ECONOMICS 207
SPRING 2007
PROBLEM SET 13

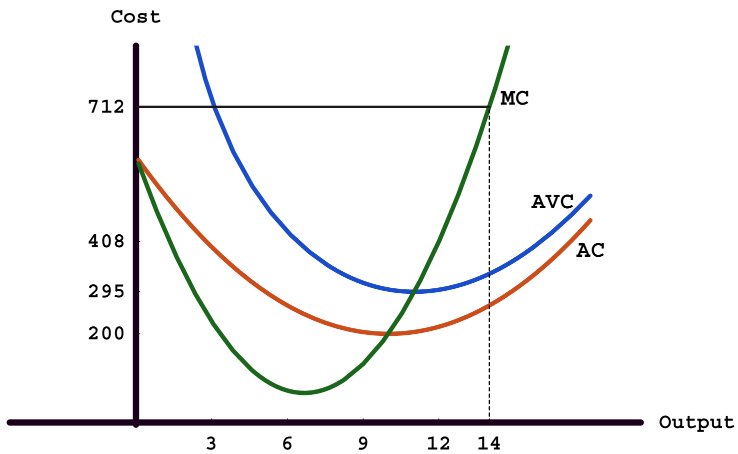
Problem 1. For the following problem, write an equation that represents profit as a function of the input x. Write it in the form π = pf(x)-wx and then simplify the expression. Then find the first and second derivatives of the function. Then find the critical points. For each critical point state whether profit is at a relative maximum, relative minimum, or otherwise. Check to see if there are points of inflection **at points other than** critical points.

$$\begin{aligned} f(x) &= 60x + 50x^2 - 2x^3 \\ p &= 2 \\ w &= 692 \end{aligned}$$

Date: 9 April 2007.

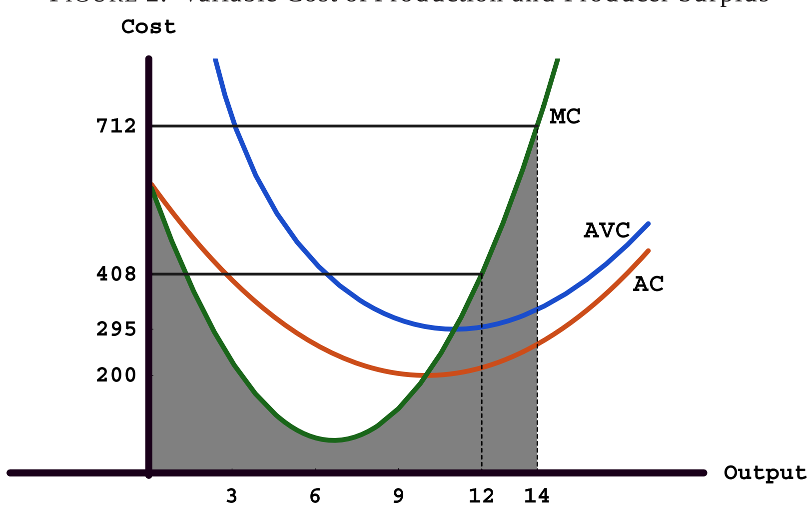
Problem 2. The cost function for a firm is a rule or mapping that tells the total cost of production of any output level produced by the firm. If the variable y represents the output of the firm, then the cost function is given by c(y). Marginal cost represents the change in the cost of production for the firm as output changes and is given by the derivative of the cost function with respect to output,i.e., Marginal Cost(MC) = $\frac{dc(y)}{dy}$. A competitive firm facing a fixed output price maximizes profit at the output level where marginal cost is equal to price as in the figure 1.

FIGURE 1. Profit Maximization



The area below the cost curve is a measure of variable cost and can be found by integrating the marginal cost curve from 0 to any given output level y. The shaded area in figure 2 represents the variable cost of production for the cost function $c(y) = 1000 + 600y - 80\,y^2 + 4y^3$.

FIGURE 2. Variable Cost of Production and Producer Surplus



Producer surplus is the area below a given price and above the marginal cost curve. Producer surplus is the unshaded area below the horizontal line at 712 in figure 2. Producer surplus can be computed by subtracting the shaded area from total revenue.

a. Find the profit maximizing level of output for the following firm. Demonstrate that the level you choose maximizes profit.

$$price \; = \; p \; = \$712$$

$$cost \; = \; c(y) \; = 1000 \; + \; 600y \; - \; 80y^2 \; + \; 4y^3$$

b. Find the profit maximizing level of output when the price is \$408. Demonstrate that the level you choose maximizes profit.

c. What is variable cost for this firm when price is $712?

d. What is producer surplus for this firm when the price is $712?

e. What is variable cost for this firm when price is $408?

f. What is producer surplus for this firm when the price is $408?

g. How much is the firm worse off when price falls from $712 to $408?

h. Cross-hatch the change in producer surplus in Figure 2.

Problem 3. For each of the following problems, write an equation that represents profit as a function of the two inputs x_1 and x_2. Write it in the form $\pi = pf(x_1,x_2)-w_1x_1-w_2x_2$ and then simplify the expression. Then find all first and second partial derivatives of the function.

a.

$$f(x_1, x_2) = 5x_1 + 4x_2 - 2x_1^2 + x_1x_2 - x_2^2$$
$$p = 2$$
$$w_1 = 6, \qquad w_2 = 2$$

$\frac{\partial \pi}{\partial x_1}$	$\frac{\partial \pi}{\partial x_2}$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2}$

Find potential profit maximizing levels of x_1 and x_2.

By evaluating the Hessian matrix of the profit equation at the critical values, verify the optimal levels of x_1 and x_2.

b.

$$\begin{aligned} f(x_1, x_2) &= 20x_1 + 20x_2 - 2x_1^2 + 2x_1x_2 - 2x_2^2 \\ p &= 5 \\ w_1 &= 10, \qquad w_2 = 100 \end{aligned}$$

$\frac{\partial \pi}{\partial x_1}$	$\frac{\partial \pi}{\partial x_2}$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2}$

Find potential profit maximizing levels of x_1 and x_2.

By evaluating the Hessian matrix of the profit equation at the critical values, verify the optimal levels of x_1 and x_2.

c.

$$
\begin{aligned}
f(x_1, x_2) &= x_1^{2/7} x_2^{1/3} \\
p &= 3024 \\
w_1 &= 81, \qquad w_2 = 448
\end{aligned}
$$

$\frac{\partial \pi}{\partial x_1}$	$\frac{\partial \pi}{\partial x_2}$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2}$

Find potential profit maximizing levels of x_1 and x_2.

By evaluating the Hessian matrix of the profit equation at the critical values, verify the optimal levels of x_1 and x_2.

d.

$$
\begin{aligned}
f(x_1, x_2) &= x_1^{1/6} x_2^{1/4} \\
p &= 972 \\
w_1 &= 2, \qquad w_2 = 27
\end{aligned}
$$

$\frac{\partial \pi}{\partial x_1}$	$\frac{\partial \pi}{\partial x_2}$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2}$

Find potential profit maximizing levels of x_1 and x_2.

By evaluating the Hessian matrix of the profit equation at the critical values, verify the optimal levels of x_1 and x_2.

Problem 4. Find the listed partial derivatives of each of the following functions.

a. $\mathcal{L}(x_1, x_2, \lambda) \; = \; 6x_1 + 2x_2 - \lambda \left(5x_1 + 4x_2 - 2x_1^2 + x_1 x_2 - x_2^2 - 9\right)$

$\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1}$	$\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2}$	$\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda}$
$\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial x_1}$	$\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial x_2}$	$-\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial \lambda}$
$\frac{\partial^2 \mathcal{L}}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \mathcal{L}}{\partial x_2 \partial x_2}$	$-\frac{\partial^2 \mathcal{L}}{\partial x_2 \partial \lambda}$
$-\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial x_1}$	$-\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial x_2}$	$-\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial \lambda}$

b. $\mathcal{L}(x_1, x_2, \lambda) \; = \; 20x_1 + 20x_2 - 2x_1^2 + 2x_1x_2 - 2x_2^2 - \lambda\,(10x_1 \; + \; 100x_2 \; - \; 360)$

$\frac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1}$	$\frac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2}$	$\frac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda}$
$\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial x_1}$	$\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial x_2}$	$-\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial \lambda}$
$\frac{\partial^2 \mathcal{L}}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \mathcal{L}}{\partial x_2 \partial x_2}$	$-\frac{\partial^2 \mathcal{L}}{\partial x_2 \partial \lambda}$
$-\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial x_1}$	$-\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial x_2}$	$-\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial \lambda}$

c. $\mathcal{L}(x_1, x_2, \lambda) \ = \ 81x_1 + 448x_2 - \lambda\left(x_1^{2/7}x_2^{1/3} - 12\right)$

$\frac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1}$	$\frac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2}$	$\frac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda}$
$\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial x_1}$	$\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial x_2}$	- $\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial \lambda}$
$\frac{\partial^2 \mathcal{L}}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \mathcal{L}}{\partial x_2 \partial x_2}$	- $\frac{\partial^2 \mathcal{L}}{\partial x_2 \partial \lambda}$
- $\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial x_1}$	- $\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial x_2}$	- $\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial \lambda}$

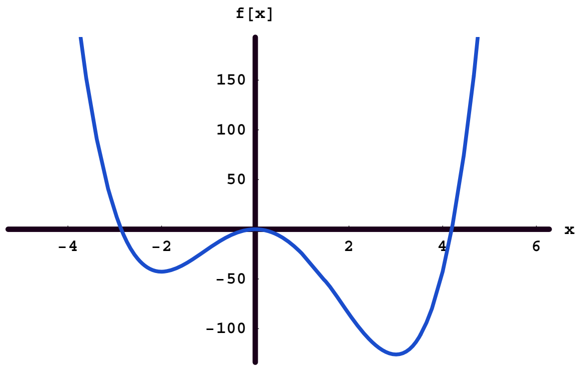
d. $\mathcal{L}(x_1, x_2, \lambda) \ = \ x_1^{1/6} x_2^{1/4} - \lambda \left(2x_1 + 27x_2 - 3645\right)$

$\frac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1}$	$\frac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2}$	$\frac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda}$
$\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial x_1}$	$\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial x_2}$	$-\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial \lambda}$
$\frac{\partial^2 \mathcal{L}}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \mathcal{L}}{\partial x_2 \partial x_2}$	$-\frac{\partial^2 \mathcal{L}}{\partial x_2 \partial \lambda}$
$-\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial x_1}$	$-\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial x_2}$	$-\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial \lambda}$

Problem 5. For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise. Check to see if there are potential points of inflection **at points other than** critical points.

a. $f(x) = 2x^4 - \frac{8}{3}x^3 - 24x^2$.

FIGURE 3. $f(x) = 2x^4 - \frac{8}{3}x^3 - 24x^2$



The inflection points are $\frac{1 \pm \sqrt{19}}{3}$.

b. $f(x) \ = \ \frac{5}{4}x^4 - \frac{20}{3}x^3 - \frac{55}{2}x^2 + 150x$

FIGURE 4. $f(x) \ = \ \frac{5}{4}x^4 - \frac{20}{3}x^3 - \frac{55}{2}x^2 + 150x$