ECONOMICS 207
SPRING 2007
PROBLEM SET 5

Problem 1. Solve the following equations for x.

a. $\dfrac{5x - 2}{4x - 28} = \dfrac{14}{11}$

b. $10x^2 - 58x + 40 = 0$

Date: 7 February 2007.

c. $-6x^2 - 16x - 8 = 0$

d. $15x_1^{-2/3} - 10x_1^{-1/6} = 0$

Problem 2. Solve the following systems of equations for x_1 and x_2 using the method of substitution.
a.

$$432x_1^{-4/5}x_2^{1/3} - 81 = 0$$

$$720x_1^{1/5}x_2^{-2/3} - 160 = 0$$

b.

$$\{x_1 = 27,\ x_2 = 32\}$$

$$\begin{aligned} 180x_1^{-2/3}x_2^{2/5} - 80 &= 0 \\ 216x_1^{1/3}x_2^{-3/5} - 81 &= 0 \end{aligned}$$

Problem 3. Solve the following systems of equation for x_1, x_2, and x_3 first using the method of substitution and then using the method of elimination.

a.

$$\{x_1 = -1,\ x_2 = 1,\ x_3 = -2\}$$

$$\begin{aligned} x_1 + 2x_2 + 5x_3 &= -9 \\ 3x_1 + 7x_2 + 16x_3 &= -28 \\ 3x_1 - x_2 + 9x_3 &= -22 \end{aligned}$$

Elimination here

b.

$$\{x_1 = 2,\ x_2 = 1,\ x_3 = 6\}$$

$$\begin{aligned} x_1 + 3x_2 + 2x_3 &= 17 \\ 2x_1 + 7x_2 + 5x_3 &= 41 \\ -4x_1 - x_2 + 4x_3 &= 15 \end{aligned}$$

Elimination here

Problem 4. Do the following problems from the book.

a. Section 6.2
   1) 1 (The comparison is with equation 6 in the text, not problem 6)
   2) 3
   3) 5

b. Section 6.3
   1) 1

c. Section 6.4
   1) 1
   2) 3
   3) 7

d. Section 6.5
   1) 1a
   2) 1b
   3) 1c
   4) 1d

e. Section 6.6
   1) 3a
   2) 3b
   3) 3c
   4) 3d
   5) 3e
   6) 3f
   7) 3g
   8) 3h

Problem 5. Find the derivatives of each of the following functions with respect to x.

a. $y = 4x^2 - 3x^3$

b. $f(x) = 4\,e^x + \frac{1}{2}x^2$

c. $f(x) = 27x^{1/3} - 2\log[x]$

d. $f(x) = 3x^2 + 2x^3 - 5^x$

e. $f(x) = 3x^{1/2}z^{1/3} - 4x^{-1}$

f. $f(x) = x^{-2} + 3x^2\, e^x$

g. $f(x) = 2x^3 \log[x]$

h. $f(x) = (2x + 5)^2$ Find in two different ways.

i. $f(x) = \frac{x^3+2x}{x^2+3}$

j. $f(x) = \frac{2x^3\, e^x}{x^2+\log[x]}$

Problem 6. Find the derivatives of each of the following functions with respect to x.

a. $f(x) = (2x^2 + 3x)^3$

b. $f(x) = (2x + 2)(2x - 4)$ Show two ways.

c. $f(x) = 2x^2\, e^{x^2+2}$

d. $f(x) = 5^x\, e^{3x^2+e^x}$

e. $f(x) = x^2\, e^{x^2-3x}$

f. $f(x) = \log[(x^2+2x)^3]$

g. $f(x) = \frac{3x\, e^{2x}}{4x^{1/2}+2}$

h. $f(x) = \frac{3x\, e^{2x^2}}{x^2 + 2\log[x]}$

Problem 7. For each of the following, take the derivative with respect to x_1, set the derivative equal to zero and solve the resulting equation for x_1.

a. $f(x) = 81x_1^{1/3} - 3x_1$

b. $f(x) = 54x_1^{2/3} - 9x_1$

c. $f(x) = 64x_1^{5/8} - 5x_1$

d. $f(x) = 432x_1^{1/4}x_2^{1/3} - 16x_1 - 27x_2$

Problem 8. For the given function find an equation of the tangent line at the specified point x = a.

a.

$$\begin{aligned} y &= 3x^2 + 2x \\ a &= 3 \end{aligned}$$

b.

$$\begin{aligned} y &= 10x + 2x^2 - 0.01x^3 \\ a &= 10 \end{aligned}$$