

ECONOMICS 207  
PROBLEM SET 6  
SPRING 2007

**Problem 1.** Find the derivatives of each of the following functions with respect to  $x$ .

a.  $y = \frac{3x^2}{(2x^3+2)^2}$

b.  $y = \frac{e^{2x^3}}{2x^2 e^{2x}}$

c. Find the derivative with respect to  $x_1$ .  $y = 180x_1^{1/2}x_2^{2/5} - 40x_1 - 81x_2$

d. Find the derivative with respect to  $x_2$ .  $y = 180x_1^{1/2}x_2^{2/5} - 40x_1 - 81x_2$

**Problem 2.** Find the second derivative of each of the following functions with respect to  $x$

a.  $y = 6x^{1/3} - 2x^{1/2} + 2x^2$

b.  $y = \frac{3x^3}{(2x^2 + 5)^2}$

c.  $y = \ln[(3x^2 - 5x)^2]$

d.  $y = 5(200x + 30x^2 - x^3) - 2125x$

**Problem 3.** In the following problems you are given a production function for a firm where  $y$  is the level of output and  $x$  is the level of the variable input. You are given the price ( $p$ ) of the output and the price ( $w$ ) of the single variable input. For each problem write down an equation that represents profit for the firm. Then maximize this function by taking its derivative with respect to the variable input  $x$  and set equal to zero. What is the optimal level of  $x$ , of output, of cost, of revenue, of profit?

a.

$$\text{output price} = p = 5$$

$$\text{input price} = w = 2125$$

$$y = \text{output} = f(x) = 200x + 30x^2 - x^3$$

b.

$$\text{output price} = p = 3$$

$$\text{input price} = w = 1872$$

$$y = \text{output} = f(x) = 400x + 50x^2 - 2x^3$$

**Problem 4.** For each of the following problems you are given a price ( $p$ ) and the cost function for a competitive firm.  $TC(y)$  stands for total cost and  $y$  represents the level of output. Marginal cost is the derivative of the cost function with respect to output. Find the profit maximizing level of output for each case.

a.

$$price = p = 392$$

$$Total\ Cost = TC = 500 + 200y - 10y^2 + y^3$$

b.

$$p = 496$$

$$TC = 1000 + 400y - 50y^2 + 3y^3$$



**Problem 5.** Solve the following systems of equations.

$$90x_1^{-1/2}x_2^{2/5} - 40 = 0$$

$$72x_1^{1/2}x_2^{-3/5} - 81 = 0$$

**Problem 6.** Find the indefinite integral of each of the following functions. Write in the form  $F(x) + c$ .

a.  $f(x) = 5x^2 + 12x + 4$

b.  $f(x) = 8x^{-1/3} - 4$

c.  $y = \frac{1}{x}$

d.  $y = 6x^{-2} - 60x^{-1/2} + 200$

**Problem 7.** Find the definite integral of each of the following functions.

a.  $\int_1^3 (3x + 4) dx$

b.  $\int_0^{15} (3x^2 - 24x + 200) dx$

c.  $\int_4^{12} (9x^2 - 60x + 400) dx$

d.  $\int_0^{11} (6x^2 - 30x + 400) dx$

**Problem 8.** Do the following problems from the book.

a. Section 9.1

- 1) 1a
- 2) 1b
- 3) 1c
- 4) 3a
- 5) 3b
- 6) 5a
- 7) 5b
- 8) 5c

b. Section 9.2

- 1) 5a
- 2) 5b
- 3) 5c
- 4) 5d
- 5) 5e
- 6) 7

c. Section 9.3

- 1) 1a
- 2) 1b
- 3) 1c
- 4) 1d
- 5) 1e