ECONOMICS 207
PROBLEM SET 11

Problem 1. Consider the following matrix and vector.

$$R = \begin{bmatrix} -3 & -2 \\ 5 & 3 \end{bmatrix}, \quad r = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$

a. Use elementary row operations to find the inverse of R and solve the equation Rx=r in one set of operations.

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b. Find the determinant of the matrix R.

$$R = \begin{bmatrix} -3 & -2 \\ 5 & 3 \end{bmatrix}, \quad r = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$

c. Find the inverse of the matrix R using the cofactor/adjoint method.

d. Solve the equation Rx=r using the inverse you found in part 1c

e. Solve the equation Rx=r using Cramer's rule.

$$R = \begin{bmatrix} -3 & -2 \\ 5 & 3 \end{bmatrix}, \quad r = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$

Problem 2. Consider the following matrix and vector.

$$S = \begin{bmatrix} 4 & -2 \\ -3 & 2 \end{bmatrix}, \quad s = \begin{bmatrix} 8 \\ -5 \end{bmatrix},$$

a. Use elementary row operations to find the inverse of S and solve the equation Sx=s in one set of operations.

b. Find the determinant of the matrix S.

$$S = \begin{bmatrix} 4 & -2 \\ -3 & 2 \end{bmatrix}, \quad s = \begin{bmatrix} 8 \\ -5 \end{bmatrix},$$

c. Find the inverse of the matrix S using the cofactor/adjoint method.

d. Solve the equation Sx=s using the inverse you found in part 2c

e. Solve the equation Sx=s using Cramer's rule.

$$S = \begin{bmatrix} 4 & -2 \\ -3 & 2 \end{bmatrix}, \quad s = \begin{bmatrix} 8 \\ -5 \end{bmatrix},$$

Problem 3. Consider the following matrix and vector.

$$G = \begin{bmatrix} 1 & 4 & -3 \\ -2 & -7 & 4 \\ 3 & 13 & -10 \end{bmatrix}, \quad g = \begin{bmatrix} 9 \\ -14 \\ 30 \end{bmatrix}$$

a. Use elementary row operations to find the inverse of G and solve the equation Gx=g in one set of operations.

b. Find the determinant of the matrix G.

$$G = \begin{bmatrix} 1 & 4 & -3 \\ -2 & -7 & 4 \\ 3 & 13 & -10 \end{bmatrix}, \quad g = \begin{bmatrix} 9 \\ -14 \\ 30 \end{bmatrix}$$

c. Find the inverse of the matrix G using the cofactor/adjoint method.

$$G = \begin{bmatrix} 1 & 4 & -3 \\ -2 & -7 & 4 \\ 3 & 13 & -10 \end{bmatrix}, \quad g = \begin{bmatrix} 9 \\ -14 \\ 30 \end{bmatrix}$$

d. Solve the equation Gx=g using the inverse you found in part 3c

$$G = \begin{bmatrix} 1 & 4 & -3 \\ -2 & -7 & 4 \\ 3 & 13 & -10 \end{bmatrix}, \quad g = \begin{bmatrix} 9 \\ -14 \\ 30 \end{bmatrix}$$

e. Solve the equation Gx=g using Cramer's rule.

Problem 4. Consider the following matrix and vector.

$$H = \begin{bmatrix} 1 & \frac{1}{2} & 2 \\ -3 & -1 & -2 \\ 3 & 1 & 4 \end{bmatrix}, \quad h = \begin{bmatrix} 0 \\ -5 \\ 3 \end{bmatrix}$$

a. Use elementary row operations to find the inverse of H and solve the equation Hx=h in one set of operations.

b. Find the determinant of the matrix H.

$$H = \begin{bmatrix} 1 & \frac{1}{2} & 2 \\ -3 & -1 & -2 \\ 3 & 1 & 4 \end{bmatrix}, \quad h = \begin{bmatrix} 0 \\ -5 \\ 3 \end{bmatrix}$$

c. Find the inverse of the matrix H using the cofactor/adjoint method.

$$H = \begin{bmatrix} 1 & \frac{1}{2} & 2 \\ -3 & -1 & -2 \\ 3 & 1 & 4 \end{bmatrix}, \quad h = \begin{bmatrix} 0 \\ -5 \\ 3 \end{bmatrix}$$

d. Solve the equation Hx=h using the inverse you found in part 4c

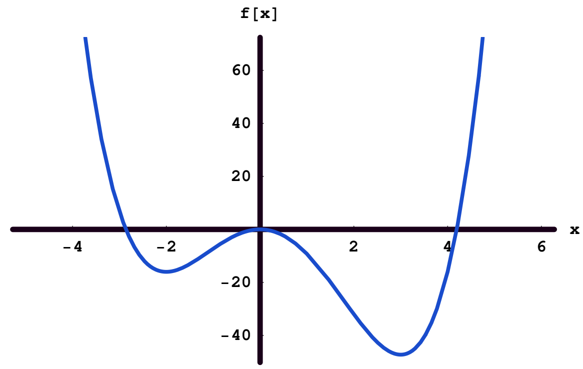
$$H = \begin{bmatrix} 1 & \frac{1}{2} & 2 \\ -3 & -1 & -2 \\ 3 & 1 & 4 \end{bmatrix}, \quad h = \begin{bmatrix} 0 \\ -5 \\ 3 \end{bmatrix}$$

e. Solve the equation Hx=h using Cramer's rule.

Problem 5. For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise. Check to see if there are potential points of inflection **at points other than** critical points.

a. $f(x) = \frac{3}{4}x^4 - x^3 - 9x^2$.

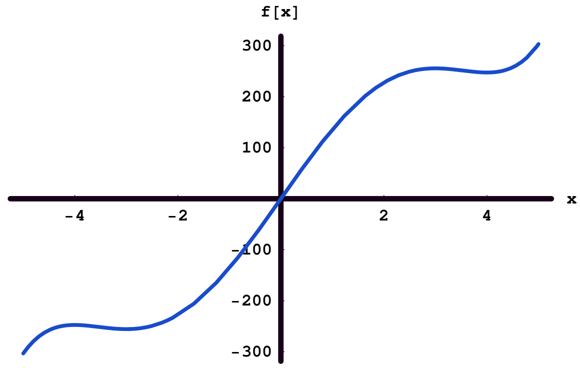
FIGURE 1. $f(x) = \frac{3}{4}x^4 - x^3 - 9x^2$



The inflection points are $\frac{1\pm\sqrt{19}}{3}$.

b. $f(x) \ = \ \dfrac{x^5}{5} - \dfrac{25}{3}x^3 + 144x$

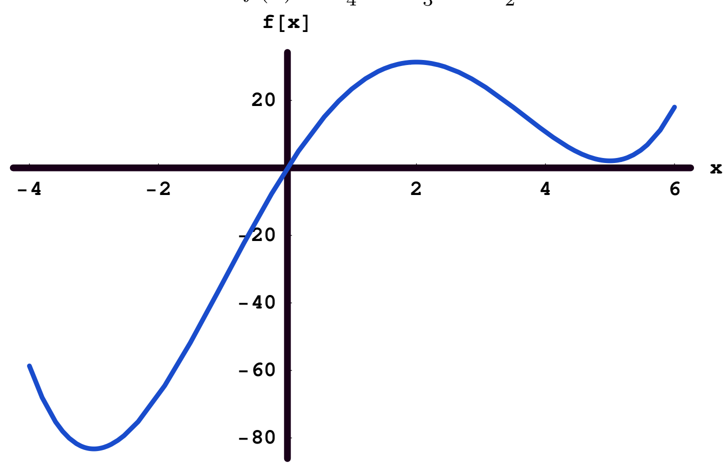
FIGURE 2. $f(x) \ = \ \frac{x^5}{5} - \frac{25}{3}x^3 + 144x$



The inflection points are $\pm\frac{5}{\sqrt{2}}$.

c. $f(x) \;=\; \frac{1}{4}x^4 - \frac{4}{3}x^3 - \frac{11}{2}x^2 + 30x$

FIGURE 3. $f(x) \;=\; \frac{1}{4}x^4 - \frac{4}{3}x^3 - \frac{11}{2}x^2 + 30x$



Problem 6. In the following problem you are given a production function for a firm where y is the level of output and x is the level of the variable input. You are given the price (p) of the output and the price (w) of the single variable input. Write down an equation that represents profit for the firm. Then maximize this function by taking its derivative with respect to the variable input x and set equal to zero. What is the optimal level of x? Show why this level of x maximizes profit.

$$output\,price \;=\; p \;= 4$$

$$input\,price \;=\; w \;= 2304$$

$$y \;=\; output \;=\; f(x) \;=\; 100x + 80x^2 - 3x^3$$

Problem 7. Consider the following production function, output price and input prices.

$$\begin{aligned} f(x_1, x_2) &= x_1^{1/4} x_2^{3/5} \\ p &= 20 \\ w_1 &= 5, \quad w_2 \;=\; 6 \end{aligned}$$

a. Write an equation representing profit for a firm using this technology and facing these prices.

$\pi =$

b. What is the derivative of π with respect to x_1?

c. What is the derivative of π with respect to x_2?

d. Set the two equations from parts b and c equal to zero and solve them for x_1 and x_2.

Problem 8. Find all first partial derivatives of each of the following

a. $f(x) \;=\; 3x_1^2 - 6x_2^3 + 45x_1x_2$

b. $y \;=\; 3x_1 + 4x_1^3x_2^4$

c. $f(x) \;=\; 96x_1^{1/6}x_2^{1/4} - x_1 - 6x_2$

d. $f(x) = 30x_1^{1/3}x_2^{2/5}x_3^{1/6}$

e. $f(x) = 60x_1^{1/6}x_2^{7/12}x_3^{1/3} - 2x_1 - 3x_2 - 5x_3$

f. $f(x) = 2\log x_1 + 3\log {x_1}^2 + 4\log x_2 + 2\log x_1 \log x_2 + 5\log {x_2}^2 + 6\log x_3 + 4\log x_1 \log x_3 + 2\log x_2 \log x_3 + 3\log {x_3}^2$

Problem 9. Do the following problems from the book.

a. Section 11.2
   1) 1a
   2) 1b
   3) 1c
   4) 1d
   5) 3a
   6) 3b
   7) 3c
   8) 5a
   9) 5b
   10) 5c
   11) 5d

b. Section 11.5
   1) 1a
   2) 1b
   3) 3a
   4) 3b