

**ECONOMICS 207**  
**SPRING 2008**  
**EXAM 2**

Arithmetic is where numbers fly like pigeons in and out of your head.

Arithmetic tells you how many you lose or win if you know how many you had before you lost or won.

Arithmetic is seven eleven all good children go to heaven—or five six bundle of sticks.

Arithmetic is numbers you squeeze from your head to your hand to your pencil to your paper till you get the answer.

Arithmetic is where the answer is right and everything is nice and you can look out of the window and see the blue sky—or the answer is wrong and you have to start all over and try again and see how it comes out this time.

If you take a number and double it and double it again and then double it a few more times, the number gets bigger and bigger and goes higher and higher and only arithmetic can tell you what the number is when you decide to quit doubling.

Arithmetic is where you have to multiply—and you carry the multiplication table in your head and hope you won't lose it.

If you have two animal crackers, one good and one bad, and you eat one and a striped zebra with streaks all over him eats the other, how many animal crackers will you have if somebody offers you five six seven and you say No no no and you say Nay nay nay and you say Nix nix nix?

If you ask your mother for one fried egg for breakfast and she gives you two fried eggs and you eat both of them, who is better in arithmetic, you or your mother?  
—Carl Sandburg

**Notes.**

- (1)  $2^8 = 256$
- (2)  $2^{11} = 2,048$
- (3)  $2^{13} = 8,192$
- (4)  $16 \times 31 = 496$
- (5)  $496 - 960 = -464$

**Problem 1** (15 Points).

Find the derivatives of each of the following functions with respect to  $x$ .

a.  $f(x) = 8x^3 - 5x^2 + 4x + 3x^{-1/3} + 10$

b.  $f(x) = 3x^2e^{3x^4-2x^2}$

c.  $f(x) = \frac{(5x^3-2x^2)^2}{2x^3+4x}$

**Problem 2** (15 Points).

Find the second derivatives of each of the following functions with respect to  $x$ .

a.  $f(x) = 12x^3 - 5x^2 + 6x + e^{2x}$

b.  $f(x) = 256x^{3/8} - 3x$

c.  $f(x) = e^{3x^2+5x}$

**Problem 3** (20 Points).

For each of the following problems find the value or values of  $x$  where the first derivative of the function is equal to zero. Then decide if the function has a maximum or a minimum or an inflection point at that value of  $x$ .

a.  $f(x) = x^3 - 2x^2 - 15x$

b.  $f(x) = 256x^{3/8} - 3x$

**Problem 4** (20 Points). In the following problem you are given a production function for a firm where  $y$  is the level of output and  $x$  is the level of the variable input. You are given the price ( $p$ ) of the output and the price ( $w$ ) of the single variable input.

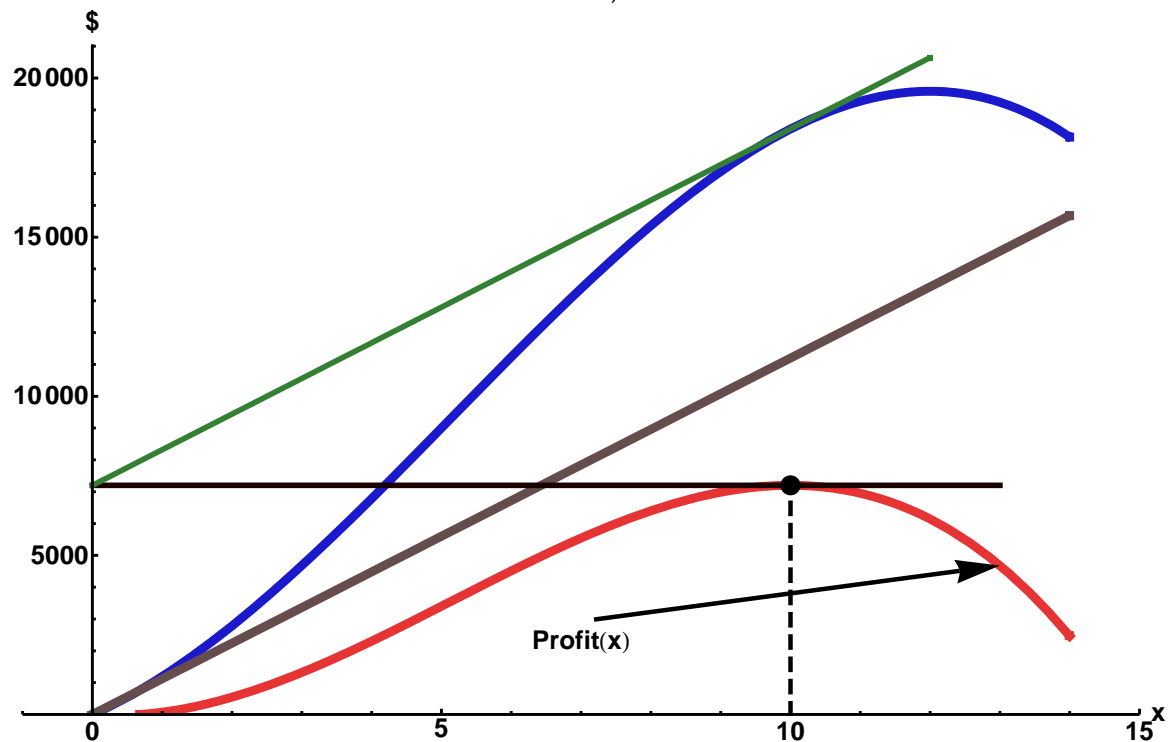
- Write down an equation that represents profit for the firm.
- Maximize this function by taking its derivative with respect to the variable input  $x$  and setting the derivative equal to zero.
- Solve the equation in part 4b for the potentially profit maximizing level of  $x$ .
- Determine using the second order conditions which of the roots represents maximum profit.

$$\text{output price} = p = 8$$

$$\text{input price} = w = 1120$$

$$y = \text{output} = f(x) = 120x + 31x^2 - 2x^3$$

FIGURE 1. Revenue, Cost and Profit



There is more space on the next page if you need it.

**Extra Space for Problem 4**

**Problem 5** (10 Points).

- a. Find the indefinite integral for the following function. Write in the form  $F(x) + c$ .

$$f(x) = 96x^{-5/8} - 3$$

- b. Find the indefinite integral for the following function. Write in the form  $F(x) + c$ .

$$f(x) = -48x^2 + 496x - 160$$

**Problem 6** (10 Points).

Solve the following system of equations for  $x_1$ ,  $x_2$ , and  $x_3$ .

$$x_1 + 2x_2 - 3x_3 = 2$$

$$-2x_1 - 3x_2 + 8x_3 = 0$$

$$3x_1 + 5x_2 - 10x_3 = 3$$



**Problem 7** (10 points).

Solve the following system of equations.

$$12x_1^{-2/3}x_2^{1/2} - 8 = 0$$

$$18x_1^{1/3}x_2^{-1/2} - 9 = 0$$