ECONOMICS 207
SPRING 2007
PROBLEM SET 12

Consider the following matrices and vectors.

$$P = \begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}, \qquad Q = \begin{bmatrix} 1 & -1 & 3 \\ 4 & -3 & 6 \\ -3 & 2 & -2 \end{bmatrix},$$

$$p = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \qquad q = \begin{bmatrix} 1 \\ -4 \\ 7 \end{bmatrix},$$

Problem 1.
Find the determinant of the matrix P.

b. Find the inverse of the matrix P using the adjoint method.

Date: 11 April 2007.

c. Using the inverse from part b, solve the system of equations

$$P \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = p$$

d. Using row reduction, find the inverse of the matrix P and the solution to the system of equations

$$P \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = p$$

Problem 2.
Find the determinant of the matrix Q.

b. Find the inverse of the matrix Q using the adjoint method.

c. Using the inverse from part b, solve the system of equations

$$Q \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = q$$

d. Using row reduction, find the inverse of the matrix Q and the solution to the system of equations

$$Q \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = q$$

Problem 3. Do the following problems from the book.

a. Section 11.6, Problem 1
b. Section 11.7
    1) 1
    2) 3
    3) 5
c. Section 11.8
    1) 1
    2) 3

Problem 4. Solve the following system of equations.

$$
\begin{aligned}
216x_1^{-3/4}x_2^{1/3} \; - \; 81 \; &= 0 \\
288x_1^{1/4}x_2^{-2/3} \; - \; 64 \; &= 0
\end{aligned}
$$

Problem 5. Find all first and second partial derivatives of each of the following

a. $f(x_1, x_2) = 1000x_1 + 600x_2 - 10x_1^2 + 20x_1x_2 - 20x_2^2 - 200x_1 - 100x_2$

$\frac{\partial f}{\partial x_1}$	$\frac{\partial f}{\partial x_2}$
$\frac{\partial^2 f}{\partial x_1 \partial x_1}$	$\frac{\partial^2 f}{\partial x_1 \partial x_2}$
$\frac{\partial^2 f}{\partial x_2 \partial x_1}$	$\frac{\partial^2 f}{\partial x_2 \partial x_2}$

b. $f(x_1, x_2) = 20(200x_1 + 90x_2 - x_1^2 + 2x_1x_2 - 2x_2^2) - 200x_1 - 80x_2$

$\frac{\partial f}{\partial x_1}$	$\frac{\partial f}{\partial x_2}$
$\frac{\partial^2 f}{\partial x_1 \partial x_1}$	$\frac{\partial^2 f}{\partial x_1 \partial x_2}$
$\frac{\partial^2 f}{\partial x_2 \partial x_1}$	$\frac{\partial^2 f}{\partial x_2 \partial x_2}$

c. $f(x_1, x_2, x_3) \;=\; 24x_1^{1/8}x_2^{1/4}x_3^{1/3} - 5x_1 - 2x_2 - 3x_3$

$\frac{\partial f}{\partial x_1}$	$\frac{\partial f}{\partial x_2}$	$\frac{\partial f}{\partial x_3}$
$\frac{\partial^2 f}{\partial x_1 \partial x_1}$	$\frac{\partial^2 f}{\partial x_1 \partial x_2}$	$\frac{\partial^2 f}{\partial x_1 \partial x_3}$
$\frac{\partial^2 f}{\partial x_2 \partial x_1}$	$\frac{\partial^2 f}{\partial x_2 \partial x_2}$	$\frac{\partial^2 f}{\partial x_2 \partial x_3}$
$\frac{\partial^2 f}{\partial x_3 \partial x_1}$	$\frac{\partial^2 f}{\partial x_3 \partial x_2}$	$\frac{\partial^2 f}{\partial x_3 \partial x_3}$

d. $f(x) = 5x_1^{1/3}x_2^{1/2}e^{\left[-\frac{(log[x_1])^2}{50}+\frac{log[x_1]log[x_2]}{10}-\frac{log[x_2]^2}{5}\right]}$, first derivatives only.

$\frac{\partial f}{\partial x_1}$	$\frac{\partial f}{\partial x_2}$
$\frac{\partial^2 f}{\partial x_1 \partial x_1}$	$\frac{\partial^2 f}{\partial x_1 \partial x_2}$
$\frac{\partial^2 f}{\partial x_2 \partial x_1}$	$\frac{\partial^2 f}{\partial x_2 \partial x_2}$

Problem 6. For each of the following problems, write an equation that represents profit as a function of the two inputs x_1 and x_2. Write it in the form π = pf(x_1,x_2)-w_1x_1-w_2x_2 and then simplify the expression. Then find all first and second partial derivatives of the function at the specified point.

a.

$$\begin{aligned} f(x_1, x_2) &= 50x_1 + 60x_2 - x_1^2 + 2x_1x_2 - 2x_2^2 \\ p &= 50 \\ w_1 &= 2000, \qquad w_2 = 400 \\ x_1 &= 36, \qquad x_2 = 31 \end{aligned}$$

$\frac{\partial \pi}{\partial x_1}$	$\frac{\partial \pi}{\partial x_2}$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2}$

b.

$$
\begin{aligned}
f(x_1, x_2) &= 16x_1 \ + \ 10x_2 \ - \ 2x_1^2 \ + \ 2x_1x_2 \ - \ x_2^2 \\
p &= 5 \\
w_1 &= 60, \qquad w_2 \ = \ 10 \\
x_1 &= 6, \qquad x_2 \ = \ 10
\end{aligned}
$$

$\frac{\partial \pi}{\partial x_1}$	$\frac{\partial \pi}{\partial x_2}$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2}$

c.

$$
\begin{aligned}
f(x_1, x_2) &= x_1^{1/2} x_2^{2/5} \\
p &= 2970 \\
w_1 &= 1215, \qquad w_2 \;=\; 484 \\
x_1 &= 121, \qquad x_2 \;=\; 243
\end{aligned}
$$

$\frac{\partial \pi}{\partial x_1}$	$\frac{\partial \pi}{\partial x_2}$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2}$

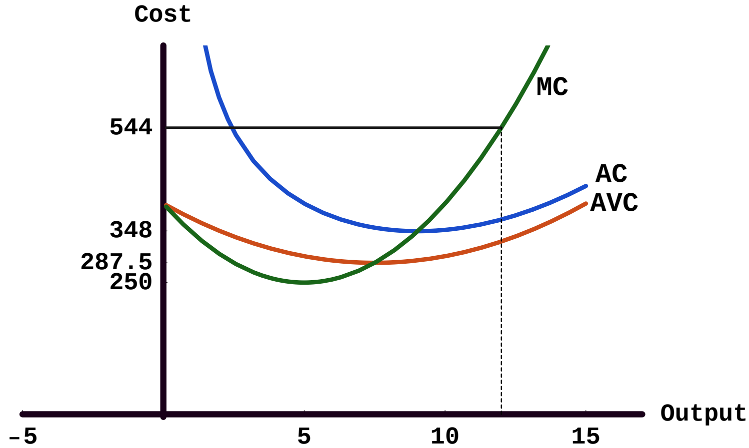
d.

$$\begin{aligned} f(x_1, x_2) &= x_1^{1/5} x_2^{3/4} \\ p &= 40 \\ w_1 &= 32, \qquad w_2 \ = \ 15 \\ x_1 &= 32, \qquad x_2 \ = \ 256 \end{aligned}$$

$\frac{\partial \pi}{\partial x_1}$	$\frac{\partial \pi}{\partial x_2}$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2}$

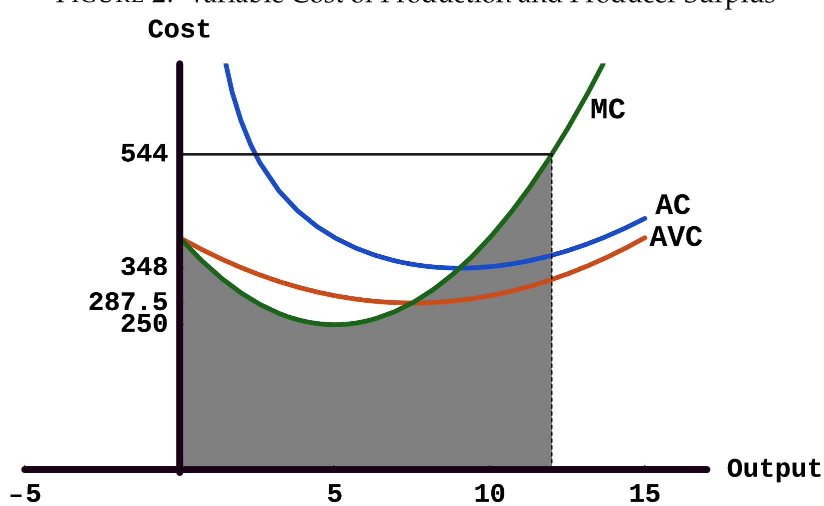
Problem 7. The cost function for a firm is a rule or mapping that tells the total cost of production of any output level produced by the firm. If the variable y represents the output of the firm, then the cost function is given by c(y). Marginal cost represents the change in the cost of production for the firm as output changes and is given by the derivative of the cost function with respect to output,i.e., Marginal Cost(MC) = $\frac{dc(y)}{dy}$. A competitive firm facing a fixed output price maximizes profit at the output level where marginal cost is equal to price as in the figure 1.

FIGURE 1. Profit Maximization



The area below the cost curve is a measure of variable cost and can be found by integrating the marginal cost curve from 0 to any given output level y. The shaded area in figure 2 represents the variable cost of production for the cost function $c(y) = 500 + 400y - 30y^2 + 2y^3$.

FIGURE 2. Variable Cost of Production and Producer Surplus



Producer surplus is the area below a given price and above the marginal cost curve. Producer surplus is the unshaded area below the horizontal line at 544 in figure 2. Producer surplus can be computed by subtracting the shaded area from total revenue.

a. Find the profit maximizing level of output for the following firm. Demonstrate that the level you choose maximizes profit.

$$price = p = \$544$$
$$cost = c(y) = 500 + 400y - 30y^2 + 2y^3$$

b. What is revenue minus variable cost for this firm when price is $544? This is the same as producer surplus.

c. Find producer surplus for this firm assuming you are only given the following marginal cost function: $MC(y) = 400 - 60y + 6y^2$ and a price of $544.