Problem 1. Do the following problems from the book.

(i) [a.]
(ii) Section 2.1
(a) [1]
(b) 1d
\[ 4x + 2(x - 4) - 3 = 2(3x - 5) - 1 \]
x \in \mathbb{R} \text{ All real numbers}

(c) 1f
\[ (8x - 7)5 - 3(6x - 4) + 5x^2 = x(5x - 1) \]
x = 1

(d) 2f
\[ \sqrt{2x+14} = 16 \]
x = 121

(e) 3c
\[ \frac{6x}{5} - \frac{5}{x} = \frac{2x-3}{3} + \frac{8x}{15} \]
x = 5

(f) 4d
\[ .1 * 15000 + .12 * x = 2100 \]
x = 5000

(iii) Section 2.2
(a) [1]
(b) 2b
\[ \frac{ax + b}{cx + d} = A \]
x = \frac{-b + Ad}{a - Ac}

(c) 2c
\[ \frac{1}{2}px^{-1/2} - w = 0 \]
x = \frac{1}{4} \frac{p^2}{w^2}

Date: 27 July 2007.
(d) 3a

\[ q = 0.15p + 0.14 \]
\[ p = \frac{20}{3}q - \frac{14}{15} \]

(e) 3b

\[ S = \alpha + \beta P \]
\[ P = \frac{1}{\beta}S - \frac{\alpha}{\beta} \]

(f) 3e

\[ AK^\alpha L^\beta = Y_0 \]
\[ L = \left( \frac{Y_0}{AK^\alpha} \right)^{\frac{1}{\beta}} \]
\[ = A^{-\frac{1}{\beta}}K^{-\frac{\alpha}{\beta}}Y_0^{\frac{1}{\beta}} \]

(g) 4d

\[ K^{1/2} \left( \frac{1}{2} \frac{r}{w} \right)^{1/4} = Q \]
\[ K = \frac{Q^2}{\sqrt{\frac{1}{2} \frac{r}{w}}} \]

(h) 4e

\[ \frac{1}{2}K^{-1/2}L^{1/4} = \frac{r}{w} \]
\[ \frac{1}{4}L^{-3/4}K^{1/2} = \frac{1}{2}K^{r/w} \]

(i) 4f

\[ \frac{1}{2}pK^{-1/4} \left( \frac{1}{2} \frac{r}{w} \right)^{1/4} = r \]

(iv) Section 2.4

(a) [1]

(b) 4a

\[ a + b = 52 \]
\[ a - b = 26 \]
\[ a = 39, b = 13 \]

(c) 4d

\[ a + b = 10,000 \]
\[ 0.05 \times x + 0.072 \times y = 676 \]
\[ a = \$2000, b = \$8,000 \]

(v) Section 2.5

(a) [1]
(b) 1b
\[ \begin{aligned} x^3 \left(1 + x^2\right) (1 - 2x) & = 0 \\ \{x : x \in \left\{0, \frac{1}{2}, \pm i\right\}\} \\ \text{meaning} & : x = 0, x = \frac{1}{2}, x = \pm i \text{ are all the solutions} \end{aligned} \]

(c) 1e
\[ \begin{aligned} \frac{x(x+1)}{x^2+1} & = 0 \\ \{x : x \in \{0, -1\}\} \\ \text{meaning} & : x = 0, x = -1 \text{ are the solutions} \end{aligned} \]

(d) 2c
\[ \begin{aligned} \frac{(x+1)^{1/3} - \frac{1}{3}x(x+1)^{-2/3}}{(x+1)^{2/3}} & = 0 \\ x & = -\frac{3}{2} \end{aligned} \]
, Solution is: $-\frac{3}{2}$
Problem 2. Solve the following equations for x.

a. \[ \frac{2x+5}{x-3} = \frac{1}{6} \]
   
   \[
   6(2x + 5) = x - 3 \\
   \Rightarrow 12x + 30 = x - 3 \\
   \Rightarrow 11x = -33 \\
   \Rightarrow x = -3
   \]

b. \[ \frac{3x-2}{3x+11} = \frac{9-5x}{x+11} \]
   
   \[
   -11(3x - 2) = 10(9 - 5x) \\
   \Rightarrow -33x + 22 = 90 - 50x \\
   \Rightarrow 17x = 68 \\
   \Rightarrow x = 4
   \]

c. \[ \frac{23}{4x+11} = \frac{9-5x}{8-5x} \]
   
   \[
   23(8 - 5x) = -7(4x + 11) \\
   \Rightarrow 184 - 115x = -28x - 77 \\
   \Rightarrow -87x = -261 \\
   \Rightarrow x = 3
   \]

d. \[ \frac{3x+14}{2x-6} = \frac{13}{9} \]
   
   \[
   9 \frac{3x + 14}{2x + 1} = 13(2x - 6) \\
   \Rightarrow 9(3x + 14) = 13(2x - 6)(2x + 1) \\
   \Rightarrow 27x + 126 = 13(4x^2 - 10x - 6) \\
   \Rightarrow 27x + 126 = 52x^2 - 130x - 78 \\
   \Rightarrow 52x^2 - 157x - 204 = 0 \\
   \Rightarrow (52x + 51)(x - 4) = 0 \\
   \Rightarrow x = -\frac{51}{52}, 4
   \]
Problem 3. Solve the following equations for x.

a. $2x^2 - 2x - 12 = 0$

$$x^2 - x - 6 = 0 \Rightarrow (x - 3)(x + 2) = 0$$
$$\Rightarrow x = 3, -2$$

b. $15x^2 - 13x - 20 = 0$

$$15x^2 - 13x - 20 = 0$$
$$\Rightarrow (3x - 5)(5x + 4) = 0$$
$$\Rightarrow x = \frac{5}{3}, -\frac{4}{5}$$

c. $-72x^2 - 21x + 18 = 0$

$$-72x^2 - 21x + 18 = 0$$
$$\Rightarrow (9x + 6)(8x - 3) = 0$$
$$\Rightarrow x = -\frac{2}{3}, \frac{3}{8}$$

d. $-3x^2 + 6x - 51 = 0$

$$-3x^2 + 6x - 51 = 0$$
$$\Rightarrow 3x^2 - 6x + 51 = 0$$
$$\Rightarrow 3(x^2 - 2x) + 51 = 0$$
$$\Rightarrow 3(x - 1)^2 - 3 + 51 = 0$$
$$\Rightarrow 3(x - 1)^2 + 48 = 0$$

Since the square of any real number is positive, there is no real number solution to the above equation. However, there exists solution of complex numbers.

$$3(x - 1)^2 + 48 = 0$$
$$\Rightarrow 3(x - 1)^2 = -48$$
$$\Rightarrow (x - 1)^2 = -16$$
$$\Rightarrow x - 1 = \pm 4i$$
$$\Rightarrow x = 1 \pm 4i$$
e. \( 7x^2 + \frac{3}{2}x - 118 = 0 \)

\[
7x^2 + \frac{3}{2}x - 118 = 0 \\
\Rightarrow 14x^2 + 3x - 236 = 0 \\
\Rightarrow (14x + 59)(x - 4) = 0
\]

\( \Rightarrow x = -\frac{59}{14}, 4 \)
**Problem 4.** Solve the following equations for \(x_1\).

a. \(12x_1^{-2/5} - 3 = 0, \ x_1 = 32\)

\[
12x_1^{-2/5} - 3 = 0 \\
\Rightarrow 4x_1^{-2/5} = 1 \\
\text{Multiply both sides by } x_1^{2/5} \Rightarrow 4 = x_1^{2/5} \\
\text{Raise both sides by power } 5/2 \Rightarrow 4^{5/2} = (x_1^{2/5})^{5/2} \\
\Rightarrow (2^2)^{5/2} = x_1^{2/5 	imes 5/2} \\
\Rightarrow 2^{2 	imes 5/2} = x_1 \\
\Rightarrow 2^5 = x_1 \\
\Rightarrow x_1 = 32
\]

b. \(81x_1^{-3/4} - 3 = 0\)

\[
81x_1^{-3/4} - 3 = 0 \\
\Rightarrow 27x_1^{-3/4} = 1 \\
\Rightarrow 27 = x_1^{3/4} \\
\Rightarrow 27^{4/3} = x_1 \\
\Rightarrow (3^3)^{4/3} = x_1 \\
\Rightarrow x_1 = 3^4 = 81
\]

c. \(162x_1^{-4/5} - 2 = 0\)

\[
162x_1^{-4/5} - 2 = 0 \\
\Rightarrow 81x_1^{-4/5} = 1 \\
\Rightarrow 81 = x_1^{4/5} \\
\Rightarrow (3^4)^{5/4} = x_1 \\
\Rightarrow x_1 = 3^5 = 243
\]
d. $486x_1^{-5/3} - 2 = 0$

\[
486x_1^{-5/3} - 2 = 0 \\
\Rightarrow 243x_1^{-5/3} = 1 \\
\Rightarrow 243 = x_1^{5/3} \\
\Rightarrow (3^5)^{3/5} = x_1 \\
\Rightarrow x_1 = 3^3 = 27
\]
Problem 5. Solve the following equations for $x_1$.

a. $512x_1^{-1/3} = 2x_1$

\[
512x_1^{-1/3} = 2x_1 \\
\Rightarrow 512x_1^{-1/3} - 2x_1 = 0 \\
\Rightarrow 256x_1^{-1/3} - x_1 = 0 \\
\Rightarrow x_1(256x_1^{-1/3} - 1) = 0 \\
\Rightarrow x_1(256x_1^{-4/3} - 1) = 0 \\
\Rightarrow x_1 = 0 \\
\text{or } 256x_1^{-4/3} - 1 = 0 \\
\Rightarrow 256 = x_1^{4/3} \\
\Rightarrow 256^{3/4} = x_1 \\
\Rightarrow x_1 = (4^4)^{3/4} = 4^3 = 64
\]

Hence, the solutions are $x_1 = 0$ or $64$.

b. $32x_1^{-2/3} = x_1$

\[
32x_1^{-2/3} = x_1 \\
\Rightarrow x_1(32x_1^{-2/3} - 1) = 0 \\
\Rightarrow x_1(32x_1^{-5/3} - 1) = 0 \\
\Rightarrow x_1 = 0 \\
\text{or } 32x_1^{-5/3} - 1 = 0 \\
\Rightarrow 32 = x_1^{5/3} \\
\Rightarrow x_1 = (2^5)^{3/5} = 8
\]

Hence, the solutions are $x_1 = 0$ or $8$. 
c. \(147x_{1}^{-3/5} = 3x_{1}^{-1/5}\)

Multiply both sides by \(x_{1}^{1/5}\) \(\Rightarrow\) \(147x_{1}^{-3/5+1/5} = 3x_{1}^{-1/5+1/5}\)

\[\Rightarrow 147x_{1}^{-2/5} = 3x_{1}^{0}\]
\[\Rightarrow 147x_{1}^{-2/5} = 3\]
\[\Rightarrow 49x_{1}^{-2/5} = 1\]
\[\Rightarrow 49 = x_{1}^{2/5} \Rightarrow x_{1} = (7^2)^{5/2} = 7^5 = 16807\]

Hence, the solutions are \(x_{1} = 16807\).

d. \(1024x_{1}^{2/3} = x_{1}^{7/3}\)

\[1024x_{1}^{2/3} = x_{1}^{7/3}\]
\[\Rightarrow 1024x_{1}^{2/3} - x_{1}^{7/3} = 0\]
\[\Rightarrow x_{1}^{2/3}(1024 - x_{1}^{5/3}) = 0\]
\[\Rightarrow x_{1}^{2/3} = 0\]
\[\Rightarrow x_{1} = 0\]

Or \(1024 - x_{1}^{5/3} = 0\)
\[\Rightarrow 1024 = x_{1}^{5/3}\]
\[\Rightarrow x_{1} = (4^5)^{3/5} = 4^3 = 64\]

Hence, the solutions are \(x_{1} = 0\) or 64.
Problem 6. Solve the following systems of equations for $x_1$ and $x_2$ using the method of substitution.

a.

\[
\begin{align*}
5x_1 + 2x_2 &= 14 \\
7x_1 - 3x_2 &= 8
\end{align*}
\]

From the first equation, we get

\[x_1 = \frac{14}{5} - \frac{2}{5}x_2.\]

We then substitute that into the second equation.

\[
\begin{align*}
7(\frac{14}{5} - \frac{2}{5}x_2) - 3x_2 &= 8 \\
\frac{98}{5} - \frac{14}{5}x_2 - 3x_2 &= 8 \\
\frac{-29}{5}x_2 &= \frac{-58}{5} \\
x_2 &= 2
\end{align*}
\]

And plugging $x_2 = 2$ back into the first equation, we get

\[x_1 = \frac{14}{5} - \frac{4}{5} \]

\[x_1 = 2\]

Therefore, the solution to the above equations are ($x_1 = 2, x_2 = 2$).

b.

\[
\begin{align*}
4x_1 - 5x_2 &= -9 \\
3x_1 + 2x_2 &= 22
\end{align*}
\]

From the first equation, we get

\[x_1 = \frac{-9}{4} + \frac{5}{4}x_2.\]

We then substitute that into the second equation.

\[
\begin{align*}
3\left(\frac{-9}{4} + \frac{5}{4}x_2\right) + 2x_2 &= 22 \\
\frac{-27}{4} + \frac{15}{4}x_2 + 2x_2 &= 22 \\
\frac{23}{4}x_2 &= \frac{115}{4} \\
x_2 &= 5
\end{align*}
\]

And plugging $x_2 = 5$ back into the first equation, we get

\[x_1 = \frac{-9}{4} + \frac{25}{4} \]

\[x_1 = 4\]

Therefore, the solution to the above equations are ($x_1 = 4, x_2 = 5$).
c.

\[-2x_1 + 5x_2 = 19\]
\[4x_1 + 2x_2 = -2\]

From the first equation, we get
\[x_1 = -\frac{19}{2} + \frac{5}{2}x_2.\]

We then substitute that into the second equation.
\[4\left(-\frac{19}{2} + \frac{5}{2}x_2\right) + 2x_2 = -2\]
\[-38 + 10x_2 + 2x_2 = -2\]
\[12x_2 = 36\]
\[x_2 = 3\]

And plugging \(x_2 = 5\) back into the first equation, we get
\[x_1 = -\frac{19}{2} + \frac{15}{2}\]
\[x_1 = -2\]

Therefore, the solution to the above equations are \((x_1 = -2, x_2 = 3)\).

d.

\[\frac{3}{2}x_1 + 3x_2 = 3\]
\[4x_1 - 2x_2 = 28\]

From the first equation, we get
\[x_1 = 2 - 2x_2.\]

We then substitute that into the second equation.
\[4(2 - 2x_2) - 2x_2 = 28\]
\[8 - 8x_2 - 2x_2 = 28\]
\[-10x_2 = 20\]
\[x_2 = -2\]

And plugging \(x_2 = 5\) back into the first equation, we get
\[x_1 = 2 - 2(-2)\]
\[x_1 = 6\]

Therefore, the solution to the above equations are \((x_1 = 6, x_2 = -2)\).
e.

\[ 2x_1 + 5x_2 = 2 \]
\[ 4x_1 + 10x_2 = 4 \]

It is easy to see that two equations are equivalent. Hence, we have one equation with two unknown variables. And there are infinitely many solutions.

f.

\[ x_1 - 2x_2 + 3x_3 = 8 \]
\[ 4x_1 - 7x_2 + 9x_3 = 26 \]
\[ -2x_1 - 2x_2 + 13x_3 = 21 \]

From the first equation, we have
\[ x_1 = 2x_2 - 3x_3 + 8 \]

Plugging that into the second equation, we have
\[
4(2x_2 - 3x_3 + 8) - 7x_2 + 9x_3 = 26 \\
8x_2 - 12x_3 + 32 - 7x_2 + 9x_3 = 26 \\
x_2 - 3x_3 = -6 \\
x_2 = -6 + 3x_3.
\]

We then substitute both \(x_1, x_2\) into the third equation.
\[
-2(2x_2 - 3x_3 + 8) - 2x_2 + 13x_3 = 21 \\
-4x_2 + 6x_3 - 16 - 2x_2 + 13x_3 = 21 \\
-6x_2 + 19x_3 = 37 \\
-6(-6 + 3x_3) + 19x_3 = 37 \\
36 - 18x_3 + 19x_3 = 37 \\
x_3 = 1
\]

And
\[
x_2 = -6 + 3x_3 \\
= -6 + 3 \\
= -3
\]
\[
x_1 = 2x_2 - 3x_3 + 8 \\
= -6 - 3 + 8 \\
= -1
\]
Problem 7. Solve the following systems of equations for \( x_1 \) and \( x_2 \) using the method of substitution.

a.

\[
\begin{align*}
40x_1^{-3/4}x_2^{1/2} - 25 &= 0 \\
80x_1^{1/4}x_2^{-1/2} - 32 &= 0
\end{align*}
\]

From the first equation, we get

\[
\begin{align*}
40x_1^{-3/4}x_2^{1/2} &= 25 \\
x_1^{3/4} &= \frac{25}{40x_2} \\
x_1 &= (\frac{5}{8}x_2^{-1/2})^{-4/3} \\
&= (\frac{5}{8})^{-4/3}x_2^{2/3}.
\end{align*}
\]

Plugging the above equation into the second one, we have.

\[
\begin{align*}
80(\frac{5}{8})^{-4/3}x_2^{2/3}1/4x_2^{-1/2} &= 32 \\
5(\frac{5}{8})^{-4/3}x_2^{2/3}1/4x_2^{-1/2} &= 2 \\
5(\frac{5}{8})^{-1/3}x_2^{1/6}x_2^{-1/2} &= 2 \\
5(\frac{5}{8})^{-1/3}x_2^{-1/3} &= 1 \\
x_2^{1/3} &= 5^{2/3} \\
x_2 &= 25.
\end{align*}
\]

And,

\[
\begin{align*}
x_1 &= (\frac{5}{8})^{-4/3}x_2^{2/3} \\
&= 5^{-4/3}8^{4/3} \cdot 5^{4/3} \\
&= 8^{4/3} \\
&= (2^3)^{4/3} \\
&= 16.
\end{align*}
\]

Hence,

\( x_1 = 25, \ x_2 = 16 \)
b.

\[1296x_1^{-4/5}x_2^{1/3} - 64 = 0\]

\[2160x_1^{1/5}x_2^{-2/3} - 405 = 0\]

From the first equation, we get

\[1296x_1^{-4/5}x_2^{1/3} = 64\]

\[81x_1^{-4/5}x_2^{1/3} = 4\]

\[x_1^{-4/5} = \frac{4}{81}x_2^{-1/3}\]

\[x_1 = \left(\frac{4}{81}x_2^{-1/3}\right)^{-5/4}\]

\[= \left(\frac{4}{81}\right)^{-5/4}x_2^{5/12}.\]

Plugging the above equation into the second one, we have.

\[2160x_1^{1/5}x_2^{-2/3} = 405\]

\[16x_1^{1/5}x_2^{-2/3} = 3\]

\[16\left(\frac{4}{81}\right)^{-5/4}x_2^{5/12}\]

\[= \left(\frac{4}{81}\right)^{-5/4}x_2^{5/12}\]

\[16\left(\frac{4}{81}\right)^{-1/4}x_2^{1/12}x_2^{-2/3} = 3\]

\[4^{2-1/4}(3-4)^{-1/4}x_2^{-7/12} = 3\]

\[4^{7/4}3x_2^{-7/12} = 3\]

\[4^{7/4} = x_2^{7/12}\]

\[x_2 = (4^{7/4})^{12/7}\]

\[= 4^3 = 64\]

And,

\[x_1 = \left(\frac{4}{81}\right)^{-5/4}x_2^{5/12}\]

\[= \left(\frac{4}{81}\right)^{-5/4}(64)^{5/12}\]

\[= 4^{-5/4}3^{5/4}\]

\[= 3^5\]

\[= 243.\]

Hence,

\[x_1 = 243, \ x_2 = 64\]