

ECONOMICS 207
SPRING 2007
PROBLEM SET 7

Problem 1. Find the second derivative of each of the following functions with respect to x

a. $f(x) = -300 + 100x + 10x^2 - x^3$

b. $f(x) = 30x^{2/3}z^{1/4} - 10x - 10z$

c. $f(x) = (4x^3 + 2x^2 + 3x)^3$

d. $f(x) = \log[x^3 - 2x^2]$

Problem 2. Find the definite integral of each of the following functions.

a. $\int_1^3 (3x^2 + 3x) dx$

b. $\int_{27}^{216} (20x^{-1/3}z^{1/4} - 10) dx, \quad z = 81.$

c. $\int_0^{10} (3x^2 - 20x + 100) dx$

d. $\int_0^{10} (250 - 5x) dx$

Problem 3. Solve the following systems of equations.

$$20x_1^{-1/3}x_2^{1/4} - 10 = 0$$

$$\frac{15}{2}x_1^{2/3}x_2^{-3/4} - 10 = 0$$

Problem 4.

- a. Find the profit maximizing level of output for the following firm. Demonstrate that the level you choose maximizes profit.

$$\text{price} = p = \$200$$

$$\text{cost} = c(y) = 300 + 100y - 10y^2 + y^3$$

- b. What is revenue minus variable cost for this firm when price is \$200?
- c. Find producer surplus for this firm assuming you are only given the following marginal cost function: $MC(y) = 100 - 20y + 3y^2$ and a price of \$200.

Problem 5.

- a. Find the profit maximizing level of output for the following firm. Demonstrate that the level you choose maximizes profit.

$$\text{price} = p = \$374$$

$$\text{cost} = c(y) = 500 + 400y - 40y^2 + 2y^3$$

- b. What is revenue minus variable cost for this firm when price is \$374?
- c. Find producer surplus for this firm assuming you are only given the following marginal cost function: $MC(y) = 400 - 80y + 6y^2$ and a price of \$374.

Problem 6. In the following problem you are given a production function for a firm where y is the level of output and x is the level of the variable input. You are given the price (p) of the output and the price (w) of the single variable input. Write down an equation that represents profit for the firm. Then maximize this function by taking its derivative with respect to the variable input x and set equal to zero. What is the optimal level of x ? Show why this x is the one that maximizes profit.

$$\text{output price} = p = 3$$

$$\text{input price} = w = 792$$

$$y = \text{output} = f(x) = 30x + 40x^2 - 2x^3$$

Problem 7. Solve the following system of equations for x_1 , x_2 , and x_3 .

$$\{x_1 = 1, x_2 = 3, x_3 = 1\}$$

$$x_1 - 2x_2 + 3x_3 = -2$$

$$4x_1 - 9x_2 + 12x_3 = -11$$

$$-2x_1 + 3x_2 - 5x_3 = 2$$

Problem 8. For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise.

a. $y = x^2$

b. $y = 9x^3 - 27x^2$

c. $f(x) = -960x + 400x^2 - 20x^3$

d. $f(x) = -20x^3 + 1500x$

e. $f(x) = -3x^5 + 5x^3$

f. $f(x) = x^2 + \frac{1}{x^2}$

Problem 9. Do the following problems from the book.

a. Section 9.4

1) 3 (Example 3 is useful to study)

2) 5

b. Section 8.5

1) 3

2) 5a

c. Section 8.6

Problem 1

d. Section 8.7

You will need to use Theorem 8.7.1 for these problems.

1) 1a

2) 1b

3) 3a

4) 3b

5) 3c