Problem 1. Consider the following matrix and vector.

\[
R = \begin{bmatrix}
-3 & -2 \\
5 & 3
\end{bmatrix}, \quad r = \begin{bmatrix}
1 \\
-1
\end{bmatrix},
\]

a. Use elementary row operations to find the inverse of \( R \) and solve the equation \( Rx = r \) in one set of operations.
b. Find the determinant of the matrix $R$.

$$R = \begin{bmatrix} -3 & -2 \\ 5 & 3 \end{bmatrix}, \quad r = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$ 

c. Find the inverse of the matrix $R$ using the cofactor/adjoint method.
d. Solve the equation $Rx=r$ using the inverse you found in part 1c
e. Solve the equation $Rx=r$ using Cramer’s rule.

$$R = \begin{bmatrix} -3 & -2 \\ 5 & 3 \end{bmatrix}, \quad r = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$
Problem 2. Consider the following matrix and vector.

\[ S = \begin{bmatrix} 4 & -2 \\ -3 & 2 \end{bmatrix}, \quad s = \begin{bmatrix} 8 \\ -5 \end{bmatrix}, \]

a. Use elementary row operations to find the inverse of \( S \) and solve the equation \( Sx = s \) in one set of operations.
b. Find the determinant of the matrix \( S \).

\[
S = \begin{bmatrix}
4 & -2 \\
-3 & 2
\end{bmatrix}, \quad s = \begin{bmatrix}
8 \\
-5
\end{bmatrix},
\]

c. Find the inverse of the matrix \( S \) using the cofactor/adjoint method.

d. Solve the equation \( Sx = s \) using the inverse you found in part 2c.
e. Solve the equation $Sx = s$ using Cramer’s rule.

$$S = \begin{bmatrix} 4 & -2 \\ -3 & 2 \end{bmatrix}, \quad s = \begin{bmatrix} 8 \\ -5 \end{bmatrix},$$
Problem 3. Consider the following matrix and vector.

\[ G = \begin{bmatrix} 1 & 4 & -3 \\ -2 & -7 & 4 \\ 3 & 13 & -10 \end{bmatrix}, \quad g = \begin{bmatrix} 9 \\ -14 \\ 30 \end{bmatrix} \]

a. Use elementary row operations to find the inverse of \( G \) and solve the equation \( Gx = g \) in one set of operations.
b. Find the determinant of the matrix $G$.

$$G = \begin{bmatrix} 1 & 4 & -3 \\ -2 & -7 & 4 \\ 3 & 13 & -10 \end{bmatrix}, \quad g = \begin{bmatrix} 9 \\ -14 \\ 30 \end{bmatrix}$$
c. Find the inverse of the matrix $G$ using the cofactor/adjoint method.

$$G = \begin{bmatrix} 1 & 4 & -3 \\ -2 & -7 & 4 \\ 3 & 13 & -10 \end{bmatrix}, \quad g = \begin{bmatrix} 9 \\ -14 \\ 30 \end{bmatrix}$$
d. Solve the equation $Gx=g$ using the inverse you found in part 3c

$$
G = \begin{bmatrix} 
1 & 4 & -3 \\
-2 & -7 & 4 \\
3 & 13 & -10 
\end{bmatrix}, \quad g = \begin{bmatrix} 
9 \\
-14 \\
30 
\end{bmatrix}
$$


e. Solve the equation $Gx=g$ using Cramer’s rule.
Problem 4. Consider the following matrix and vector.

\[ H = \begin{bmatrix} 1 & \frac{1}{2} & 2 \\ -3 & -1 & -2 \\ 3 & 1 & 4 \end{bmatrix}, \quad h = \begin{bmatrix} 0 \\ -5 \\ 3 \end{bmatrix} \]

a. Use elementary row operations to find the inverse of H and solve the equation Hx=h in one set of operations.
b. Find the determinant of the matrix $H$.

$$H = \begin{bmatrix} 1 & \frac{1}{2} & 2 \\ -3 & -1 & -2 \\ 3 & 1 & 4 \end{bmatrix}, \quad h = \begin{bmatrix} 0 \\ -5 \\ 3 \end{bmatrix}$$
c. Find the inverse of the matrix $H$ using the cofactor/adjoint method.

$$H = \begin{bmatrix} 1 & \frac{1}{2} & 2 \\ -3 & -1 & -2 \\ 3 & 1 & 4 \end{bmatrix}, \quad h = \begin{bmatrix} 0 \\ -5 \\ 3 \end{bmatrix}$$
d. Solve the equation $Hx = h$ using the inverse you found in part 4c

\[
H = \begin{bmatrix}
1 & \frac{1}{2} & 2 \\
-3 & -1 & -2 \\
3 & 1 & 4 \\
\end{bmatrix}, \quad h = \begin{bmatrix}
0 \\
-5 \\
3 \\
\end{bmatrix}
\]

e. Solve the equation $Hx = h$ using Cramer’s rule.
Problem 5. For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise. Check to see if there are potential points of inflection at points other than critical points.

a. \( f(x) = \frac{3}{4}x^4 - x^3 - 9x^2 \).

The inflection points are \( \frac{1 \pm \sqrt{19}}{3} \).

\[ \text{Figure 1. } f(x) = \frac{3}{4}x^4 - x^3 - 9x^2 \]
b. \( f(x) = \frac{x^5}{5} - \frac{25}{3}x^3 + 144x \)

**Figure 2.** \( f(x) = \frac{x^5}{5} - \frac{25}{3}x^3 + 144x \)

The inflection points are \( \pm \frac{5}{\sqrt{2}} \).
c. \( f(x) = \frac{1}{4}x^4 - \frac{4}{3}x^3 - \frac{11}{2}x^2 + 30x \)

**Figure 3.** \( f(x) = \frac{1}{4}x^4 - \frac{4}{3}x^3 - \frac{11}{2}x^2 + 30x \)
**Problem 6.** In the following problem you are given a production function for a firm where \( y \) is the level of output and \( x \) is the level of the variable input. You are given the price \((p)\) of the output and the price \((w)\) of the single variable input. Write down an equation that represents profit for the firm. Then maximize this function by taking its derivative with respect to the variable input \( x \) and set equal to zero. What is the optimal level of \( x \)? Show why this level of \( x \) maximizes profit.

- **Output price** \( p = 4 \)
- **Input price** \( w = 2304 \)

\[
y = \text{output} = f(x) = 100x + 80x^2 - 3x^3
\]
Problem 7. Consider the following production function, output price and input prices.

\[ f(x_1, x_2) = x_1^{1/4} x_2^{3/5} \]
\[ p = 20 \]
\[ w_1 = 5, \quad w_2 = 6 \]

a. Write an equation representing profit for a firm using this technology and facing these prices.

\[ \pi = \]

b. What is the derivative of \( \pi \) with respect to \( x_1 \)?

c. What is the derivative of \( \pi \) with respect to \( x_2 \)?
d. Set the two equations from parts b and c equal to zero and solve them for $x_1$ and $x_2$. 
Problem 8. Find all first partial derivatives of each of the following

a. \( f(x) = 3x_1^2 - 6x_2^3 + 45x_1x_2 \)

b. \( y = 3x_1 + 4x_1^3x_2^4 \)

c. \( f(x) = 96x_1^{1/6}x_2^{1/4} - x_1 - 6x_2 \)
d. \( f(x) = 30x_1^{1/3}x_2^{2/3}x_3^{1/6} \)

e. \( f(x) = 60x_1^{1/6}x_2^{7/12}x_3^{1/3} - 2x_1 - 3x_2 - 5x_3 \)

f. \( f(x) = 2\log x_1 + 3\log x_1^2 + 4\log x_2 + 2\log x_1 \log x_2 + 5\log x_2^2 + 6\log x_3 + 4\log x_1 \log x_3 + 2\log x_2 \log x_3 + 3\log x_3^2 \)
Problem 9. Do the following problems from the book.

a. Section 11.2
   1) 1a
   2) 1b
   3) 1c
   4) 1d
   5) 3a
   6) 3b
   7) 3c
   8) 5a
   9) 5b
  10) 5c
  11) 5d

b. Section 11.5
   1) 1a
   2) 1b
   3) 3a
   4) 3b