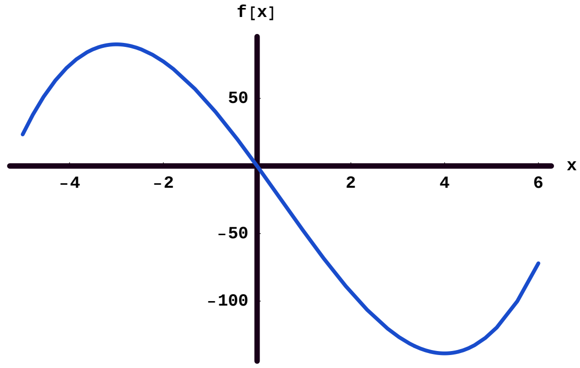
$$D = \begin{bmatrix} 1 & 1 & -1 \\ -5 & -3 & 4 \\ 2 & -1 & 0 \end{bmatrix}, \quad d = \begin{bmatrix} -1 \\ 5 \\ 2 \end{bmatrix}$$

e. Solve the equation Dx=d using Cramer's rule.

Problem 3. For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise. Check to see if there are potential points of inflection **at points other than** critical points.

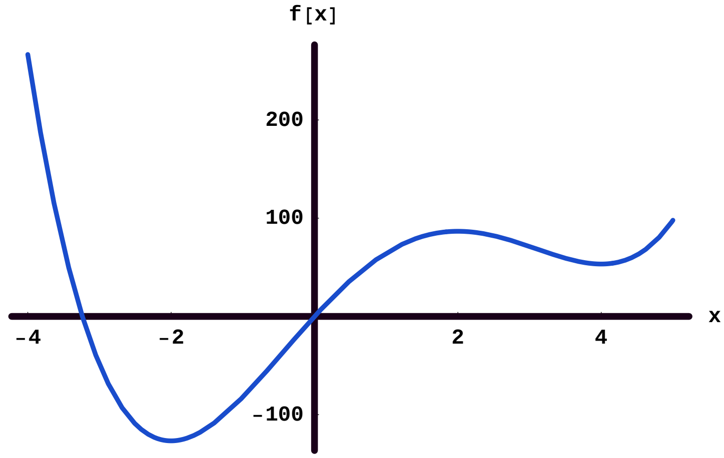
a. $f(x) = \dfrac{4}{3}x^3 - 2x^2 - 48x.$

FIGURE 1. $f(x) = \frac{4}{3}x^3 - 2x^2 - 48x$



b. $f(x) = \frac{5}{4}x^4 - \frac{20}{3}x^3 - 10x^2 + 80x$

FIGURE 2. $f(x) = \frac{5}{4}x^4 - \frac{20}{3}x^3 - 10x^2 + 80x$



The inflection points are $\frac{2}{3}\left(2 \pm \sqrt{7}\right)$

Problem 4. Solve the following systems of equations for x_1 and x_2 using the method of substitution.

$$360x_1^{-2/3}x_2^{2/5} \ - \ 160 \ = 0 \tag{4.1}$$

$$432x_1^{1/3}x_2^{-3/5} \ - \ 162 \ = 0 \tag{4.2}$$

The answers are x_1 = 27 and x_2 = 32.

Problem 5. In the following problem you are given a production function for a firm where y is the level of output and x is the level of the variable input. You are given the price (p) of the output and the price (w) of the single variable input. Write down an equation that represents profit for the firm. Then maximize this function by taking its derivative with respect to the variable input x and set equal to zero. What is the optimal level of x? Show why this level of x maximizes profit.

$$output\ price \ = \ p \ = 6$$

$$input\ price \ = \ w \ = 2916$$

$$y \ = \ output \ = \ f(x) \ = \ 200x + 50x^2 - 2x^3$$

Problem 6. Find all first and second partial derivatives of each of the following

a. $f(x_1, x_2) = -2x_1^2 - x_2^2 + 100x_1 + 50x_2$

$\frac{\partial f}{\partial x_1}$	$\frac{\partial f}{\partial x_2}$
$\frac{\partial^2 f}{\partial x_1 \partial x_1}$	$\frac{\partial^2 f}{\partial x_1 \partial x_2}$
$\frac{\partial^2 f}{\partial x_2 \partial x_1}$	$\frac{\partial^2 f}{\partial x_2 \partial x_2}$

b. $f(x_1, x_2) = -4x_1^2 + 3x_1x_2 - 2x_2^2 + 80x_1 + 40x_2$

$\frac{\partial f}{\partial x_1}$	$\frac{\partial f}{\partial x_2}$
$\frac{\partial^2 f}{\partial x_1 \partial x_1}$	$\frac{\partial^2 f}{\partial x_1 \partial x_2}$
$\frac{\partial^2 f}{\partial x_2 \partial x_1}$	$\frac{\partial^2 f}{\partial x_2 \partial x_2}$

c. $f(x_1, x_2, x_3) = -2x_1^2 + x_1x_2 + 3x_2^2 + 40x_1 - 60x_2$

$\frac{\partial f}{\partial x_1}$	$\frac{\partial f}{\partial x_2}$
$\frac{\partial^2 f}{\partial x_1 \partial x_1}$	$\frac{\partial^2 f}{\partial x_1 \partial x_2}$
$\frac{\partial^2 f}{\partial x_2 \partial x_1}$	$\frac{\partial^2 f}{\partial x_2 \partial x_2}$

d. $f(x_1, x_2, x_3) = 100x_1^{1/2}x_2^{1/5}x_3^{1/3} - 4x_1 - 3x_2 - 5x_3$

$\frac{\partial f}{\partial x_1}$	$\frac{\partial f}{\partial x_2}$	$\frac{\partial f}{\partial x_3}$
$\frac{\partial^2 f}{\partial x_1 \partial x_1}$	$\frac{\partial^2 f}{\partial x_1 \partial x_2}$	$\frac{\partial^2 f}{\partial x_1 \partial x_3}$
$\frac{\partial^2 f}{\partial x_2 \partial x_1}$	$\frac{\partial^2 f}{\partial x_2 \partial x_2}$	$\frac{\partial^2 f}{\partial x_2 \partial x_3}$
$\frac{\partial^2 f}{\partial x_3 \partial x_1}$	$\frac{\partial^2 f}{\partial x_3 \partial x_2}$	$\frac{\partial^2 f}{\partial x_3 \partial x_3}$

Problem 7. For each of the following problems, write an equation that represents profit as a function of the two inputs x_1 and x_2. Write it in the form π = $pf(x_1,x_2)-w_1x_1-w_2x_2$ and then simplify the expression. Then find all first and second partial derivatives of the function at the specified point.

a.

$$\begin{aligned} f(x_1, x_2) &= 60x_1 + 42x_2 - x_1^2 + x_1x_2 - x_2^2 \\ p &= 4 \\ w_1 &= 60, \qquad w_2 = 12 \\ x_1 &= 43, \qquad x_2 = 41 \end{aligned}$$

$\frac{\partial \pi}{\partial x_1}$	$\frac{\partial \pi}{\partial x_2}$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2}$

b.

$$
\begin{aligned}
f(x_1, x_2) &= 30x_1 \; + \; 15x_2 \; - \; 2x_1^2 \; + \; x_1x_2 \; - \; x_2^2 \\
p &= 4 \\
w_1 &= 60, \qquad w_2 \; = \; 12 \\
x_1 &= 6, \qquad x_2 \; = \; 9
\end{aligned}
$$

$\frac{\partial \pi}{\partial x_1}$	$\frac{\partial \pi}{\partial x_2}$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2}$

c.

$$
\begin{aligned}
f(x_1, x_2) &= 40x_1 \ + \ 10x_2 \ - \ 2x_1^2 \ + \ 2x_1x_2 \ - \ x_2^2 \\
p &= 1 \\
w_1 &= 34, \qquad w_2 \ = \ 2 \\
x_1 &= 7, \qquad x_2 \ = \ 11
\end{aligned}
$$

$\frac{\partial \pi}{\partial x_1}$	$\frac{\partial \pi}{\partial x_2}$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2}$

d.

$$\begin{aligned} f(x_1, x_2) &= x_1^{3/5} x_2^{1/3} \\ p &= 240 \\ w_1 &= 64, \qquad w_2 \ = \ 135 \\ x_1 &= 243, \qquad x_2 \ = \ 64 \end{aligned}$$

$\frac{\partial \pi}{\partial x_1}$	$\frac{\partial \pi}{\partial x_2}$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1} = -\frac{128}{1215}$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2} = 48 x_1^{-2/5} x_2^{-2/3}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2} = -\frac{45}{32}$

e.

$$
\begin{aligned}
f(x_1, x_2) &= x_1^{1/4} x_2^{3/7} \\
p &= 224 \\
w_1 &= 7, \qquad w_2 \ = \ 24 \\
x_1 &= 256, \qquad x_2 \ = \ 128
\end{aligned}
$$

$\frac{\partial \pi}{\partial x_1}$	$\frac{\partial \pi}{\partial x_2}$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1} = -\frac{21}{1024}$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2} = -\frac{3}{28}$