

ECONOMICS 207
SPRING 2007
PROBLEM SET 1 KEY

Problem 1. Do the following problems from the book.

a. Section 1.2

- 1) 4a
- 2) 4b
- 3) 4c
- 4) 13a

b. Section 1.3

- 1) 1b
- 2) 2a
- 3) 4a
- 4) 4c
- 5) 9a
- 6) 12a
- 7) 12c
- 8) 15c
- 9) 15f

c. Section 1.4

- 1) 2b
- 2) 3a
- 3) 3c
- 4) 5b
- 5) 6f

d. Section 1.5

- 1) 1d
- 2) 1f
- 3) 3d
- 4) 4a
- 5) 4g
- 6) 5b
- 7) 6a
- 8) 6b
- 9) 6c
- 10) 9a
- 11) 9d

Date: July 30, 2007.

Problem 2. Carry out the following operations.

a. $3^2 * 3^3$

$$3^2 * 3^3 = 3^5 = 243$$

b. $(3^2)^3$

$$(3^2)^3 = 3^6 = 729$$

c. $(3 + 3)^2$

$$(3 + 3)^2 = 6^2 = 36$$

d. $3^2 + 3^2$

$$3^2 + 3^2 = 9 + 9 = 18$$

e. $(a^2)^2$

$$(a^2)^2 = a^{2 \cdot 2} = a^4$$

f. $a^3 a^2 b^3$

$$a^3 a^2 b^3 = a^{3+2} b^3 = a^5 b^3$$

g. $2^3 2^x 2^4$

$$2^3 2^x 2^4 = 2^{3+x+4} = 2^{7+x}$$

h. $(2^3 2^x)^2 2^4$

$$(2^3 2^x)^2 2^4 = (2^{3+x})^2 2^4 = 2^{6+2x} 2^4 = 2^{10+2x}$$

i. $(Ax_1^{\alpha_1} x_2^{\alpha_2})^2$

$$(Ax_1^{\alpha_1} x_2^{\alpha_2})^2 = A^2 x_1^{2\alpha_1} x_2^{2\alpha_2}$$

j. $(Ax_1^{\alpha_1}x_1^{\alpha_2})^2$

$$(Ax_1^{\alpha_1}x_1^{\alpha_2})^2 = A^2x_1^{2\alpha_1}x_1^{2\alpha_2} = A^2x_1^{2(\alpha_1+\alpha_2)}$$

Problem 3. Simplify the following expressions.

a.

$$\begin{aligned}\frac{3^2}{3} &= \frac{9}{3} \\ &= 3\end{aligned}$$

b.

$$\begin{aligned}\frac{3^2 a^2}{3a^3} &= \frac{9a^2}{3a^3} \\ &= \frac{3}{a}\end{aligned}$$

c.

$$\begin{aligned}\left(\frac{3^2 a^2}{3a^3}\right)^2 &= \left(\frac{3}{a}\right)^2 \\ &= \frac{9}{a^2}\end{aligned}$$

d.

$$\begin{aligned}\frac{3^2 a^2 a^4}{3a^3 3^2} &= \frac{3^2 a^{2+4}}{3^{1+2} a^3} \\ &= \frac{3^2 a^6}{3^3 a^3} \\ &= \frac{a^{6-3}}{3} \\ &= \frac{a^3}{3}\end{aligned}$$

Problem 4. Simplify, add, subtract, multiply or divide the following fractions. Express all answers in reduced form.

a. $\frac{21}{35}$

$$\begin{aligned}\frac{21}{35} &= \frac{3 \cdot 7}{5 \cdot 7} \\ &= \frac{3}{5}\end{aligned}$$

b. $(\frac{252}{462})(\frac{4}{\frac{1}{2}})$

$$\begin{aligned}\left(\frac{252}{462}\right)\left(\frac{4}{\frac{1}{2}}\right) &= \left(\frac{252}{462}\right)4 \cdot 2 \\ &= \frac{252}{2 \cdot 231}4 \cdot 2 \\ &= \frac{252 \cdot 4}{231} \\ &= \frac{4 \cdot 63 \cdot 4}{3 \cdot 77} \\ &= \frac{4 \cdot 7 \cdot 9 \cdot 4}{3 \cdot 7 \cdot 11} \\ &= \frac{2^4 \cdot 3}{11}\end{aligned}$$

c. $\frac{\frac{693}{2520}}{\frac{93}{231}}$

$$\begin{aligned}
\frac{\frac{693}{2520}}{\frac{93}{231}} &= \frac{693}{2520} \cdot \frac{231}{93} \\
&= \frac{693 \cdot 231}{2520 \cdot 93} \\
&= \frac{3 \cdot 231 \cdot 231}{2520 \cdot 3 \cdot 31} \\
&= \frac{3 \cdot (3 \cdot 7 \cdot 11)^2}{3 \cdot 840 \cdot 31} \\
&= \frac{3^3 \cdot 7^2 \cdot 11^2}{3 \cdot 3 \cdot 280 \cdot 31} \\
&= \frac{3^3 \cdot 7^2 \cdot 11^2}{3^2 \cdot 4 \cdot 7 \cdot 10 \cdot 31} \\
&= \frac{3 \cdot 7 \cdot 11^2}{4 \cdot 2 \cdot 5 \cdot 31} \\
&= \frac{3 \cdot 7 \cdot 11^2}{2^3 \cdot 5 \cdot 31}
\end{aligned}$$

d. $\frac{3}{8} + \frac{5}{10} + \frac{2}{5}$

$$\begin{aligned}
\frac{3}{8} + \frac{5}{10} + \frac{2}{5} &= \frac{3}{2^3} + \frac{5}{2 \cdot 5} + \frac{2}{5} \\
&= \frac{3 \cdot 5}{2^3 \cdot 5} + \frac{5 \cdot 2^2}{2^3 \cdot 5} + \frac{2^4}{2^3 \cdot 5} \\
&= \frac{15 + 20 + 16}{40} \\
&= \frac{51}{40}
\end{aligned}$$

e. $\frac{7}{8} + \frac{5}{11} + \frac{17}{22}$

$$\begin{aligned}
\frac{7}{8} + \frac{5}{11} + \frac{17}{22} &= \frac{7}{2^3} + \frac{5}{11} + \frac{17}{2 \cdot 11} \\
&= \frac{7 \cdot 11}{2^3 \cdot 11} + \frac{5 \cdot 2^3}{2^3 \cdot 11} + \frac{17 \cdot 2^2}{2^3 \cdot 11} \\
&= \frac{77 + 40 + 68}{88} \\
&= \frac{185}{88}
\end{aligned}$$

f. $\frac{3}{16} + \frac{1}{3} + \frac{7}{12} - \frac{5}{6}$

$$\begin{aligned} \frac{3}{16} + \frac{1}{3} + \frac{7}{12} - \frac{5}{6} &= \frac{3}{2^4} + \frac{1}{3} + \frac{7}{2^2 \cdot 3} - \frac{5}{2 \cdot 3} \\ &= \frac{3^2}{2^4 \cdot 3} + \frac{2^4}{2^4 \cdot 3} + \frac{7 \cdot 2^2}{2^4 \cdot 3} - \frac{5 \cdot 2^3}{2^4 \cdot 3} \\ &= \frac{9 + 16 + 28 - 40}{48} \\ &= \frac{13}{48} \end{aligned}$$

g. $\frac{3}{16} \frac{1}{3} + \frac{1}{4} \frac{1}{9} - \frac{7}{14} + \left(\frac{4}{7}\right)^{-2}$

$$\begin{aligned} \frac{3}{16} \frac{1}{3} + \frac{1}{4} \frac{1}{9} - \frac{7}{14} + \left(\frac{4}{7}\right)^{-2} &= \frac{3}{16 \cdot 3} + \frac{1 \cdot 9}{4 \cdot 4} - \frac{1}{2} + \frac{7^2}{4^2} \\ &= \frac{1}{16} + \frac{9}{16} - \frac{1}{2} + \frac{49}{16} \\ &= \frac{1 + 9 - 8 + 49}{16} \\ &= \frac{6}{16} \\ &= \frac{3}{8} \end{aligned}$$

h. $\frac{3}{13} + \frac{3}{4} - \frac{5}{12}$

$$\begin{aligned} \frac{3}{13} + \frac{3}{4} - \frac{5}{12} &= \frac{3 \cdot 39}{13} + \frac{3 \cdot 52}{4 \cdot 4} - \frac{5 \cdot 65}{12 \cdot 5} \\ &= 9 + \frac{3 \cdot 13}{4} - \frac{65}{12} \\ &= \frac{9 \cdot 12 + 3 \cdot 3 \cdot 13 - 65}{12} \\ &= \frac{108 + 117 - 65}{12} \\ &= \frac{160}{12} \\ &= \frac{40}{3} \end{aligned}$$

i. $\frac{\frac{1}{2}}{\frac{4}{12}} - \frac{3}{12} \left(\frac{3}{16}\right)^2$

$$\begin{aligned} \frac{\frac{1}{2}}{\frac{4}{12}} - \frac{3}{12} \left(\frac{3}{16}\right)^2 &= \frac{\frac{1}{2}}{\frac{1}{3}} - \frac{1}{4} \frac{3^2}{16^2} \\ &= \frac{3}{2} - \frac{9}{2^{10}} \\ &= \frac{3 \cdot 2^9 - 9}{2^{10}} \\ &= \frac{1527}{1024} \end{aligned}$$

Problem 5. Complete the following operations.

a. $e^{2x}e^{3x} = e^{5x}$

$$e^{2x}e^{3x} = e^{5x}$$

b. $(e^{2x})^2$

$$(e^{2x})^2 = e^{4x}$$

c. $e^{\log(x)}$

By the definition of log and anti-log, $e^{\log(x)} = x$

d. $\frac{1}{3} \frac{Ax_1^{1/3} x_2^{1/2}}{x_1}$

$$\frac{1}{3} \frac{Ax_1^{1/3} x_2^{1/2}}{x_1} = \frac{1}{3} Ax_1^{-2/3} x_2^{1/2}$$

e. $(3 + 2x)(3 - 2x)$

$$(3 + 2x)(3 - 2x) = 9 - 4x^2$$

f. $(3 + 2x)(3 - 2x)(3 + 2x)$

$$\begin{aligned} (3 + 2x)(3 - 2x)(3 + 2x) &= (9 - 4x^2)(3 + 2x) \\ &= 27 - 12x^2 + 18x - 8x^3 \\ &= -8x^3 - 12x^2 + 18x + 27 \end{aligned}$$

g. $(x + 2)^3$

$$(x + 2)^3 = x^3 + 4x^2 + 8x + 8$$

h. $\frac{(x^2 - 6x + 9)(x - 2)}{(x - 3)(x + 3)}$

$$\begin{aligned}\frac{(x^2 - 6x + 9)(x - 2)}{(x - 3)(x + 3)} &= \frac{(x - 3)^2(x - 2)}{(x - 3)(x + 3)} \\ &= \frac{(x - 3)(x - 2)}{x + 3} \\ &= \frac{x^2 - 5x + 6}{x + 3}\end{aligned}$$

i. $\frac{(x^2 + 7x + 12)}{x^2 + 2x - 8}$

$$\begin{aligned}\frac{(x^2 + 7x + 12)}{x^2 + 2x - 8} &= \frac{(x + 3)(x + 4)}{(x + 4)(x - 2)} \\ &= \frac{x + 3}{x - 2}\end{aligned}$$

Problem 6. Factor the following.

a. $x^2 - 6x + 9$

$$x^2 - 6x + 9 = (x - 3)^2$$

b. $x^2 - 9$

$$x^2 - 9 = (x + 3)(x - 3)$$

c. $4x^2 + 12x + 9$

$$4x^2 + 12x + 9 = (2x + 3)^2$$

d. $6x^2 - x - 2$

$$6x^2 - x - 2 = (2x + 1)(3x - 2)$$

e. $30x^2 + 34x - 8$

$$\begin{aligned} 30x^2 + 34x - 8 &= 2(15x^2 + 17x - 4) \\ &= 2(3x + 4)(5x - 1) \end{aligned}$$

f. $x^2 + \sqrt{-1}x + 2$

$$x^2 + \sqrt{-1}x + 2 = (x + 2\sqrt{-1})(x - \sqrt{-1})$$