

**ECONOMICS 207**  
**SPRING 2007**  
**PROBLEM SET 2 KEY**

**Problem 1.** Do the following problems from the book.

a. Section 2.3

- 1) 2a
- 2) 2d
- 3) 2e

b. Section 2.3

- 1) 3a
- 2) 3b
- 3) 4a

c. Section 3.1

- 1) 1a
- 2) 1b
- 3) 1c

d. Section 3.2

- 1) 1
- 2) 2

e. section 4.2

- 1) 1a
- 2) 3a
- 3) 6

**Problem 2.** Carry out the following long division operations.

$$\begin{array}{r} 23 \\ 13 \overline{) 299} \\ \underline{26} \\ 39 \\ \underline{39} \\ 0 \end{array}$$

a.

$$\begin{array}{r} 92.5 \\ 10 \overline{) 925} \\ \underline{90} \\ 25 \\ \underline{20} \\ 50 \\ \underline{50} \\ 0 \end{array}$$

b.

$$\begin{array}{r} 49 \\ 42 \overline{) 2058} \\ \underline{168} \\ 378 \\ \underline{378} \\ 0 \end{array}$$

c.

$$\begin{array}{r} 58 \\ 213 \overline{) 12412} \\ \underline{1065} \\ 1762 \\ \underline{1704} \\ 58 \end{array}$$

d.

**Problem 3.** Complete the square in the following and then write in the following form  $(x + a)^2 + c$ . For example the first problem would be written  $(x + 1)^2 - 1$ .

a.

$$x^2 + 2x = (x - 1)^2 - 1$$

b.

$$\begin{aligned}x^2 - 8x &= (x^2 - 2 \cdot 4x + 4^2) - 4^2 \\ &= (x - 4)^2 - 16\end{aligned}$$

c.

$$\begin{aligned}3x^2 + 12x &= 3(x^2 + 4x) \\ &= 3(x^2 + 2 \cdot 2x + 2^2) - 3 \cdot 2^2 \\ &= 3(x + 2)^2 - 12\end{aligned}$$

d.

$$\begin{aligned}4x^2 - 16x &= 4(x^2 - 4x) \\ &= 4(x^2 - 2 \cdot 2x + 2^2) - 4 \cdot 2^2 \\ &= 4(x - 2)^2 - 16\end{aligned}$$

e.

$$\begin{aligned}8x^2 - 16x + 1 &= 8(x^2 - 2x) + 1 \\ &= 8(x^2 - 2x + 1) + 1 - 8 \\ &= 8(x - 1)^2 - 7\end{aligned}$$

f.

$$\begin{aligned}25x^2 - 20x + 7 &= 25\left(x^2 - \frac{4}{5}x\right) + 7 \\ &= 25\left(x^2 - 2\frac{2}{5}x + \frac{2^2}{5}\right) + 7 - 25\frac{2^2}{5} \\ &= 25\left(x - \frac{2}{5}\right)^2 + 3\end{aligned}$$

**Problem 4.** Simplify, add, subtract, multiply or divide the following fractions. Express all answers in reduced form.

a.

$$\begin{aligned}\frac{54}{80} + \frac{7}{16} &= \frac{54}{80} + \frac{7 \cdot 5}{16 \cdot 5} \\ &= \frac{89}{80}\end{aligned}$$

b.

$$\begin{aligned}\frac{17}{21} + \frac{3}{49} &= \frac{17}{3 \cdot 7} + \frac{3}{7 \cdot 7} \\ &= \frac{17 \cdot 7}{3 \cdot 7 \cdot 7} + \frac{3 \cdot 3}{7 \cdot 7 \cdot 3} \\ &= \frac{121}{147}\end{aligned}$$

c.

$$\begin{aligned}\left(\frac{131}{524}\right) \left(\frac{\frac{18}{5}}{\frac{9}{10}}\right) &= \left(\frac{131}{124}\right) \cdot \left(\frac{18}{5}\right) \cdot \left(\frac{10}{9}\right) \\ &= \left(\frac{131}{124}\right) \cdot \left(\frac{2 \cdot 9}{5}\right) \cdot \left(\frac{2 \cdot 5}{9}\right) \\ &= \frac{131 \cdot 4}{124} \\ &= \frac{131 \cdot 4}{4 \cdot 31} \\ &= \frac{131}{31}\end{aligned}$$

d.

$$\begin{aligned}\frac{2}{11} + \frac{2}{3} + \frac{33}{77} - \frac{8}{21} &= \frac{28}{3 \cdot 11} + \frac{33}{7 \cdot 11} - \frac{8}{3 \cdot 7} \\ &= \frac{28 \cdot 6 + 33 \cdot 3 - 8 \cdot 11}{3 \cdot 7 \cdot 11} \\ &= \frac{207}{231}\end{aligned}$$

e.

$$\begin{aligned}\frac{8a}{3b} + \frac{3b}{a} + \frac{5}{b} &= \frac{8a \cdot a + 3b \cdot 3b + 5 \cdot 3a}{3ab} \\ &= \frac{8a^2 + 9b^2 + 15a}{3ab}\end{aligned}$$

$$\text{f. } \frac{a}{ab-b^2} - \frac{2}{a-b} + \frac{ab+b^2}{a^3-ab^2}$$

$$ab - b^2 = b(a - b)$$

$$a^3 - ab^2 = a(a^2 - b^2) = a(a + b)(a - b)$$

The common denominator is  $ab(a + b)(a - b)$

$$\begin{aligned} \frac{a}{ab-b^2} - \frac{2}{a-b} + \frac{ab+b^2}{a^3-ab^2} &= \frac{a(a(a+b))}{ab(a+b)(a-b)} - \frac{2ab(a+b)}{ab(a+b)(a-b)} + \frac{b(ab+b^2)}{ab(a+b)(a-b)} \\ &= \frac{a^2(a+b) - 2ab(a+b) + b^2(a+b)}{ab(a+b)(a-b)} \\ &= \frac{(a^2 - 2ab + b^2)(a+b)}{ab(a+b)(a-b)} \\ &= \frac{(a^2 - 2ab + b^2)}{ab(a-b)} \\ &= \frac{(a-b)^2}{ab(a-b)} \\ &= \frac{(a-b)}{ab} \end{aligned}$$

**Problem 5.** Factor the following.

a.

$$6x^2 - 11x - 10 = (2x - 5)(3x + 2)$$

b.

$$\begin{aligned} 12x^2 - 16x - 28 &= 4(3x^2 - 4x - 7) \\ &= 4(3x - 7)(x + 1) \end{aligned}$$

c.

$$\begin{aligned} 9x^2 - 3x - 30 &= 3(3x^2 - x - 10) \\ &= 3(x - 2)(3x + 5) \end{aligned}$$

d.

$$18x^2 + 3x - 10 = (3x - 2)(6x + 5)$$

e.

$$\begin{aligned} 24x^2 - 20x - 16 &= 4(6x^2 - 5x - 4) \\ &= 4(2x + 1)(3x - 4) \end{aligned}$$

f.

$$8x^2 + 29x - 12 = (x + 4)(8x - 3)$$

**Problem 6.** Solve the following equations for x.

a.

$$\begin{aligned}2x + 3 &= 13 \\ \Rightarrow 2x &= 10 \\ \Rightarrow x &= 5\end{aligned}$$

b.

$$\begin{aligned}7x + 3 &= 27 - 5x \\ \Rightarrow 12x &= 24 \\ \Rightarrow x &= 2\end{aligned}$$

c.

$$\begin{aligned}\frac{x + 1}{2x + 3} &= 22 \\ \Rightarrow x + 1 &= 44x + 66 \\ \Rightarrow 43x &= -65 \\ \Rightarrow x &= -\frac{65}{43}\end{aligned}$$

d.

$$\begin{aligned}\frac{2x - 5}{x + 6} &= \frac{5}{11} \\ \Rightarrow 11(2x - 5) &= 5(x + 6) \\ \Rightarrow 22x - 55 &= 5x + 30 \\ \Rightarrow 17x &= 85 \\ \Rightarrow x &= \frac{85}{17}\end{aligned}$$

e.

$$\begin{aligned}\frac{x + 3}{(x + 3)(x - 2)} &= \frac{1}{8} \\ \Rightarrow \frac{1}{x - 2} &= \frac{1}{8} \\ \Rightarrow x - 2 &= 8 \\ \Rightarrow x &= 10\end{aligned}$$

f.

$$\begin{aligned}\frac{x - 2}{(x - 3)(x + 2)} &= \frac{1}{3} \\ \Rightarrow 3(x - 2) &= (x - 3)(x + 2) \\ \Rightarrow 3x - 6 &= x^2 - x - 6 \\ \Rightarrow x^2 - 4x &= 0 \\ \Rightarrow x(x - 4) &= 0 \\ \Rightarrow x &= 0, 4\end{aligned}$$



**Problem 7.** Solve the following equations for x.

a.

$$\begin{aligned}x^2 + 2x + 1 &= 0 \\ \Rightarrow (x + 1)^2 &= 0 \\ \Rightarrow x &= -1\end{aligned}$$

b.

$$\begin{aligned}25x^2 - 20x - 320 &= 0 \\ \Rightarrow 5x^2 - 4x - 64 &= 0 \\ \Rightarrow (x - 4)(5x + 16) &= 0 \\ \Rightarrow x &= 4, -\frac{16}{5}\end{aligned}$$

c.  $3x^2 + 15x - 9 = 0$

$$\begin{aligned}3x^2 + 15x - 9 &= 0 \\ \Rightarrow x^2 + 5x - 3 &= 0 \\ \Rightarrow x^2 + 5x &= 3 \\ \Rightarrow x^2 + 5x + \frac{25}{4} - \frac{25}{4} &= 3 \\ \Rightarrow \left(x + \frac{5}{2}\right)^2 &= 3 + \frac{25}{4} = \frac{37}{4} \\ \Rightarrow x + \frac{5}{2} &= \pm \frac{\sqrt{37}}{2} \\ \Rightarrow x &= \frac{-5 \pm \sqrt{37}}{2}\end{aligned}$$

d.

$$\begin{aligned}
 -4x^2 - 6x + 54 &= 0 \\
 \Rightarrow 2x^2 + 3x - 27 &= 0 \\
 \Rightarrow (2x + 9)(x - 3) &= 0 \\
 \Rightarrow x &= -\frac{9}{2}, 3
 \end{aligned}$$

e.

$$\begin{aligned}
 8x^2 - 12x - 56 &= 0 \\
 \Rightarrow 2x^2 - 3x - 7 &= 0 \\
 \Rightarrow 2(x^2 - 3/2x) - 7 &= 0 \\
 \Rightarrow 2(x^2 - 2 \cdot \frac{3}{4}x + (\frac{3}{4})^2) - 7 - 2 \cdot (\frac{3}{4})^2 &= 0 \\
 \Rightarrow 2(x - \frac{3}{4})^2 &= \frac{65}{8} \\
 \Rightarrow (x - \frac{3}{4})^2 &= \frac{65}{16} \\
 \Rightarrow x - \frac{3}{4} &= \pm \sqrt{\frac{65}{16}} \\
 \Rightarrow x &= \frac{3 \pm \sqrt{65}}{4}
 \end{aligned}$$

f.  $4x^2 - 2x + 5 = 0$ 

Use the quadratic formula.

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 \Rightarrow x &= \frac{-(-2) \pm \sqrt{2^2 - 4(4)(5)}}{2(4)} \\
 &= \frac{2 \pm \sqrt{4 - 80}}{8} = \frac{2 \pm \sqrt{-76}}{8} \\
 &= \frac{2 \pm \sqrt{(4)(-19)}}{8} \\
 &= \frac{2 \pm 2\sqrt{-19}}{8} \\
 &= \frac{1 \pm \sqrt{-19}}{4} \\
 &= \frac{1 \pm i\sqrt{19}}{4}
 \end{aligned}$$

**Problem 8.** Solve the following equations for  $x_1$ .

a.

$$9x_1^{-1/4} - 3 = 0$$

$$\rightarrow 3x_1^{-1/4} = 1$$

Multiply both sides by  $x_1^{1/4} \rightarrow 3x_1^{1/4} x_1^{-1/4} = x_1^{1/4}$

$$\rightarrow 3x_1^{1/4-1/4} = x_1^{1/4}$$

$$\rightarrow 3x_1^0 = x_1^{1/4}$$

$$\rightarrow 3 = x_1^{1/4}$$

$$\rightarrow (3)^4 = (x_1^{1/4})^4$$

$$\rightarrow 81 = x_1^{1/4 \cdot 4}$$

$$\rightarrow 81 = x_1$$

b.

$$36x_1^{-1/4} - 6 = 0$$

$$\rightarrow 6x_1^{-1/4} = 1$$

$$\rightarrow 6 = x_1^{1/4}$$

$$\rightarrow 6^4 = x_1$$

$$\rightarrow x_1 = 1296$$

c.

$$48x_1^{-4/7} - 3 = 0$$

$$\rightarrow 48x_1^{-4/7} = 3$$

$$\rightarrow 16x_1^{-4/7} = 1$$

$$\rightarrow 16 = x_1^{4/7}$$

$$\rightarrow (16)^{7/4} = x_1$$

$$\rightarrow (2^4)^{7/4} = x_1$$

$$\rightarrow x_1 = 2^7 = 128$$

d.

$$\begin{aligned}32x_1^{-2/3} - 2 &= 0 \\ \rightarrow 32x_1^{-2/3} &= 2 \\ \rightarrow 16x_1^{-2/3} &= 1 \\ \rightarrow 16 &= x_1^{2/3} \\ \rightarrow 16^{3/2} &= x_1 \\ \rightarrow x_1 &= (4^2)^{3/2} = 4^3 = 64\end{aligned}$$

e.

$$\begin{aligned}162x_1^{-4/5} - 2 &= 0 \\ \rightarrow 81 &= x_1^{4/5} \\ \rightarrow (81)^{5/4} &= x_1 \\ \rightarrow x_1 &= (3^4)^{5/4} = 3^5 = 243\end{aligned}$$

f.

$$\begin{aligned}1458x_1^{-6/7} - 2 &= 0 \\ \rightarrow 729 &= x_1^{6/7} \\ \rightarrow 729^{7/6} &= x_1 \\ \rightarrow x_1 &= (3^6)^{7/6} = 3^7 = 2187\end{aligned}$$

g.

$$\begin{aligned}507x_1^{-2/3} - 3 &= 0 \\ \rightarrow 169 &= x_1^{2/3} \\ \rightarrow (13^2)^{3/2} &= x_1 \\ \rightarrow x_1 &= 13^3 = 507\end{aligned}$$

h.

$$\begin{aligned}1250x_1^{-4/3} - 2 &= 0 \\ \rightarrow 625 &= x_1^{4/3} \\ \rightarrow (5^4)^{3/4} &= x_1 \\ \rightarrow x_1 &= 5^3 = 125\end{aligned}$$

**Problem 9.** Consider the following quadratic equation in  $x_1$ .

$$3pa_3x_1^2 + 2pa_2x_1 + pa_1 = w_1$$

In each of the problems below solve the equation for  $x_1$  for the given values of  $a_3, a_2, a_1, p$  and  $w_1$ .

a.  $a_3 = -1, a_2 = 20, a_1 = 100, p = 20$  and  $w_1 = 500$

$$\begin{aligned} 3pa_3x_1^2 + 2pa_2x_1 + pa_1 &= w_1 \\ \rightarrow 3(20)(-1)x_1^2 + 2(20)(20)x_1 + 20(100) &= 500 \\ \rightarrow -60x_1^2 + 800x_1 - 1500 &= 0 \\ \rightarrow 3x_1^2 - 40x_1 - 75 &= 0 \\ \rightarrow (x_1 - 15)(3x_1 + 5) &= 0 \\ \rightarrow x_1 &= 15, -\frac{5}{3} \end{aligned}$$

b.  $a_3 = -1, a_2 = 5, a_1 = 50, p = 10$  and  $w_1 = 20$

$$\begin{aligned} 3pa_3x_1^2 + 2pa_2x_1 + pa_1 &= w_1 \\ \rightarrow 3(10)(-1)x_1^2 + 2(10)(5)x_1 + 10(50) &= 20 \\ \rightarrow -30x_1^2 + 100x_1 + 480 &= 0 \\ \rightarrow 3x_1^2 - 10x_1 - 48 &= 0 \\ \rightarrow (x_1 - 6)(3x_1 + 8) &= 0 \\ \rightarrow x_1 &= 6, -\frac{8}{3} \end{aligned}$$

c.  $a_3 = -2$ ,  $a_2 = 50$ ,  $a_1 = 200$ ,  $p = 10$  and  $w_1 = 4860$

$$\begin{aligned}
 3pa_3x_1^2 + 2pa_2x_1 + pa_1 &= w_1 \\
 \Rightarrow 3(10)(-2)x_1^2 + 2(10)(50)x_1 + (10)(200) &= 4860 \\
 \Rightarrow -60x_1^2 + 1000x_1 + 2000 &= 4860 \\
 \Rightarrow -6x_1^2 + 100x_1 + 200 &= 486 \\
 \Rightarrow -6x_1^2 + 100x_1 - 286 &= 0 \\
 \Rightarrow -3x_1^2 + 50x_1 - 143 &= 0 \\
 \Rightarrow (-3x + 11)(x - 13) &= 0 \\
 \Rightarrow x = 13 \text{ and } 3x &= 11 \\
 \Rightarrow x = 13 \text{ and } x &= \frac{11}{3}
 \end{aligned}$$

d.  $a_3 = -3$ ,  $a_2 = 50$ ,  $a_1 = 100$ ,  $p = 5$  and  $w_1 = 1000$

$$\begin{aligned}
 3pa_3x_1^2 + 2pa_2x_1 + pa_1 &= w_1 \\
 \rightarrow 3(5)(-3)x_1^2 + 2(5)(50)x_1 + 5(100) &= 1000 \\
 \rightarrow -45x_1^2 + 500x_1 - 500 &= 0 \\
 \rightarrow 9x_1^2 - 100x_1 + 100 &= 0 \\
 \rightarrow (x_1 - 10)(9x_1 - 10) &= 0 \\
 \rightarrow x_1 = 10, -\frac{10}{9}
 \end{aligned}$$