ECONOMICS 207
SPRING 2007
PROBLEM SET 3

Problem 1. Do the following problems from the book.

(i) [a.]
(ii) Section 2.1
  (a) [1)]
  (b) 1d
$$\begin{aligned} 4x + 2(x-4) - 3 &= 2(3x-5) - 1 \\ x &\in \mathbb{R} \text{ All real numbers} \end{aligned}$$
  (c) 1f
$$\begin{aligned} (8x-7)5 - 3(6x-4) + 5x^2 &= x(5x-1) \\ x &= 1 \end{aligned}$$
  (d) 2f
$$\begin{aligned} \sqrt{2x+14} &= 16 \\ x &= 121 \end{aligned}$$
  (e) 3c
$$\begin{aligned} \frac{6x}{5} - \frac{5}{x} &= \frac{2x-3}{3} + \frac{8x}{15} \\ x &= 5 \end{aligned}$$
  (f) 4d
$$\begin{aligned} .1 * 15000 + .12 * x &= 2100 \\ x &= 5000 \end{aligned}$$
(iii) Section 2.2
  (a) [1)]
  (b) 2b
$$\begin{aligned} \frac{ax+b}{cx+d} &= A \\ x &= \frac{-b+Ad}{a-Ac} \end{aligned}$$
  (c) 2c
$$\begin{aligned} \frac{1}{2}px^{-1/2} - w &= 0 \\ x &= \frac{1}{4}\frac{p^2}{w^2} \end{aligned}$$

Date: 27 July 2007.

(d) 3a

$$\begin{aligned} q &= .15p + .14 \\ p &= \frac{20}{3}q - \frac{14}{15} \end{aligned}$$

(e) 3b

$$\begin{aligned} S &= \alpha + \beta P \\ P &= \frac{1}{\beta}S - \frac{\alpha}{\beta} \end{aligned}$$

(f) 3e

$$\begin{aligned} AK^{\alpha}L^{\beta} &= Y_0 \\ L &= \left(\frac{Y_0}{AK^{\alpha}}\right)^{\frac{1}{\beta}} \\ &= A^{-\frac{1}{\beta}}K^{-\frac{\alpha}{\beta}}Y_0^{\frac{1}{\beta}} \end{aligned}$$

(g) 4d

$$\begin{aligned} K^{1/2}\left(\frac{1}{2}\frac{r}{w}\right)^{1/4} &= Q \\ K &= \frac{Q^2}{\sqrt{\frac{1}{2}\frac{r}{w}}} \end{aligned}$$

(h) 4e

$$\begin{aligned} \frac{\frac{1}{2}K^{-1/2}L^{1/4}}{\frac{1}{4}L^{-3/4}K^{1/2}} &= \frac{r}{w} \\ L &= \frac{1}{2}K\frac{r}{w} \end{aligned}$$

(i) 4f

$$\frac{1}{2}pK^{-1/4}\left(\frac{1}{2}\frac{r}{w}\right)^{1/4} = r$$

(iv) Section 2.4
(a) [1)]
(b) 4a

$$\begin{aligned} a + b &= 52 \\ a - b &= 26 \\ a &= 39,\ b = 13 \end{aligned}$$

(c) 4d

$$\begin{aligned} a + b &= 10,000 \\ .05 * x + .072 * y &= 676 \\ a &= \$2000,\ b = \$8,000 \end{aligned}$$

(v) Section 2.5
(a) [1)]

(b) 1b

$$\begin{aligned} x^3\left(1+x^2\right)(1-2x) &= 0 \\ \{x &: x \in \left\{0, \frac{1}{2}, \pm\, i\right\}\} \\ \text{meaning} &: x=0,\ x=\frac{1}{2},\ x=\pm i \text{ are all the solutions} \end{aligned}$$

(c) 1e

$$\begin{aligned} \frac{x\,(x+1)}{x^2+1} &= 0 \\ \{x &: x \in \{0,-1\}\} \\ \text{meaning} &: x=0,\ x=-1 \text{ are the solutions} \end{aligned}$$

(d) 2c

$$\begin{aligned} \frac{(x+1)^{1/3}-\frac{1}{3}x\,(x+1)^{-2/3}}{(x+1)^{2/3}} &= 0 \\ x &= -\frac{3}{2} \end{aligned}$$

, Solution is: $-\frac{3}{2}$

Problem 2. Solve the following equations for x.

a. $\frac{2x+5}{x-3} = \frac{1}{6}$

$$\begin{aligned} 6(2x+5) &= x-3 \\ \Rightarrow 12x+30 &= x-3 \\ \Rightarrow 11x &= -33 \\ \Rightarrow x &= -3 \end{aligned}$$

b. $\frac{3x-2}{10} = \frac{9-5x}{-11}$

$$\begin{aligned} -11(3x-2) &= 10(9-5x) \\ \Rightarrow -33x+22 &= 90-50x \\ \Rightarrow 17x &= 68 \\ \Rightarrow x &= 4 \end{aligned}$$

c. $\frac{23}{4x+11} = \frac{-7}{8-5x}$,

$$\begin{aligned} 23(8-5x) &= -7(4x+11) \\ \Rightarrow 184-115x &= -28x-77 \\ \Rightarrow -87x &= -261 \\ \Rightarrow x &= 3 \end{aligned}$$

d. $\frac{\frac{3x+14}{2x+1}}{2x-6} = \frac{13}{9}$

$$\begin{aligned} 9\frac{3x+14}{2x+1} &= 13(2x-6) \\ \Rightarrow 9(3x+14) &= 13(2x-6)(2x+1) \\ \Rightarrow 27x+126 &= 13(4x^2-10x-6) \\ \Rightarrow 27x+126 &= 52x^2-130x-78 \\ \Rightarrow 52x^2-157x-204 &= 0 \\ \Rightarrow (52x+51)(x-4) &= 0 \\ \Rightarrow x &= -\frac{51}{52}, 4 \end{aligned}$$

Problem 3. Solve the following equations for x.

a. $2x^2 - 2x - 12 = 0$

$$\begin{aligned} x^2 - x - 6 = 0 \Rightarrow (x-3)(x+2) =& \quad 0 \\ \Rightarrow x = 3, -2 & \end{aligned}$$

b. $15x^2 - 13x - 20 = 0$

$$\begin{aligned} 15x^2 - 13x - 20 &= 0 \\ \Rightarrow (3x-5)(5x+4) &= 0 \\ \Rightarrow x &= \frac{5}{3}, -\frac{4}{5} \end{aligned}$$

c. $-72x^2 - 21x + 18 = 0$

$$\begin{aligned} -72x^2 - 21x + 18 &= 0 \\ \Rightarrow (9x+6)(8x-3) &= 0 \\ \Rightarrow x &= -\frac{2}{3}, \frac{3}{8} \end{aligned}$$

d. $-3x^2 + 6x - 51 = 0$

$$\begin{aligned} -3x^2 + 6x - 51 &= 0 \\ \Rightarrow 3x^2 - 6x + 51 &= 0 \\ \Rightarrow 3(x^2 - 2x) + 51 &= 0 \\ \Rightarrow 3(x-1)^2 - 3 + 51 &= 0 \\ \Rightarrow 3(x-1)^2 + 48 &= 0 \end{aligned}$$

Since the square of any real number is positive, there is no real number solution to the above equation. However, there exists solution of complex numbers.

$$\begin{aligned} 3(x-1)^2 + 48 &= 0 \\ \Rightarrow 3(x-1)^2 &= -48 \\ \Rightarrow (x-1)^2 &= -16 \\ \Rightarrow x - 1 &= \pm 4i \\ \Rightarrow x &= 1 \pm 4i \end{aligned}$$

e. $7x^2 \ + \ \frac{3}{2}x \ - \ 118 \ = \ 0$

$$\begin{aligned} 7x^2 \ + \ \frac{3}{2}x \ - \ 118 \ &= 0 \\ \Rightarrow 14x^2 + 3x - 236 &= 0 \\ \Rightarrow (14x + 59)(x - 4) &= 0 \\ \Rightarrow x &= -\frac{59}{14}, 4 \end{aligned}$$

Problem 4. Solve the following equations for x_1.

a. $12x_1^{-2/5} - 3 = 0, \; x_1 = 32$

$$
\begin{aligned}
12x_1^{-2/5} - 3 &= 0 \\
\Rightarrow 4x_1^{-2/5} &= 1 \\
\text{Multiply both sides by } x_1^{2/5} \Rightarrow 4 &= x_1^{2/5} \\
\text{Raise both sides by power } 5/2 \Rightarrow 4^{5/2} &= (x_1^{2/5})^{5/2} \\
\Rightarrow (2^2)^{5/2} &= x_1^{2/5\times 5/2} \\
\Rightarrow 2^{2\times 5/2} &= x_1 \\
\Rightarrow 2^5 &= x_1 \\
\Rightarrow x_1 &= 32
\end{aligned}
$$

b. $81x_1^{-3/4} - 3 = 0$

$$
\begin{aligned}
81x_1^{-3/4} - 3 &= 0 \\
\Rightarrow 27x_1^{-3/4} &= 1 \\
\Rightarrow 27 &= x_1^{3/4} \\
\Rightarrow 27^{4/3} &= x_1 \\
\Rightarrow (3^3)^{4/3} &= x_1 \\
\Rightarrow x_1 &= 3^4 = 81
\end{aligned}
$$

c. $162x_1^{-4/5} - 2 = 0$

$$
\begin{aligned}
162x_1^{-4/5} - 2 &= 0 \\
\Rightarrow 81x_1^{-4/5} &= 1 \\
\Rightarrow 81 &= x_1^{4/5} \\
\Rightarrow (3^4)^{5/4} &= x_1 \\
\Rightarrow x_1 &= 3^5 = 243
\end{aligned}
$$

d. $486x_1^{-5/3} - 2 = 0$

$$\begin{aligned} 486x_1^{-5/3} - 2 &= 0 \\ \Rightarrow 243x_1^{-5/3} &= 1 \\ \Rightarrow 243 &= x_1^{5/3} \\ \Rightarrow (3^5)^{3/5} &= x_1 \\ \Rightarrow x_1 &= 3^3 = 27 \end{aligned}$$

Problem 5. Solve the following equations for x_1.

a. $512x_1^{-1/3} = 2x_1$

$$
\begin{aligned}
512x_1^{-1/3} &= 2x_1 \\
\Rightarrow 512x_1^{-1/3} - 2x_1 &= 0 \\
\Rightarrow 256x_1^{-1/3} - x_1 &= 0 \\
\Rightarrow x_1(256x_1^{-1/3-1} - 1) &= 0 \\
\Rightarrow x_1(256x_1^{-4/3} - 1) &= 0 \\
\Rightarrow x_1 &= 0 \\
\text{or } 256x_1^{-4/3} - 1 &= 0 \\
\Rightarrow 256 &= x_1^{4/3} \\
\Rightarrow 256^{3/4} &= x_1 \\
\Rightarrow x_1 &= (4^4)^{3/4} = 4^3 = 64
\end{aligned}
$$

Hence, the solutions are $x_1 = 0$ or 64.

b. $32x_1^{-2/3} = x_1$

$$
\begin{aligned}
32x_1^{-2/3} &= x_1 \\
\Rightarrow x_1(32x_1^{-2/3-1} - 1) &= 0 \\
\Rightarrow x_1(32x_1^{-5/3} - 1) &= 0 \\
\Rightarrow x_1 &= 0 \\
\text{or } 32x_1^{-5/3} - 1 &= 0 \\
\Rightarrow 32 &= x_1^{5/3} \\
\Rightarrow x_1 &= (2^5)^{3/5} = 8
\end{aligned}
$$

Hence, the solutions are $x_1 = 0$ or 8.

c. $147x_1^{-3/5} = 3x_1^{-1/5}$

$$\begin{aligned}
147x_1^{-3/5} &= 3x_1^{-1/5} \\
\text{Multiply both sides by } x_1^{1/5} \Rightarrow 147x_1^{-3/5+1/5} &= 3x_1^{-1/5+1/5} \\
\Rightarrow 147x_1^{-2/5} &= 3x_1^{0} \\
\Rightarrow 147x_1^{-2/5} &= 3 \\
\Rightarrow 49x_1^{-2/5} &= 1 \\
\Rightarrow 49 = x_1^{2/5} \Rightarrow x_1 = & \quad (7^2)^{5/2} = 7^5 = 16807
\end{aligned}$$

Hence, the solutions are $x_1 = 16807$.

d. $1024x_1^{2/3} = x_1^{7/3}$

$$\begin{aligned}
1024x_1^{2/3} &= x_1^{7/3} \\
\Rightarrow 1024x_1^{2/3} - x_1^{7/3} &= 0 \\
\Rightarrow x_1^{2/3}(1024 - x_1^{5/3}) &= 0 \\
\Rightarrow x_1^{2/3} &= 0 \\
\Rightarrow x_1 &= 0 \\
\text{Or } 1024 - x_1^{5/3} &= 0 \\
\Rightarrow 1024 &= x_1^{5/3} \\
\Rightarrow x_1 &= (4^5)^{3/5} = 4^3 = 64
\end{aligned}$$

Hence, the solutions are $x_1 = 0$ or 64.

Problem 6. Solve the following systems of equations for x_1 and x_2 using the method of substitution
a.

$$\begin{aligned} 5x_1 + 2x_2 &= 14 \\ 7x_1 - 3x_2 &= 8 \end{aligned}$$

From the first equation, we get

$$x_1 = \frac{14}{5} - \frac{2}{5}x_2.$$

We then substitute that into the second equation.

$$\begin{aligned} 7(\frac{14}{5} - \frac{2}{5}x_2) - 3x_2 &= 8 \\ \frac{98}{5} - \frac{14}{5}x_2 - 3x_2 &= 8 \\ -\frac{29}{5}x_2 &= -\frac{58}{5} \\ x_2 &= 2 \end{aligned}$$

And plugging $x_2 = 2$ back into the first equation, we get

$$\begin{aligned} x_1 &= \frac{14}{5} - \frac{4}{5} \\ x_1 &= 2 \end{aligned}$$

Therefore, the solution to the above equations are $(x_1 = 2, x_2 = 2)$.

$$x_1 = 26,\ x_2 = -58$$

b.

$$\begin{aligned} 4x_1 - 5x_2 &= -9 \\ 3x_1 + 2x_2 &= 22 \end{aligned}$$

From the first equation, we get

$$x_1 = -\frac{9}{4} + \frac{5}{4}x_2.$$

We then substitute that into the second equation.

$$\begin{aligned} 3(-\frac{9}{4} + \frac{5}{4}x_2) + 2x_2 &= 22 \\ -\frac{27}{4} + \frac{15}{4}x_2 + 2x_2 &= 22 \\ \frac{23}{4}x_2 &= \frac{115}{4} \\ x_2 &= 5 \end{aligned}$$

And plugging $x_2 = 5$ back into the first equation, we get

$$\begin{aligned} x_1 &= -\frac{9}{4} + \frac{25}{4} \\ x_1 &= 4 \end{aligned}$$

Therefore, the solution to the above equations are $(x_1 = 4, x_2 = 5)$.

c.

$$\begin{aligned} -2x_1 \ + \ 5x_2 \ &= 19 \\ 4x_1 \ + \ 2x_2 \ &= -2 \end{aligned}$$

From the first equation, we get

$$x_1 = -\frac{19}{2} + \frac{5}{2}x_2.$$

We then substitute that into the second equation.

$$\begin{aligned} 4(-\frac{19}{2} + \frac{5}{2}x_2) + 2x_2 &= -2 \\ -38 + 10x_2 + 2x_2 &= -2 \\ 12x_2 &= 36 \\ x_2 &= 3 \end{aligned}$$

And plugging $x_2 = 5$ back into the first equation, we get

$$\begin{aligned} x_1 &= -\frac{19}{2} + \frac{15}{2} \\ x_1 &= -2 \end{aligned}$$

Therefore, the solution to the above equations are $(x_1 = -2, x_2 = 3)$.

d.

$$\begin{aligned} \frac{3}{2}x_1 \ + \ 3x_2 \ &= 3 \\ 4x_1 \ - \ 2x_2 \ &= 28 \end{aligned}$$

From the first equation, we get

$$x_1 = 2 - 2x_2.$$

We then substitute that into the second equation.

$$\begin{aligned} 4(2 - 2x_2) - 2x_2 &= 28 \\ 8 - 8x_2 - 2x_2 &= 28 \\ -10x_2 &= 20 \\ x_2 &= -2 \end{aligned}$$

And plugging $x_2 = 5$ back into the first equation, we get

$$\begin{aligned} x_1 &= 2 - 2(-2) \\ x_1 &= 6 \end{aligned}$$

Therefore, the solution to the above equations are $(x_1 = 6, x_2 = -2)$.

e.

$$\begin{aligned} 2x_1 + 5x_2 &= 2 \\ 4x_1 + 10x_2 &= 4 \end{aligned}$$

It is easy to see that two equations are equivalent. Hence, we have one equation with two unknown variables. And there are infinitely many solutions.

f.

$$\begin{aligned} x_1 - 2x_2 + 3x_3 &= 8 \\ 4x_1 - 7x_2 + 9x_3 &= 26 \\ -2x_1 - 2x_2 + 13x_3 &= 21 \end{aligned}$$

From the first equation, we have

$$x_1 = 2x_2 - 3x_3 + 8$$

Plugging that into the second equation, we have

$$\begin{aligned} 4(2x_2 - 3x_3 + 8) - 7x_2 + 9x_3 &= 26 \\ 8x_2 - 12x_3 + 32 - 7x_2 + 9x_3 &= 26 \\ x_2 - 3x_3 &= -6 \\ x_2 &= -6 + 3x_3. \end{aligned}$$

We then substitute both x_1, x_2 into the third equation.

$$\begin{aligned} -2(2x_2 - 3x_3 + 8) - 2x_2 + 13x_3 &= 21 \\ -4x_2 + 6x_3 - 16 - 2x_2 + 13x_3 &= 21 \\ -6x_2 + 19x_3 &= 37 \\ -6(-6 + 3x_3) + 19x_3 &= 37 \\ 36 - 18x_3 + 19x_3 &= 37 \\ x_3 &= 1 \end{aligned}$$

And

$$\begin{aligned} x_2 &= -6 + 3x_3 \\ &= -6 + 3 \\ &= -3 \\ x_1 &= 2x_2 - 3x_3 + 8 \\ &= -6 - 3 + 8 \\ &= -1 \end{aligned}$$

Problem 7. Solve the following systems of equations for x_1 and x_2 using the method of substitution.

a.

$$\begin{aligned} 40x_1^{-3/4}x_2^{1/2} - 25 &= 0 \\ 80x_1^{1/4}x_2^{-1/2} - 32 &= 0 \end{aligned}$$

From the first equation, we get

$$\begin{aligned} 40x_1^{-3/4}x_2^{1/2} &= 25 \\ x_1^{-3/4} &= \frac{25}{40}x_2^{-1/2} \\ x_1 &= (\frac{5}{8}x_2^{-1/2})^{-4/3} \\ &= (\frac{5}{8})^{-4/3}x_2^{2/3}. \end{aligned}$$

Plugging the above equation into the second one, we have.

$$\begin{aligned} 80((\frac{5}{8})^{-4/3}x_2^{2/3})^{1/4}x_2^{-1/2} &= 32 \\ 5((\frac{5}{8})^{-4/3}x_2^{2/3})^{1/4}x_2^{-1/2} &= 2 \\ 5(\frac{5}{8})^{-1/3}x_2^{1/6}x_2^{-1/2} &= 2 \\ 5(\frac{5}{8})^{-1/3}x_2^{-1/3} &= 2 \\ 5^{1-1/3}x_2^{-1/3} &= 1 \\ x_2^{1/3} &= 5^{2/3} \\ x_2 &= 25. \end{aligned}$$

And,

$$\begin{aligned} x_1 &= (\frac{5}{8})^{-4/3}x_2^{2/3} \\ &= 5^{-4/3}8^{4/3}5^{4/3} \\ &= 8^{4/3} \\ &= (2^3)^{4/3} \\ &= 16. \end{aligned}$$

Hence,

$$x_1 = 25,\ x_2 = 16$$

b.

$$\begin{aligned} 1296x_1^{-4/5}x_2^{1/3} \; - \; 64 \; &= 0 \\ 2160x_1^{1/5}x_2^{-2/3} \; - \; 405 \; &= 0 \end{aligned}$$

From the first equation, we get

$$\begin{aligned} 1296x_1^{-4/5}x_2^{1/3} \; &= 64 \\ 81x_1^{-4/5}x_2^{1/3} \; &= 4 \\ x_1^{-4/5} &= \frac{4}{81}x_2^{-1/3} \\ x_1 &= (\frac{4}{81}x_2^{-1/3})^{-5/4} \\ &= (\frac{4}{81})^{-5/4}x_2^{5/12}. \end{aligned}$$

Plugging the above equation into the second one, we have.

$$\begin{aligned} 2160x_1^{1/5}x_2^{-2/3} \; &= 405 \\ 16x_1^{1/5}x_2^{-2/3} \; &= 3 \\ 16((\frac{4}{81})^{-5/4}x_2^{5/12})^{1/5}x_2^{-2/3} \; &= 3 \\ 16(\frac{4}{81})^{-1/4}x_2^{1/12}x_2^{-2/3} \; &= 3 \\ 4^{2-1/4}(3^{-4})^{-1/4}x_2^{-7/12} \; &= 3 \\ 4^{7/4}3x_2^{-7/12} \; &= 3 \\ 4^{7/4} &= x_2^{7/12} \\ x_2 &= (4^{7/4})^{12/7} \\ &= 4^3 = 64 \end{aligned}$$

And,

$$\begin{aligned} x_1 &= (\frac{4}{81})^{-5/4}x_2^{5/12} \\ &= (\frac{4}{81})^{-5/4}(64)^{5/12} \\ &= 4^{-5/4}3^5 4^{5/4} \\ &= 3^5 \\ &= 243. \end{aligned}$$

Hence,

$$x_1 = 243, \; x_2 = 64$$