# ECONOMICS 207
# SPRING 2008
# EXAM 3

**Problem 1** (21 points)**.**

Consider the following matrices.

$$A = \begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}, \quad F = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}$$

$$G = \begin{bmatrix} 1 & -1 & 2 \\ -4 & 5 & -2 \\ 3 & -4 & -1 \end{bmatrix}, \quad H = \begin{bmatrix} 13 & 9 & 8 \\ 10 & 7 & 6 \\ -1 & -1 & -1 \end{bmatrix}$$

$$x = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}, \quad b = \begin{bmatrix} -5 \\ 10 \\ -3 \end{bmatrix}, \quad d = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Compute the following

a. A + B

b. BF

c. d'B

*Date*: March 12, 2008.

$$A = \begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}, \quad F = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}$$

$$G = \begin{bmatrix} 1 & -1 & 2 \\ -4 & 5 & -2 \\ 3 & -4 & -1 \end{bmatrix}, \quad H = \begin{bmatrix} 13 & 9 & 8 \\ 10 & 7 & 6 \\ -1 & -1 & -1 \end{bmatrix}$$

$$x = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}, \quad b = \begin{bmatrix} -5 \\ 10 \\ -3 \end{bmatrix}, \quad d = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

d. GA

e. FA

f. Gx

$$\begin{bmatrix} 1 & -1 & 2 \\ -4 & 5 & -2 \\ 3 & -4 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} =$$

g. Hb

$$\begin{bmatrix} 13 & 9 & 8 \\ 10 & 7 & 6 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} -5 \\ 10 \\ -3 \end{bmatrix} =$$

**Problem 2.** [18 Points]

Use elementary row operations to solve the following system of equations.
Hint: $x_3 = -2$.

$$Gx = b$$

$$\begin{pmatrix} 1 & -1 & 2 \\ -4 & 5 & -2 \\ 3 & -4 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ -3 \end{pmatrix}$$

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More Space for Problem 2

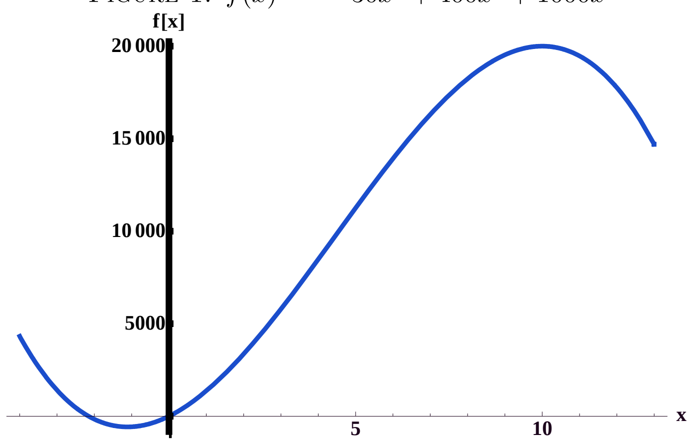
**Problem 3** (30 Points)**.**

**You may skip part a or part b.**

For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise. Also find the points of inflection for each function.

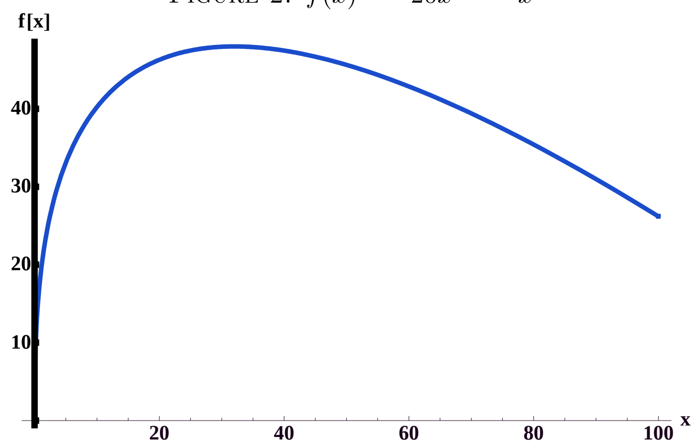
a. $f(x) = -30x^3 + 400x^2 + 1000x$

FIGURE 1. $f(x) = -30x^3 + 400x^2 + 1000x$



b. $f(x) = 20x^{2/5} - x$

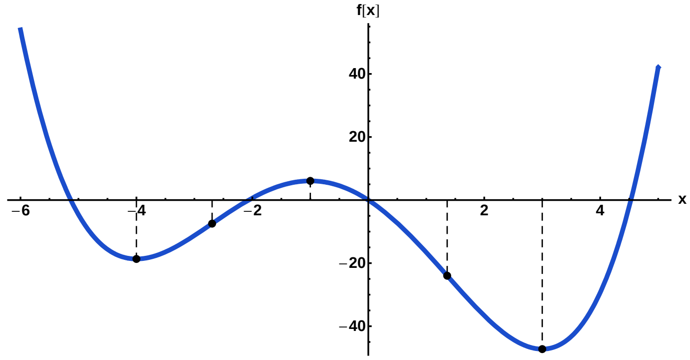
FIGURE 2. $f(x) = 20x^{2/5} - x$



c. $f(x) \;=\; \frac{x^4}{4} + \frac{2}{3}x^3 - \frac{11}{2}x^2 - 12x$

Hint: The inflection points are $x = \frac{-2\pm\sqrt{37}}{3}$, but you still need to show how to get them. You will only lose one point for not showing this, however.

FIGURE 3. $f(x) \;=\; \frac{x^4}{4} + \frac{2}{3}x^3 - \frac{11}{2}x^2 - 12x$



**Problem 4** (11 points)**.**

Solve the following system of equations.

$$\frac{27}{2} x_1^{-3/4} x_2^{2/3} \; - \; 8 \; = 0$$

$$36 x_1^{1/4} x_2^{-1/3} \; - \; 27 = 0$$

**Problem 5** (20 points)**.**

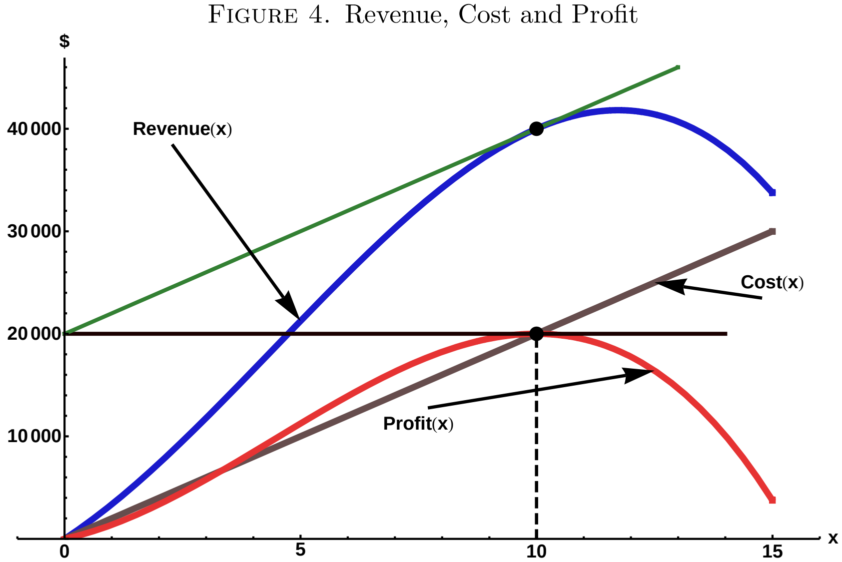
In the following problem you are given a production function for a firm where y is the level of output and x is the level of the variable input. You are given the price (p) of the output and the price (w) of the single variable input.

$$output\ price = p = 10$$

$$input\ price = w = 2000$$

$$y \ = \ output \ = \ f(x) \ = \ 300x + 40x^2 - 3x^3$$

Figure 4. Revenue, Cost and Profit




a. Write down an equation that represents profit for the firm.

b. Maximize this function by taking its derivative with respect to the variable input x and setting the resulting equation equal to zero.

c. If you identify more than one critical value from setting the first derivative of profit equal to zero, show which ones, if any, maximize profit.

d. Explain in words why the value of the marginal product for this firm is equal to the price of the single variable input at the profit maximizing level of input use. You can use the following information in explaining this phenomenon. Say something about the benefits of using an input not being less than the cost of the input.

$$\begin{aligned}
Output =& y = f(x) \\
MP =& \text{Marginal Product} = \frac{d\,f(x)}{dx} = f'(x) = \frac{\Delta y}{\Delta x} \\
Revenue =& pf(x) \\
Cost =& wx \\
Profit = \pi =& Revenue - Cost = pf(x) - wx \\
\frac{d\pi}{dx} =&
\end{aligned}$$