ECONOMICS 207
SPRING 2007
LABORATORY EXERCISE 13

Problem 1. For the following problem, write an equation that represents profit as a function of the input x. Write it in the form π = pf(x)-wx and then simplify the expression. Then find the first and second derivatives of the function. Then find the critical points. For each critical point state whether profit is at a relative maximum, relative minimum, or otherwise. Check to see if there are points of inflection **at points other than** critical points.

$$\begin{aligned} f(x) &= 30x + 30x^2 - 2x^3 \\ p &= 2 \\ w &= 168 \end{aligned}$$

Profit is the difference between revenue and cost. Revenue for this firm is price $\times$ output or py. Output depends on the amount of the input used so we can write revenue as

$$Revenue = py = pf(x) = 2(30x + 30x^2 - 2x^3) = 60x + 60x^2 - 4x^3$$

There is only one input for this firm so the cost of production is just the amount of the input used multiplied by its price.

$$Cost = wx = 168x$$

Profits are as follows.

$$\begin{aligned} Profit = \pi &= Revenue - Cost = 60x + 60x^2 - 4x^3 - 168x \\ &= -4x^3 + 60x^2 - 108x \end{aligned}$$

To maximize profit we take the derivative of the profit equation with respect to x, set the equation equal to zero and solve for x.

$$\begin{aligned} \pi &= -4x^3 + 60x^2 - 108x \\ \frac{d\pi}{dx} &= -12x^2 + 120x - 108 = 0 \\ &\Rightarrow -12(x^2 - 10x + 9) = 0 \\ &\Rightarrow (x^2 - 10x + 9) = 0 \\ &\Rightarrow (x - 9)(x - 1) = 0 \\ &\Rightarrow x = 9 \text{ and } x = 1 \end{aligned}$$

The second derivative of profit is given by

$$\frac{d^2\pi}{dx^2} = -24x + 120$$

We then evaluate the second derivative at each root.

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$$\frac{d^2\pi}{dx^2}(9) = (-24)(9) + 120 = -216 + 120 = -96$$

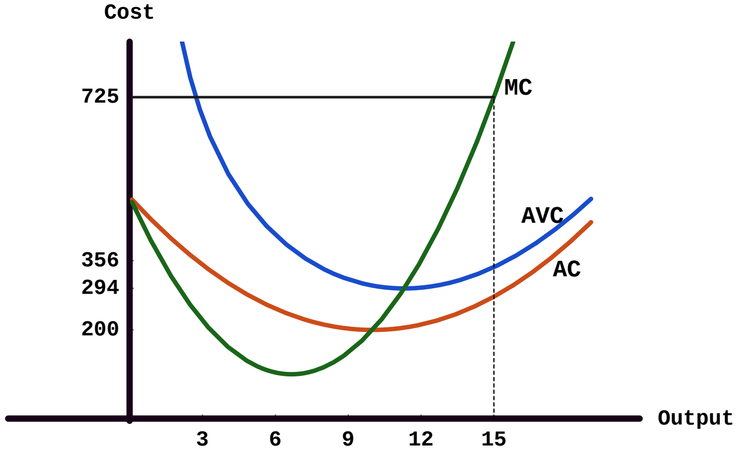
$\Rightarrow x = 9$ is a maximum

$$\frac{d^2\pi}{dx^2}(1) = (-24)(1) + 120 = 96$$

$\Rightarrow x = 1$ is a minimum

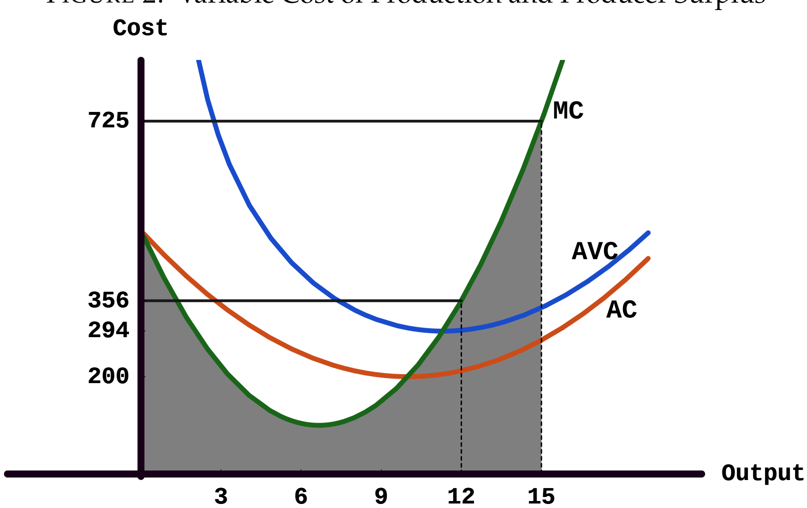
Problem 2. The cost function for a firm is a rule or mapping that tells the total cost of production of any output level produced by the firm. If the variable y represents the output of the firm, then the cost function is given by c(y). Marginal cost represents the change in the cost of production for the firm as output changes and is given by the derivative of the cost function with respect to output,i.e., Marginal Cost(MC) = $\frac{dc(y)}{dy}$. A competitive firm facing a fixed output price maximizes profit at the output level where marginal cost is equal to price as in the figure 1.

FIGURE 1. Profit Maximization



The area below the cost curve is a measure of variable cost and can be found by integrating the marginal cost curve from 0 to any given output level y. The shaded area in figure 2 represents the variable cost of production for the cost function $c(y) = 1000 + 500y - 60\,y^2 + 3y^3$.

FIGURE 2. Variable Cost of Production and Producer Surplus



Producer surplus is the area below a given price and above the marginal cost curve. Producer surplus is the unshaded area below the horizontal line at 725 in figure 2. Producer surplus can be computed by subtracting the shaded area from total revenue.

a. Find the profit maximizing level of output for the following firm. Demonstrate that the level you choose maximizes profit.

$$price \;=\; p \;= \$725$$

$$cost \;=\; c(y) \;= 1000 \;+\; 500y \;-\; 60y^2 \;+\; 3y^3$$

Profit is the difference between revenue and cost. Revenue for this firm is price $\times$ output or py. The cost for this firm as a function of y is given by $1000 \;+\; 500y \;-\; 60y^2 \;+\; 3y^3$.

Profits are as follows.

$$\begin{aligned} Profit =& \pi = Revenue - Cost = 725y - (1000 \;+\; 500y \;-\; 60y^2 \;+\; 3y^3) \\ =& 725y - 1000 \;-\; 500y \;+\; 60y^2 \;-\; 3y^3 \\ =& 225y - 1000 \;+\; 60y^2 \;-\; 3y^3 \end{aligned}$$

To maximize profit we take the derivative of the profit equation with respect to y, set the equation equal to zero and solve for y.

$$\begin{aligned} \pi = 225y - 1000 \;+\; 60y^2 \;-\; 3y^3) \\ \frac{d\pi}{dy} = 225 + 120y - 9y^2 =& 0 \\ \Rightarrow -3(3y^2 - 40y - 75) =& 0 \\ \Rightarrow 3y^2 - 40y - 75 =& 0 \\ \Rightarrow (3y+5)(y-15) =& 0 \\ \Rightarrow y = -\frac{5}{3} \text{ and } y =& 15 \end{aligned}$$

The second derivative of profit is given by

$$\frac{d^2\pi}{dy^2} = 120 - 18y$$

We then evaluate the second derivative at each root.

$$\begin{aligned} \frac{d^2\pi}{dy^2}(15) =& 120 - (18)(15) = 120 - 270 = -150 \\ \Rightarrow y =& 15 \text{ is a maximum} \\ \frac{d^2\pi}{dy^2}(1) =& 120 - (18)(-\frac{5}{3}) = 120 - (-30) = 150 \\ \Rightarrow y =& -\frac{5}{3} \text{is a minimum} \end{aligned}$$

b. Find the profit maximizing level of output when the price is \$356. Demonstrate that the level you choose maximizes profit.

Profit is the difference between revenue and cost. Revenue for this firm is price × output or py or 356y. The cost for this firm as a function of y is given by $1000 + 500y - 60y^2 + 3y^3$.

Profits are as follows.

$$\begin{aligned} Profit =&\pi = Revenue - Cost = 356y - (1000 + 500y - 60y^2 + 3y^3) \\ =&356y - 1000 - 500y + 60y^2 - 3y^3 \\ =&-144y - 1000 + 60y^2 - 3y^3 \end{aligned}$$

To maximize profit we take the derivative of the profit equation with respect to y, set the equation equal to zero and solve for y.

$$\begin{aligned} \pi = -144y - 1000 + 60y^2 - 3y^3) \\ \frac{d\pi}{dy} = -144 + 120y - 9y^2 =&0 \\ \Rightarrow -3(3y^2 - 40y - 48) =&0 \\ \Rightarrow 3y^2 - 40y + 48 =&0 \\ \Rightarrow (3y - 4)(y - 12) =&0 \\ \Rightarrow y = \frac{4}{3} \text{ and } y =&12 \end{aligned}$$

The second derivative of profit is given by

$$\frac{d^2\pi}{dy^2} = 120 - 18y$$

We then evaluate the second derivative at each root.

$$\begin{aligned} \frac{d^2\pi}{dy^2}(12) =&120 - (18)(12) = 120 - 216 = -96 \\ \Rightarrow y =&12 \text{ is a maximum} \\ \frac{d^2\pi}{dy^2}(\frac{4}{3}) =&120 - (18)(\frac{4}{3}) = 120 - (24) = 96 \\ \Rightarrow y =&\frac{4}{3} \text{is a minimum} \end{aligned}$$

c. Show that variable cost for this firm is \$4125 when price is \$725? When the price is \$725, the firm produces 15 units of output.

$$\begin{aligned} Variable\ cost &= 500y - 60y^2 + 3y^3 \\ &= 500 \times 15 - 60 \times 225 + 3 \times 3375 \\ &= 4125 \end{aligned}$$

d. What is producer surplus for this firm when the price is \$725?
Producer surplus is revenue minus variable cost. For this firm revenue is given by 725 × 15 = \$10,875. So producer surplus is given by

$$Producer\ surplus = 10875 - 4125 = 6750.$$

e. What is variable cost for this firm when price is \$356? When the price is \$725, the firm produces 12 units of output.

$$\begin{aligned} Variable\ cost &= 500y - 60y^2 + 3y^3 \\ &= 500 \times 12 - 60 \times 144 + 3 \times 1728 \\ &= 2544 \end{aligned}$$

f. What is producer surplus for this firm when the price is \$356?
Producer surplus is revenue minus variable cost. For this firm revenue is given by 356 × 12 = \$4,272. So producer surplus is given by

$$Producer\ surplus = 4272 - 2544 = 1728.$$

g. Show that the firm is \$5022 worse off when price falls from \$725 to \$356?
We subtract the producer surplus when the price is \$356 from producer surplus when the price is \$725.

$$\Delta Producer\ surplus = 6750 - 1728 = 5022.$$

h. Cross-hatch the change in producer surplus in Figure 2.

Problem 3. For each of the following problems, write an equation that represents profit as a function of the two inputs x_1 and x_2. Write it in the form π = $pf(x_1,x_2)-w_1x_1-w_2x_2$ and then simplify the expression. Then find all first and second partial derivatives of the function.

a.

$$
\begin{aligned}
f(x_1,x_2) &= 10x_1 + 40x_2 - x_1^2 + x_1x_2 - x_2^2 \\
p &= 4 \\
w_1 &= 60, \qquad w_2 \;=\; 24 \\
\pi &= 4(10x_1 + 40x_2 - x_1^2 + x_1x_2 - x_2^2) - 60x_1 - 24x_2 \\
&= 40x_1 + 160x_2 - 4x_1^2 + 4x_1x_2 - 4x_2^2 - 60x_1 - 24x_2 \qquad (1) \\
&= -\,20x_1 + 136x_2 - 4x_1^2 + 4x_1x_2 - 4x_2^2
\end{aligned}
$$

$\frac{\partial \pi}{\partial x_1} = -20 - 8x_1 + 4x_2$	$\frac{\partial \pi}{\partial x_2} = 136 + 4x_1 - 8x_2$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1} = -8$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2} = 4$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1} = 4$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2} = -8$

Find potential profit maximizing levels of x_1 and x_2.
We maximize profit by taking the derivatives of (1) setting them equal to zero and solving for x_1 and x_2.

$$\frac{\partial \pi}{\partial x_1} = -20 - 8x_1 + 4x_2 = 0 \tag{2a}$$

$$\frac{\partial \pi}{\partial x_2} = 136 + 4x_1 - 8x_2 = 0 \tag{2b}$$

Multiply equation 2a by 2 and add to 2b to obtain

$$\begin{aligned} -40 - 16x_1 \quad +8x_2 &= \quad 0 \\ 136 + 4x_1 \quad -8x_2 &= \quad 0 \\ \Rightarrow 96 - 12x_1 \quad +0x_2 &= \quad 0 \\ \Rightarrow \qquad\qquad x_1 &= \quad 8 \end{aligned} \tag{3}$$

Now substitute x_1 in equation 2a and solve for x_2.

$$\begin{aligned} -20 - 8x_1 + 4x_2 &= 0 \\ \Rightarrow -20 - 8(8) + 4x_2 &= 0 \\ \Rightarrow 4x_2 &= 84 \\ \Rightarrow x_2 &= 21 \end{aligned} \tag{4}$$

By evaluating the Hessian matrix of the profit equation at the critical values, verify the optimal levels of x_1 and x_2.

$$\begin{vmatrix} \frac{\partial^2 \pi}{\partial x_1 \partial x_1} = -8 & \frac{\partial^2 \pi}{\partial x_1 \partial x_2} = 4 \\ \frac{\partial^2 \pi}{\partial x_2 \partial x_1} = 4 & \frac{\partial^2 \pi}{\partial x_2 \partial x_2} = -8 \end{vmatrix} = 64 - 14 = 48 > 0.$$

Both diagonal elements are negative and the determinant of the Hessian is positive, so the input levels $x_1 = 8$, $x_2 = 21$ represent a point of profit maximization.

b.

$$
\begin{aligned}
f(x_1, x_2) &= 45x_1 + 15x_2 - 2x_1^2 + 2x_1x_2 - x_2^2 \\
p &= 10 \\
w_1 &= 150, \qquad w_2 \ = \ 10
\end{aligned}
$$

$\frac{\partial \pi}{\partial x_1}$	$\frac{\partial \pi}{\partial x_2}$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2}$

Find potential profit maximizing levels of x_1 and x_2.

By evaluating the Hessian matrix of the profit equation at the critical values, verify the optimal levels of x_1 and x_2.

c.

$$\begin{aligned} f(x_1, x_2) &= x_1^{1/2} x_2^{1/3} \\ p &= 594 \\ w_1 &= 81, \qquad w_2 \;=\; 242 \\ x_1 &= 121, \qquad x_2 \;=\; 27 \end{aligned}$$

$$\pi \;=\; 594 x_1^{1/2} x_2^{1/3} - 81 x_1 - 242 x_2 \tag{5}$$

$\frac{\partial \pi}{\partial x_1} = 297 x_1^{-1/2} x_2^{1/3} - 81$	$\frac{\partial \pi}{\partial x_2} = 198 x_1^{1/2} x_2^{-2/3} - 242$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1} = -\frac{297}{2} x_1^{-3/2} x_2^{1/3}$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2} = 99 x_1^{-1/2} x_2^{-2/3}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1} = 99 x_1^{-1/2} x_2^{-2/3}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2} - 132 x_1^{-1/2} x_2^{-5/3}$

Find potential profit maximizing levels of x_1 and x_2.
From equation (5) we have

$$
\begin{aligned}
297 x_1^{-1/2} x_2^{1/3} - 81 &= 0 \qquad (5.1) \\
198 x_1^{1/2} x_2^{-2/3} - 242 &= 0 \qquad (5.2)
\end{aligned}
$$

Rearrange the first equation 5.1 to obtain

$$
\begin{aligned}
x_1^{-1/2} x_2^{1/3} &= \frac{81}{297} = \frac{3}{11} \\
\Rightarrow x_1^{1/2} x_1^{-1/2} x_2^{1/3} &= \frac{3}{11} x_1^{1/2} \\
\Rightarrow x_2^{1/3} &= \frac{3}{11} x_1^{1/2} \qquad (5.1.a) \\
\Rightarrow x_2 &= \left(\frac{3}{11}\right)^3 \left(x_1^{1/2}\right)^3 \\
&= \left(\frac{3}{11}\right)^3 x_1^{3/2}
\end{aligned}
$$

Rearrange the second equation 5.2 slightly to obtain

$$
x_1^{1/2} x_2^{-2/3} = \frac{242}{198} = \frac{11}{9} \qquad (5.2')
$$

Now substitute x_2 from equation 5.1.a into equation 5.2′ to obtain

$$
\begin{aligned}
x_1^{1/2} \left(\left(\frac{3}{11}\right)^3 x_1^{3/2} \right)^{-2/3} &= \frac{11}{9} \\
\Rightarrow x_1^{1/2} \left(\frac{3}{11}\right)^{-2} x_1^{-1} &= \frac{11}{9} \\
\Rightarrow x_1^{-1/2} \left(\frac{3}{11}\right)^{-2} &= \frac{11}{9} \qquad (5.2.a) \\
\Rightarrow x_1^{-1/2} &= \frac{11}{9} \left(\frac{3}{11}\right)^2 \\
\Rightarrow x_1 &= \left(\frac{11}{9} \left(\frac{3}{11}\right)^2 \right)^{-2} = \left(\frac{11}{9}\right)^{-2} \left(\frac{3}{11}\right)^{-4} \\
&= 11^{-2} 3^4 3^{-4} 11^4 = 11^2 = 121
\end{aligned}
$$

Now substitute x_1 from equation 5.2.a into equation 5.1.a to obtain

$$\begin{aligned} x_2 &= \left(\frac{3}{11}\right)^3 x_1^{3/2} \\ &= \left(\frac{3}{11}\right)^3 (121)^{3/2} \\ &= 3^3\ 11^{-3}\ (1)^3 \ = \ 3^3 \ = \ 27 \end{aligned}$$

By evaluating the Hessian matrix of the profit equation at the critical values, verify the optimal levels of x_1 and x_2. Remember that $x_1 = 121, \quad x_2 = 27$. Substituting in the Hessian matrix we obtain the following.

$$\begin{aligned}
\frac{\partial^2\pi}{\partial x_1 \partial x_1} &= -\frac{297}{2} x_1^{-3/2} x_2^{1/3} \\
&= -\frac{297}{2}(121)^{-3/2}(27)^{1/3} \\
&= -\frac{3^3 \times 11}{2} 11^{-3} \times 3 \\
&= -\frac{3^4}{2 \times 11^2} \\
&= -\frac{81}{242}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2\pi}{\partial x_1 \partial x_2} &= 99 x_1^{-1/2} x_2^{-2/3} \\
&= 99(121)^{-1/2}(27)^{-2/3} \\
&= 3^2 \times 11 \times 11^{-1} \times 3^{-2} \\
&= 1
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2\pi}{\partial x_2 \partial x_1} &= 99 x_1^{-1/2} x_2^{-2/3} \\
&= 99(121)^{-1/2}(27)^{-2/3} \\
&= 3^2 \times 11 \times 11^{-1} \times 3^{-2} \\
&= 1
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2\pi}{\partial x_2 \partial x_2} &= -132 x_1^{1/2} x_2^{-(5/3} = \frac{-484}{81} \\
&= -132(121)^{1/2}(27)^{-5/3} \\
&= -2^2 \times 3 \times 11 \times 11 \times 3^{-5} \\
&= -\frac{4 \times 121}{3^4} \\
&= -\frac{484}{81}
\end{aligned}$$

$$= \left(\frac{-81}{242}\right)\left(\frac{-484}{81}\right) - (1)(1) = 2 - 1 = 1 > 0.$$

Both diagonal elements are negative and the determinant of the Hessian is positive, so the input levels $x_1 = 121, \; x_2 = 27$ represent a point of profit maximization.

d.

$$\begin{aligned} f(x_1, x_2) &= x_1^{1/3} x_2^{2/7} \\ p &= 84 \\ w_1 &= 7, \qquad w_2 \;=\; 3 \end{aligned}$$

$\frac{\partial \pi}{\partial x_1}$	$\frac{\partial \pi}{\partial x_2}$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1} = 8 x_1^{-2/3} x_2^{-5/7}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2} = -\frac{120}{7} x_1^{1/3} x_2^{-12/7}$

Find potential profit maximizing levels of x_1 and x_2.

By evaluating the Hessian matrix of the profit equation at the critical values, verify the optimal levels of x_1 and x_2.

Problem 4. Find the listed partial derivatives of each of the following functions.

a. $\mathcal{L}(x_1, x_2, \lambda) \ = \ 60x_1 + 24x_2 - \lambda\left(10x_1 + 40x_2 - x_1^2 + x_1x_2 - x_2^2 - 583\right)$

$\dfrac{\partial\mathcal{L}(x_1, x_2, \lambda)}{\partial x_1} = 60 - 10\lambda + 2\lambda x_1 - \lambda x_2$	$\dfrac{\partial\mathcal{L}(x_1, x_2, \lambda)}{\partial x_2} = 24 - 40\lambda - \lambda x_1 + 2\lambda x_2$	$\dfrac{\partial\mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda} = -10x_1 - 40x_2 + x_1^2 - x_1x_2 + x_2^2 + 583$
$\frac{\partial^2\mathcal{L}}{\partial x_1 \partial x_1} = 2\lambda$	$\frac{\partial^2\mathcal{L}}{\partial x_1 \partial x_2}$	-$\frac{\partial^2\mathcal{L}}{\partial x_1 \partial \lambda} = 10 - 2x_1 + x_2$
$\frac{\partial^2\mathcal{L}}{\partial x_2 \partial x_1} = -\lambda$	$\frac{\partial^2\mathcal{L}}{\partial x_2 \partial x_2}$	- $\frac{\partial^2\mathcal{L}}{\partial x_2 \partial \lambda}$
- $\frac{\partial^2\mathcal{L}}{\partial \lambda \partial x_1}$	-$\frac{\partial^2\mathcal{L}}{\partial \lambda \partial x_2} = 40 + x_1 - 2x_2$	-$\frac{\partial^2\mathcal{L}}{\partial \lambda \partial \lambda}$

b. $\mathcal{L}(x_1, x_2, \lambda) \;=\; 45x_1 + 15x_2 - 2x_1^2 + 2x_1x_2 - x_2^2 - \lambda\,(150x_1 \;+\; 10x_2 \;-\; 3590)$

$\frac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1}$	$\frac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2}$	$\frac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda}$
$\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial x_1}$	$\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial x_2}$	$-\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial \lambda}$
$\frac{\partial^2 \mathcal{L}}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \mathcal{L}}{\partial x_2 \partial x_2}$	$-\frac{\partial^2 \mathcal{L}}{\partial x_2 \partial \lambda}$
$-\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial x_1}$	$-\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial x_2}$	$-\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial \lambda}$

c. $\mathcal{L}(x_1, x_2, \lambda) \;=\; 81x_1 + 242x_2 - \lambda \left(x_1^{1/2} x_2^{1/3} - 33 \right)$

$\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1}$	$\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2}$	$\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda}$
$\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial x_1} = \frac{1}{4} \lambda x_2^{1/3} x_1^{-3/2}$	$\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial x_2} = -\frac{\lambda}{6} x_2^{-2/3} x_1^{-1/2}$	$-\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial \lambda} = \frac{1}{2} x_2^{1/3} x_1^{-1/2}$
$\frac{\partial^2 \mathcal{L}}{\partial x_2 \partial x_1} =$	$\frac{\partial^2 \mathcal{L}}{\partial x_2 \partial x_2} = \frac{2}{9} \lambda x_2^{-5/3} x_1^{1/2}$	$-\frac{\partial^2 \mathcal{L}}{\partial x_2 \partial \lambda} = \frac{1}{3} x_2^{-2/3} x_1^{1/2}$
$-\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial x_1}$	$-\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial x_2}$	$-\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial \lambda}$

d. $\mathcal{L}(x_1, x_2, \lambda) \;=\; x_1^{1/3} x_2^{2/7} - \lambda\,(7x_1 + 3x_2 - 832)$

$\frac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1}$	$\frac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2}$	$\frac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda}$
$\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial x_1} = -\frac{2x_2^{2/7}}{9x_1^{5/3}}$	$\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial x_2} = \frac{2}{21} x_1^{-2/3} x_2^{-5/7}$	$-\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial \lambda} = 7$
$\frac{\partial^2 \mathcal{L}}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \mathcal{L}}{\partial x_2 \partial x_2} = -\frac{10}{49} x_1^{1/3} x_2^{-12/7}$	$-\ \frac{\partial^2 \mathcal{L}}{\partial x_2 \partial \lambda}$
$-\ \frac{\partial^2 \mathcal{L}}{\partial \lambda \partial x_1}$	$-\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial x_2}$	$-\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial \lambda}$

Consider the following matrix and vector.

$$V = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 4 & 6 \\ -2 & -3 & -3 \end{bmatrix},$$

$$v = \begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix},$$

Problem 5.
Find the determinant of the matrix V.

b. Find the inverse of the matrix V using the adjoint method.

c. Using the inverse from part b, solve the system of equations

$$V \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = v$$

$$V = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 4 & 6 \\ -2 & -3 & -3 \end{bmatrix}, \quad v = \begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix}$$

d. Using Cramer's rule, solve the system of equations

$$V \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = v$$

$$V = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 4 & 6 \\ -2 & -3 & -3 \end{bmatrix}, \quad v = \begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix}$$

e. Using row reduction, find the inverse of the matrix V and the solution to the system of equations

$$V \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = v$$

$$V = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 4 & 6 \\ -2 & -3 & -3 \end{bmatrix}, \quad v = \begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix}$$