ECONOMICS 207
SPRING 2007
PROBLEM SET 8

Consider the following matrices.

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 & 6 \\ 3 & 4 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix},$$

$$D = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}, \quad F = \begin{bmatrix} -2 & 3 \\ -3 & 1 \\ 4 & 2 \end{bmatrix}$$

$$a = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \quad c = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Problem 1. Compute the following

a. A + B

b. D'

c. AB

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d. AD

e. ADB

f. FA

g. FAD

h. Ca

i. DACa

j. CF

k. b’F

l. c’C

Problem 2. For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise. Also find the points of inflection for each function.

a. $f(x) = 180x^{1/3} - 15x$

b. $y = 42x^2 - 2x^3 + 600$

c. $f(x) = 84x - 27x^2 + 2x^3$

d. $f(x) = 6x^4 - 4x^3$

e. $\frac{x^4 + 1}{x^2 + 1}$, Substitute $z = x^2$ in the first order conditions and then solve for the roots. One will be zero and there will be four others, two are complex numbers. You need not find inflection points for the function.

f. $f(x) = \frac{x^2-2x+1}{x+1}$

g. $f(x) = 36x + 3x^2 - 8x^3 + \frac{3}{2}x^4$. (The critical values are x = -1, x = 2, and x = 3).

Problem 3. Find the definite integral of each of the following functions.

a. $\int_2^4 \left(2x^3 - 6x^2 + 2x\right)$ dx.

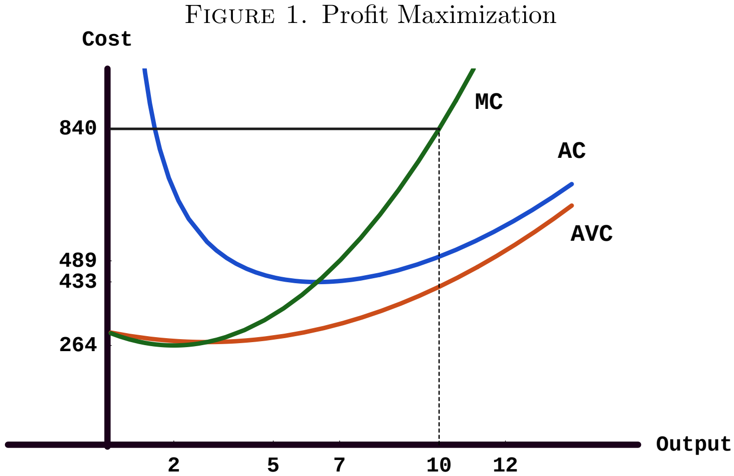
b. $\int_7^{10} \left(300 - 36y + 9y^2\right)$ dy.

Problem 4. Solve the following systems of equations (The answers are $x_1 = 49$ and $x_2 = 32$.)

$$70x_1^{-1/2}x_2^{2/5} \; - \; 40 \; = 0$$
$$56x_1^{1/2}x_2^{-3/5} \; - \; 49 \; = 0$$

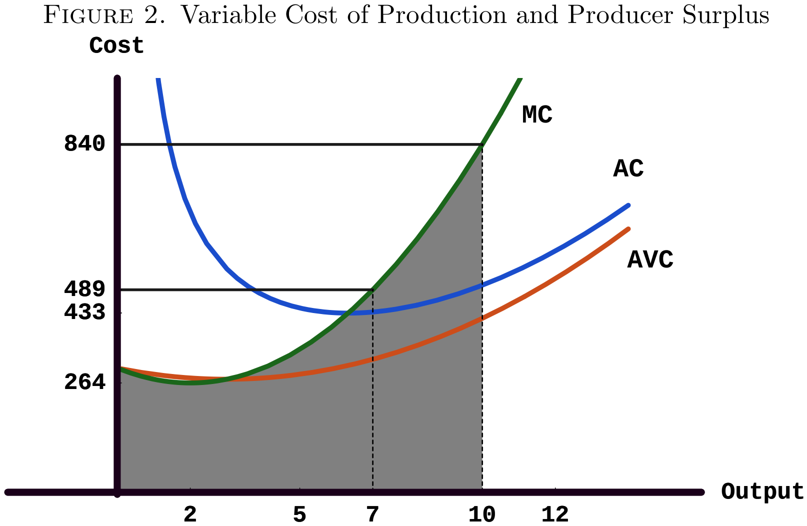
Problem 5. The cost function for a firm is a rule or mapping that tells the total cost of production of any output level produced by the firm. If the variable y represents the output of the firm, then the cost function is given by c(y). Marginal cost represents the change in the cost of production for the firm as output changes and is given by the derivative of the cost function with respect to output,i.e., Marginal Cost(MC) $= \frac{dc(y)}{dy}$. A competitive firm facing a fixed output price maximizes profit at the output level where marginal cost is equal to price. as in the figure 1.

Figure 1. Profit Maximization



The area below the cost curve is a measure of variable cost and can be found by integrating the marginal cost curve from 0 to any given output level y. The shaded area in figure 2 represents the variable cost of production for the cost function $c(y) = 800 + 300y - 18y^2 + 3y^3$.

Figure 2. Variable Cost of Production and Producer Surplus



Producer surplus is the area below a given price and above the marginal cost curve. Producer surplus is the unshaded area below the horizontal line at 840 in figure 2. Producer surplus can be computed by subtracting the shaded area from total revenue.

a. Find the profit maximizing level of output for the following firm. Demonstrate that the level you choose maximizes profit.

$$price = p = \$840$$
$$cost = c(y) = 800 + 300y - 18y^2 + 3y^3$$

b. Find the profit maximizing level of output when the price is \$489. Demonstrate that the level you choose maximizes profit.

c. What is variable cost for this firm when price is $840?

d. What is producer surplus for this firm when the price is $840?

e. What is variable cost for this firm when price is $489?

f. What is producer surplus for this firm when the price is $489?

g. How much does variable cost change when output falls from 10 to 7 units?

h. How much is the firm worse off when price falls from $840 to $489?

i. Cross-hatch the change in producer surplus in Figure 2.

Problem 6. In the following problem you are given a production function for a firm where y is the level of output and x is the level of the variable input. You are given the price (p) of the output and the price (w) of the single variable input. Write down an equation that represents profit for the firm. Then maximize this function by taking its derivative with respect to the variable input x and set equal to zero. What is the optimal level of x? Show why this x is the one one that maximizes profit.

$$\begin{aligned} output\ price &= p = 1 \\ input\ price &= w = 972 \\ y = output &= f(x) = 400x + 100x^2 - 4x^3 \end{aligned}$$

Problem 7. Solve the following system of equations for x_1, x_2, and x_3.

$$\{x_1 \ = \ 2, \ x_2 \ = \ 2, \ x_3 \ = \ -1\}$$

$$\begin{aligned} x_1 + 4x_2 + 2x_3 \ &= 8 \\ -2x_1 - 9x_2 - 4x_3 \ &= \ -18 \\ 3x_1 + 6x_2 + 5x_3 \ &= 13 \end{aligned}$$