ECONOMICS 207
SPRING 2008
EXAM 1
KEY

Problem 1 (15 points)**.** Carry out the following operations. Simplify.
a. $2^3 * 4^2 = 2^3 \times 4^4 = 2^7 = 128$

b. $(3^2)^3 = 3^6 = 3^4 \times 3^2 = 81 \times 9 = 729$

c. $(a+b)(a-b) = (a^2 - b^2)$

d. $b^{-2}a^2a^{1/2}b^{\frac{5}{2}} \quad = \quad a^{5/2}b^{1/2}$

e. $\left(27x_1^{2/3}x_1^{1/4}\right)^{\frac{1}{3}} \quad = \quad 3x_1^{2/9}x_1^{1/12} = 3x_1^{8/36}x_2 3/36 = 3x_2^{11/36}$

Date: February 4, 2008.

Problem 2 (15 points)**.** Simplify, add, subtract, multiply or divide the following fractions. Express all answers in reduced form.

a. $\frac{198}{66} + \frac{72}{24} \quad = \quad 3+3=6$

b. $\left(\frac{133}{224}\right)\left(\frac{8}{\frac{1}{4}}\right) = \frac{133}{224} \times 8 \times 4 = \frac{133 \times 4}{28} = \frac{133}{7} = 19$

c. $\frac{1}{3}+\frac{5}{6} + \frac{7}{12} = \frac{4}{12}+\frac{10}{12} + \frac{7}{12} = \frac{21}{12} = \frac{7}{4}$

d. $\frac{2}{3}+\frac{10}{27} + \frac{5}{54}$ - $\frac{5}{12}$

1. First find the prime factorization of each denominator.
   i) $3 = 3 \times 1 = 3$
   ii) $27 = 3^3$
   iii) $54 = (3^3) \times 2$
   iv) $12 = 2^2 \times 3$
2. To obtain the least common denominator consider multiplying together all these prime numbers. But only multiply the highest power of each prime number that appears in the list. For this example we obtain $2^2 \times 3^3 = 4 \times 27 = 108$.
3. Multiply the numerator in each term by the prime factors in the overall product that do not appear in the denominator for that term. For the first term we need 2^2 and 3^2, so we multiply 3 by 36 to obtain 108 and 2 by 36 to obtain 72. So the first fraction is $\frac{72}{108}$. The term in the overall product missing for 27 is 2^2 so we multiply 27 by 4 to obtain 108 and 10 by 4 to obtain 40. So the second fraction is $\frac{40}{108}$. The term in the overall product missing for 52 is 2, so we multiply 54 by 2 to obtain 108 and 5 by 2 to obtain 10. So the third fraction is $\frac{10}{108}$. The term in the overall product missing for 12 is 3^2, so we multiply 12 by 9 to obtain 108 and 5 by 9 to obtain 45. So the last fraction is $\frac{45}{108}$
4. Combining the terms together we obtain
   $\frac{72}{108} + \frac{40}{108} + \frac{10}{108} - \frac{45}{108} = \frac{77}{108}$
   This prime factorization of 77 is 7×11 and the prime factorization of 108 is $2^2 \times 3^3$ so the fraction does not simplify.

e. $\frac{1}{4}\frac{5}{6}+\frac{\frac{5}{6}}{\frac{2}{5}}-\frac{1}{6}+\left(\frac{9}{4}\frac{64}{25}\right)^{-1/2}$

$$\begin{aligned}
\frac{1}{4}\frac{5}{6}+\frac{\frac{5}{6}}{\frac{2}{5}}-\frac{1}{6}+\left(\frac{9}{4}\frac{64}{25}\right)^{-1/2} &= \frac{5}{24}+\frac{25}{12}-\frac{4}{24}+\left(\frac{4}{9}\frac{25}{64}\right)^{1/2} \\
&= \frac{5}{24}+\frac{50}{24}-\frac{4}{24}+\left(\frac{4}{9}\right)^{1/2}\left(\frac{25}{64}\right)^{1/2} \\
&= \frac{5}{24}+\frac{50}{24}-\frac{4}{24}+\left(\frac{2}{3}\right)\left(\frac{5}{8}\right) \\
&= \frac{5}{24}+\frac{50}{24}-\frac{4}{24}+\frac{10}{24} \\
&= \frac{61}{24}
\end{aligned}$$

And 61 is a prime number.

Problem 3 (12 points)**.** Complete the following operations. Simplify as possible.

a. $e^x e^{2x} = e^{3x}$

b. $\frac{1}{4}\frac{16x_1^{1/3}x_2^{1/4}}{x_2} = 4x_1^{1/3}x_2^{-3/4}$

c. $(2+3x)(5x-14) = 15x^2 - 32x - 28$

d. $\frac{(10x^2+11x-6)}{(20x^2-23x+6)} = \frac{\cancel{(5x-2)}(2x+3)}{\cancel{(5x-2)}(4x-3)} = \frac{2x+3}{4x-3}$

Problem 4 (16 points)**.** Factor the following.

a. $9x^2 - 16 = (3x+4)(3x-4)$

b. $30x^2 + 33x - 18 = 3(10x^2 + 11x - 6) = 3(5x-2)(2x+3)$

c. $12x^2 - 23x - 24 = (4x+3)(3x-8)$

d. $12x^2 + 28x + 15 = (6x+5)(2x+3)$

Problem 5 (15 points)**.** Solve the following equations for x.

a. $\dfrac{3x-5}{4x+7} = \dfrac{7}{23}$

$$\frac{3x-5}{4x+7} = \frac{7}{23}$$

$\Rightarrow 69x - 115 = 28x + 49$ Cross multiply.

$\Rightarrow 41x - 115 = 49$ Subtract 28x from both sides of the equation.

$\Rightarrow 41x = 164$ Add 115 to both sides of the equation.

$\Rightarrow x = 4$ Divide both sides of the equation by 41.

b. $\dfrac{2x^2+3x-9}{x+3} = 11$

$$\frac{2x^2+3x-9}{x+3} = 11$$

$\Rightarrow \dfrac{(2x-3)(x+3)}{x+3} = 11$ Factor numerator.

$\Rightarrow \dfrac{(2x-3)\cancel{(x+3)}}{\cancel{x+3}} = 11$ Cancel like terms.

$\Rightarrow 2x = 14$ Add 3 to both sides of equation.

$\Rightarrow x = 7$ Divide both sides of the equation by 2.

c. $\dfrac{2x^2+3x+9}{x+3} = 11$

$$\frac{2x^2+3x+9}{x+3} = 11$$

$\Rightarrow 2x^2 + 3x + 9 = 11x + 33$ Cross multiply.

$\Rightarrow 2x^2 - 8x + 9 = 33$ Subtract 11x from both sides of the equation.

$\Rightarrow 2x^2 - 8x - 24 = 0$ Subtract 33 from both sides of the equation.

$\Rightarrow 2(x^2 - 4x - 12) = 0$ Factor out a 2.

$\Rightarrow 2(x+2)(x-6) = 0$ Factor the quadratic.

$\Rightarrow x + 2 = 0$ or $(x-6) = 0$

$\Rightarrow x = -2$ Subtract 2 from both sides of the equation. And

$\Rightarrow x = 6$ Add 6 to both sides of the equation.

Problem 6 (18 points)**.** Solve the following equations for x.

a. $20x^2 - 23x + 6 = 0$

$20x^2 - 23x + 6 = 0$

$\Rightarrow (5x - 2)(4x - 3) = 0$ Factor.

$\Rightarrow 5x - 2 = 0$ or $(4x - 3) = 0$

$\Rightarrow x = \frac{2}{5}$ Add 2 to both sides of the equation and divide by 5. And

$\Rightarrow x = \frac{3}{4}$ Add 3 to both sides of the equation and divide by 4.

b. $6x^2 + 19x - 20 = 0$

$6x^2 + 19x - 20 = 0$

$\Rightarrow (6x - 5)(x + 4) = 0$ Factor.

$\Rightarrow 6x - 5 = 0$ or $(x + 4) = 0$

$\Rightarrow x = \frac{5}{6}$ Add 5 to both sides of the equation and divide by 6. And

$\Rightarrow x = -4$ Subtract 4 from both sides of the equation.

c. $24x^2 - 46x + 20 = 5$

$24x^2 - 46x + 15 = 0$

$\Rightarrow (12x - 5)(2x - 3) = 0$ Factor.

$\Rightarrow 12x - 5 = 0$ or $(2x - 3) = 0$

$\Rightarrow x = \frac{5}{12}$ Add 5 to both sides of the equation and divide by 12. And

$\Rightarrow x = \frac{3}{2}$ Add 3 to both sides of the equation and divide by 2.

Problem 7. [9 points] Solve the following systems of equations for x_1 and x_2 using the method of substitution

a.

$$x_1 + 5x_2 = -5 \tag{a.1}$$

$$3x_1 + 2x_2 = 11 \tag{a.2}$$

Rewrite the first equation (a.1) as follows

$$x_1 = -5 - 5x_2 \tag{a.1a}$$

Now substitute in the second equation (a.2), obtain one equation in one unknown, and solve to obtain

$$\begin{aligned} 3(-5-5x_2) + 2x_2 &= 11 \\ \Rightarrow \; -15 - 15x_2 + 2x_2 &= 11 \\ \Rightarrow -13x_2 &= 26 \\ \Rightarrow \; x_2 &= -2 \end{aligned}$$

Now substitute x_2 = -2 in equation a.1a to obtain

$$\begin{aligned} x_1 &= -5 - 5x_2 \\ &= -5 - 5(-2) = -5 - -10 = \quad = 5 + 10 = 5 \end{aligned}$$

We can verify the answer as

$$\begin{aligned} 5 + 5(-2) &= -5 \\ (3)(5) + (2)(-2) &= 11 \end{aligned}$$

b.

$$16x_1^{-2/3}x_2^{1/4} - 8 = 0 \tag{7b.1}$$

$$12x_1^{1/3}x_2^{-3/4} - 3 = 0 \tag{7b.2}$$

Rearrange the first equation 7b.1 to obtain

$$\begin{aligned} x_1^{-2/3}x_2^{1/4} &= \frac{8}{16} = \frac{1}{2} \\ \Rightarrow x_1^{2/3}x_1^{-2/3}x_2^{1/4} &= \frac{1}{2}x_1^{2/3} \\ \Rightarrow x_2^{1/4} &= \frac{1}{3}x_1^{2/3} \\ \Rightarrow x_2 &= \left(\frac{1}{2}\right)^4 \left(x_1^{2/3}\right)^4 \\ &= \left(\frac{1}{2}\right)^4 x_1^{8/3} \end{aligned} \tag{7b.1.a}$$

Rearrange the second equation 7b.2 slightly to obtain

$$x_1^{1/3}x_2^{-3/4} = \frac{3}{12} = \frac{1}{4} \tag{7b.2$'$}$$

Now substitute x_2 from equation 7b.1.a into equation 7b.2$'$ to obtain

$$\begin{aligned} x_1^{1/3}\left(\left(\frac{1}{2}\right)^4 x_1^{8/3}\right)^{-3/4} &= \frac{1}{4} \\ \Rightarrow x_1^{1/3}\left(\frac{1}{2}\right)^{-3} x_1^{-2} &= \frac{1}{4} \\ \Rightarrow x_1^{-5/3}\left(\frac{1}{2}\right)^{-3} &= \frac{1}{4} \\ \Rightarrow x_1^{-5/3} &= \frac{1}{4}\left(\frac{1}{2}\right)^3 \\ \Rightarrow x_1 &= \left(\frac{1}{4}\left(\frac{1}{2}\right)^3\right)^{-3/5} \\ &= \left(\frac{1}{4}\right)^{-3/5}\left(\frac{1}{2}\right)^{-9/5} \\ &= (2^2)^{3/5}2^{9/5} \\ &= 2^{6/5}2^{9/5} = 2^{15/5} = 2^3 = 8 \end{aligned} \tag{7b.2.a}$$

Now substitute x_1 from equation 7b.2.a into equation 7b.1.a to obtain

$$\begin{aligned} x_2 &= \left(\frac{1}{2}\right)^4 x_1^{8/3} \\ &= \left(\frac{1}{2}\right)^4 8^{8/3} \\ &= \left(\frac{1}{2}\right)^4 (2^3)^{8/3} \\ &= \left(\frac{1}{2}\right)^4 2^8 = 2^{-4}2^8 = 2^4 = 16 \end{aligned}$$