

ECONOMICS 207
SPRING 2007
PROBLEM SET 4

Problem 1. Do the following problems from the book.

- a. Section 3.6
 - 1) 1a
 - 2) 1b
 - 3) 3
 - 4) 5
- b. Section 4.2
 - 1) 3b
 - 2) 7a
 - 3) 7b
 - 4) 13
- c. Section 4.4
 - 1) 1a
 - 2) 1b
 - 3) 3
 - 4) 7a
 - 5) 7b
 - 6) 7c
 - 7) 7d
- d. Section 4.5
 - 1) 3
 - 2) 5
- e. Section 4.6 (Besides equation 2 on page 105 of the text, equation 5 on page 106 of the text is useful.)
 - 1) 3a
 - 2) 3c
 - 3) 3d
 - 4) 3e
 - 5) 3f

Problem 2. Solve the following equations for x .

a. $12x^2 - 26x - 16 = 0$

$$\begin{aligned} 12x^2 - 26x - 16 &= 0 \\ \Rightarrow 6x^2 - 13x - 8 &= 0 \\ \Rightarrow x &= \frac{13 \pm \sqrt{169 + 4 \cdot 6 \cdot 8}}{12} \\ \Rightarrow x &= \frac{13 \pm \sqrt{361}}{12} \end{aligned}$$

b. $24x^2 - 121x + 42 = 0$

$$\begin{aligned} 24x^2 - 121x + 42 &= 0 \\ \Rightarrow x &= \frac{121 \pm \sqrt{121^2 - 4 \cdot 24 \cdot 42}}{48} \\ \Rightarrow x &= \frac{121 \pm \sqrt{10609}}{148} \end{aligned}$$

c. $32x^2 + 60x - 27 = 0$

$$\begin{aligned} 32x^2 + 60x - 27 &= 0 \\ \Rightarrow x &= \frac{-60 \pm \sqrt{60^2 + 4 \cdot 32 \cdot 27}}{64} \\ \Rightarrow x &= \frac{-60 \pm \sqrt{7056}}{64} \\ \Rightarrow x &= \frac{-60 \pm 84}{64} \\ \Rightarrow x &= -\frac{9}{4}, \frac{3}{16}. \end{aligned}$$

Problem 3. Solve the following equations for x_1 .

a. $81x_1^{-3/5} - 3 = 0$

$$\begin{aligned}81x_1^{-3/5} - 3 &= 0 \\ \Rightarrow \frac{81}{3} &= x_1^{3/5} \\ \Rightarrow 27 &= x_1^{3/5} \\ \Rightarrow 27^{5/3} &= (x_1^{3/5})^{5/3} \\ \Rightarrow (3^3)^{5/3} &= x_1 \\ \Rightarrow x_1 &= 3^5 = 243\end{aligned}$$

b. $98x_1^{-2/3} - 2 = 0$

$$\begin{aligned}98x_1^{-2/3} - 2 &= 0 \\ \Rightarrow \frac{98}{2} &= x_1^{2/3} \\ \Rightarrow 49 &= x_1^{2/3} \\ \Rightarrow (7^2)^{3/2} &= x_1 \\ \Rightarrow x_1 &= 7^3 = 343\end{aligned}$$

c. $128x_1^{-5/6} - 4 = 0$

$$\begin{aligned}128x_1^{-5/6} - 4 &= 0 \\ \Rightarrow \frac{128}{4} &= x_1^{5/6} \\ \Rightarrow 32 &= x_1^{5/6} \\ \Rightarrow (2^5)^{6/5} &= x_1 \\ \Rightarrow x_1 &= 2^6 = 64\end{aligned}$$

Problem 4. Solve the following equations for x_1 .

a. $81x_1^{2/5} = 3x_1$

$$\begin{aligned} 81x_1^{2/5} &= 3x_1 \\ 27x_1^{2/5} &= x_1 \\ \Rightarrow 27x_1^{2/5} - x_1 &= 0 \\ \Rightarrow x_1(27x_1^{-3/5} - 1) &= 0 \\ \Rightarrow x_1 &= 0 \\ 27x_1^{-3/5} - 1 &= 0 \\ \Rightarrow x_1^{3/5} &= 27 \\ \Rightarrow x_1 &= (3^3)^{5/3} \\ \Rightarrow x_1 &= 3^5 = 243 \end{aligned}$$

b. $98x_1^{1/3} - 2x_1 = 0$

$$\begin{aligned} 98x_1^{1/3} - 2x_1 &= 0 \\ \Rightarrow x_1(49x_1^{-2/3} - 1) &= 0 \\ \Rightarrow x_1 &= 0 \\ 49x_1^{-2/3} - 1 &= 0 \\ \Rightarrow 49 &= x_1^{2/3} \\ \Rightarrow 49^{3/2} &= x_1 \\ \Rightarrow x_1 &= (7^2)^{3/2} = 7^3 = 343 \end{aligned}$$

c. $128x_1^{1/6} - 4x_1 = 0$

$$\begin{aligned} 128x_1^{1/6} - 4x_1 &= 0 \\ \Rightarrow x_1(32x_1^{-5/6} - 1) &= 0 \\ \Rightarrow x_1 &= 0 \\ x_1^{5/6} &= 32 \\ \Rightarrow x_1 &= (2^5)^{6/5} = 2^6 = 64 \end{aligned}$$

Problem 5. Solve the following systems of equations for x_1 and x_2 using the method of elimination
a.

$$5x_1 + 2x_2 = 14$$

$$7x_1 - 3x_2 = 8$$

Multiply the first equation by 3 and the second one by 2, we get

$$15x_1 + 6x_2 = 42$$

$$14x_1 - 6x_2 = 16.$$

Then we add two equations.

$$29x_1 = 58$$

$$x_1 = 2$$

Plugging the solution of x_2 into the first equation, we can solve that $x_1 = 2$.

b.

$$4x_1 - 5x_2 = -9$$

$$3x_1 + 2x_2 = 22$$

First, multiply the first equation by 2 and the second one by 5.

$$8x_1 - 10x_2 = -18$$

$$15x_1 + 10x_2 = 110$$

Adding the above two equations, we have,

$$23x_1 = 92$$

$$x_1 = 4$$

And plugging $x_1 = 4$ into the second equation, we have,

$$3x_1 + 2x_2 = 22$$

$$12 + 2x_2 = 22$$

$$x_2 = 5$$

c.

$$\begin{aligned} -2x_1 + 5x_2 &= 19 \\ 4x_1 + 2x_2 &= -2 \end{aligned}$$

Multiply the first equation by 2 and adding it to the second one, we have,

$$\begin{aligned} (-4x_1 + 10x_2) + 4x_1 + 2x_2 &= 38 + (-2) \\ 12x_2 &= 36 \\ x_2 &= 3 \end{aligned}$$

Plugging $x_2 = 3$ into the second equation, we have,

$$\begin{aligned} 4x_1 + 2 \times 3 &= -2 \\ 4x_1 &= -8 \\ x_1 &= -2. \end{aligned}$$

d.

$$\begin{aligned} \frac{3}{2}x_1 + 3x_2 &= 3 \\ 4x_1 - 2x_2 &= 28 \end{aligned}$$

Multiply the first equation by 2 and the second one by 3.

$$\begin{aligned} 3x_1 + 6x_2 &= 6 \\ 12x_1 - 6x_2 &= 84 \end{aligned}$$

Adding these two, we have

$$\begin{aligned} 15x_1 &= 90 \\ x_1 &= 6. \end{aligned}$$

And

$$\begin{aligned} 4 \times 6 - 2x_2 &= 28 \\ 24 - 2x_2 &= 28 \\ x_2 &= -2. \end{aligned}$$

e.

$$2x_1 + 5x_2 = 2$$

$$4x_1 + 10x_2 = 4$$

It is easy to see that the first equation is equivalent to the second one. Hence, we have one equation with two unknown variables. There are multiple solutions.

Problem 6. Solve the following systems of equations for x_1 , x_2 and x_3 using the method of elimination

a.

$$x_1 - 2x_2 + 3x_3 = 8 \quad (6a.1)$$

$$4x_1 - 7x_2 + 9x_3 = 26 \quad (6a.2)$$

$$-2x_1 - 2x_2 + 13x_3 = 21 \quad (6a.3)$$

We first subtract equations (6a.1) and (6a.3).

$$3x_1 - 10x_3 = -13 \quad (6a.1.a)$$

Next, we multiply the equation (6a.2) by 2 and equation (6a.3) by 7.

$$8x_1 - 14x_2 + 18x_3 = 52$$

$$-14x_1 - 14x_2 + 91x_3 = 147$$

We then subtract these two equations.

$$22x_1 - 73x_3 = -95 \quad (6a.1.b)$$

Furthermore, we multiply (6a.1.a) by 22 and (6a.1.b) by 3 and subtract those.

$$66x_1 - 220x_3 - (66x_1 - 219x_3) = -286 - (-275)$$

$$-x_3 = -11 \quad (6a.1.c)$$

$$x_3 = 11$$

Substitute $x_3 = 11$ into (6a.1.a), we have

$$3x_1 - 110 = -13$$

$$3x_1 = 97$$

$$x_1 = \frac{97}{3}$$

Finally, substitute x_3 and x_1 into (6a.1), we have

$$\frac{97}{3} - 2x_2 + 33 = 8$$

$$2x_2 = \frac{172}{3}$$

$$x_2 = \frac{86}{3}$$

b.

$$x_1 + 3x_2 - 3x_3 = 8 \quad (6b.1)$$

$$-2x_1 - 7x_2 + 9x_3 = -24 \quad (6b.2)$$

$$3x_1 + 4x_2 + 7x_3 = -19 \quad (6b.3)$$

We first multiply the equation (6b.1) by 2 and add it to the equation (6b.2).

$$\begin{aligned} 2x_1 + 6x_2 - 6x_3 + (-2x_1 - 7x_2 + 9x_3) &= 16 - 24 \\ -x_2 + 3x_3 &= -8 \end{aligned} \quad (6b.1.a)$$

Then multiply equation (6b.1) by 3 and subtract that from equation (6b.3).

$$5x_2 - 16x_3 = 43 \quad (6b.1.b)$$

Next, we multiply (6b.1.a) by 5 and add it to equation (6b.1.b).

$$\begin{aligned} -x_3 &= 3 \\ x_3 &= -3. \end{aligned}$$

Substitute x_3 back into equation (6b.1.a), we get

$$\begin{aligned} -x_2 - 9 &= -8 \\ x_2 &= -1. \end{aligned}$$

Finally, plugging x_2 and x_3 into equation (6b.1), we have

$$\begin{aligned} x_1 - 3 + 9 &= 8 \\ x_1 &= 2. \end{aligned}$$

Problem 7. Solve the following systems of equations for x_1 and x_2 using the method of substitution.

a.

$$448x_1^{-3/4}x_2^{1/2} - 49 = 0 \quad (7a.1)$$

$$896x_1^{1/4}x_2^{-1/2} - 512 = 0 \quad (7a.2)$$

Rearrange the first equation 7a.1 to obtain

$$\begin{aligned} x_1^{-3/4}x_2^{1/2} &= \frac{49}{448} = \frac{7}{64} \\ \Rightarrow x_1^{3/4}x_1^{-3/4}x_2^{1/2} &= \frac{7}{64}x_1^{3/4} \\ \Rightarrow x_2^{1/2} &= \frac{7}{64}x_1^{3/4} \\ \Rightarrow x_2 &= \left(\frac{7}{64}\right)^2 (x_1^{3/4})^2 \\ &= \left(\frac{7}{64}\right)^2 x_1^{3/2} \end{aligned} \quad (7a.1.a)$$

Rearrange the second equation 7a.2 slightly to obtain

$$x_1^{1/4}x_2^{-1/2} = \frac{512}{896} = \frac{2^9}{2^7 \times 7} = \frac{4}{7} \quad (7a.2')$$

Now substitute x_2 from equation 7a.1.a into equation 7a.2' to obtain

$$\begin{aligned} x_1^{1/4} \left(\left(\frac{7}{64}\right)^2 x_1^{3/2} \right)^{-1/2} &= \frac{4}{7} \\ \Rightarrow x_1^{1/4} \left(\frac{7}{64}\right)^{-1} x_1^{-3/4} &= \frac{4}{7} \\ \Rightarrow x_1^{-1/2} \left(\frac{7}{64}\right)^{-1} &= \frac{4}{7} \\ \Rightarrow x_1^{-1/2} &= \frac{7}{64} \frac{4}{7} = \frac{4}{64} = \frac{1}{16} \\ \Rightarrow x_1 &= \left(\frac{1}{16}\right)^{-2} = 16^2 = 256 \end{aligned} \quad (7a.2.a)$$

Now substitute x_1 from equation 7a.2.a into equation 7a.1.a to obtain

$$\begin{aligned} x_2 &= \left(\frac{7}{64}\right)^2 256^{3/2} \\ &= 7^2 2^{-12} (2^8)^{3/2} \\ &= 49 2^{-12} 2^{24/2} = 49 \end{aligned}$$

b.

$$18x_1^{-3/5}x_2^{1/3} - 2 = 0 \quad (7b.1)$$

$$15x_1^{2/5}x_2^{-2/3} - 15 = 0 \quad (7b.2)$$

Rearrange the first equation 7b.1 to obtain

$$\begin{aligned} x_1^{-3/5}x_2^{1/3} &= \frac{2}{18} = \frac{1}{9} \\ \Rightarrow x_1^{3/5}x_1^{-3/5}x_2^{1/3} &= \frac{1}{9}x_1^{3/5} \\ \Rightarrow x_2^{1/3} &= \frac{1}{9}x_1^{3/5} \end{aligned} \quad (7b.1.a)$$

$$\begin{aligned} \Rightarrow x_2 &= \left(\frac{1}{9}\right)^3 \left(x_1^{3/5}\right)^3 \\ &= \left(\frac{1}{9}\right)^3 x_1^{9/5} \end{aligned}$$

Rearrange the second equation 7b.2 slightly to obtain

$$x_1^{2/5}x_2^{-2/3} = \frac{15}{15} = 1 \quad (7b.2')$$

Now substitute x_2 from equation 7b.1.a into equation 7b.2' to obtain

$$\begin{aligned} x_1^{2/5} \left(\left(\frac{1}{9}\right)^3 x_1^{9/5} \right)^{-2/3} &= 1 \\ \Rightarrow x_1^{2/5} \left(\frac{1}{9}\right)^{-2} x_1^{-6/5} &= 1 \\ \Rightarrow x_1^{-4/5} \left(\frac{1}{9}\right)^{-2} &= 1 \quad (7b.2.a) \\ \Rightarrow x_1^{-4/5} &= \left(\frac{1}{9}\right)^2 \\ \Rightarrow x_1 &= \left(\left(\frac{1}{9}\right)^2 \right)^{-5/4} = \left(\frac{1}{9}\right)^{-5/2} = (9)^{5/2} = 3^5 = 243 \end{aligned}$$

Now substitute x_1 from equation 7b.2.a into equation 7b.1.a to obtain

$$x_2 = \left(\frac{1}{9}\right)^3 243^{9/5} = 3^{-6} 3^9 = 3^3 = 27$$