ECONOMICS 207
SPRING 2008
EXAM 3
KEY

Problem 1. [21 points]

Consider the following matrices.

$$A = \begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}, \quad F = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}$$

$$G = \begin{bmatrix} 1 & -1 & 2 \\ -4 & 5 & -2 \\ 3 & -4 & -1 \end{bmatrix}, \quad H = \begin{bmatrix} 13 & 9 & 8 \\ 10 & 7 & 6 \\ -1 & -1 & -1 \end{bmatrix}$$

$$x = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}, \quad b = \begin{bmatrix} -5 \\ 10 \\ -3 \end{bmatrix}, \quad d = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Compute the following
a. A + B

$$\begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 5 & 5 \end{bmatrix}$$

b. BF

$$\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

c. d'B

$$\begin{bmatrix} 1 & -2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -4 & -1 \end{bmatrix}$$

Date: March 14, 2008.

d. GA

These two matrices are not conformable.

e. FA

$$\begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$$

f. Gx

$$\begin{bmatrix} 1 & -1 & 2 \\ -4 & 5 & -2 \\ 3 & -4 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} = \begin{bmatrix} -5 \\ 10 \\ -3 \end{bmatrix}$$

g. Hb

$$\begin{bmatrix} 13 & 9 & 8 \\ 10 & 7 & 6 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} -5 \\ 10 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$$

Problem 2. [18 Points]

Use elementary row operations to solve the following system of equations.
Hint: $x_3 = -2$.

$$Gx = b$$

$$\begin{pmatrix} 1 & -1 & 2 \\ -4 & 5 & -2 \\ 3 & -4 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ -3 \end{pmatrix} \tag{2.1}$$

First create the augmented matrix $\tilde{A}$.

$$\tilde{A} = \begin{pmatrix} 1 & -1 & 2 & -5 \\ -4 & 5 & -2 & 10 \\ 3 & -4 & -1 & -3 \end{pmatrix}$$

Use row operations to "knock out" a_{21} by turning it into a zero. This would then make the second row have one more leading zero than the first row. We can turn the a_{21} element into a zero by adding 4 times the first row to the second row.

$$4 \times \begin{pmatrix} 1 & -1 & 2 & -5 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 4 & -4 & 8 & -20 \end{pmatrix}$$

$$\begin{array}{cccc} 4 & -4 & 8 & -20 \\ -4 & 5 & -2 & 10 \\ -- & -- & -- & -- \\ 0 & 1 & 6 & -10 \end{array}$$

This will give a new matrix on which to operate. Call it $\tilde{A}_{21}$.

$$\tilde{A}_{21} = \begin{pmatrix} 1 & -1 & 2 & -5 \\ 0 & 1 & 6 & -10 \\ 3 & -4 & -1 & -3 \end{pmatrix} \tag{2.2}$$

Now use row operations to "knock out" a_{31} by turning it into a zero. This would then make the third row have one more leading zero than the first row. We can turn the a_{31} element into a zero by adding negative 3 times the first row to the third row.

$$-3\times \begin{pmatrix} 1 & -1 & 2 & -5 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -3 & 3 & -6 & 15 \end{pmatrix}$$

$$\begin{array}{cccc} -3 & 3 & -6 & 15 \\ 3 & -4 & -1 & -3 \\ -- & -- & -- & -- \\ 0 & -1 & -7 & 12 \end{array}$$

This will give a new matrix on which to operate. Call it $\tilde{A}_{31}$.

$$\tilde{A}_{31} = \begin{pmatrix} 1 & -1 & 2 & -5 \\ 0 & 1 & 6 & -10 \\ 0 & -1 & -7 & 12 \end{pmatrix} \tag{2.3}$$

Use row operations to "knock out" a_{32} by turning it into a zero. This would then make the third row have one more leading zero than the second row. We can turn the a_{32} element into a zero by adding negative the second row of $\tilde{A}_{31}$ to the third row of $\tilde{A}_{31}$.

$$\begin{array}{cccc} 0 & 1 & 6 & -10 \\ 0 & -1 & -7 & 12 \\ -- & -- & -- & -- \\ 0 & 0 & -1 & 2 \end{array}$$

This will give a new matrix on which to operate. Call it $\tilde{A}_{32}$.

$$\tilde{A}_{32} = \begin{pmatrix} 1 & -1 & 2 & -5 \\ 0 & 1 & 6 & -10 \\ 0 & 0 & -1 & 2 \end{pmatrix} \tag{2.4}$$

Now multiply $\tilde{A}_{32}$ by negative one to obtain

$$\tilde{A}_{33} = \begin{pmatrix} 1 & -1 & 2 & -5 \\ 0 & 1 & 6 & -10 \\ 0 & 0 & 1 & -2 \end{pmatrix} \tag{2.5}$$

This means, then that $x_3 = -2$.

We now need to use row operations to obtain a zero in the a_{23} position in the matrix. We can turn the a_{13} element into a zero by adding negative 6 times the third row to the second row.

$$-6 \times \begin{pmatrix} 0 & 0 & 1 & -2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 0 & 0 & -6 & 12 \end{pmatrix}$$

$$\begin{array}{cccc} 0 & 0 & -6 & 12 \\ 0 & 1 & 6 & -10 \\ -- & -- & -- & -- \\ 0 & 1 & 0 & 2 \end{array}$$

This will give a new matrix on which to operate. Call it $\tilde{A}_{32}$.

$$\tilde{A}_{32} = \begin{pmatrix} 1 & -1 & 2 & -5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{pmatrix} \tag{2.6}$$

Now use row operations to "knock out" a_{13} by turning it into a zero. We can turn the a_{13} element into a zero by adding negative 2 times the third row to the first row.

$$-2 \times \begin{pmatrix} 0 & 0 & 1 & -2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 0 & 0 & -2 & 4 \end{pmatrix}$$

$$\begin{array}{cccc} 0 & 0 & -2 & 4 \\ 1 & -1 & 2 & -5 \\ -- & -- & -- & -- \\ 1 & -1 & 0 & -1 \end{array}$$

This will give a new matrix on which to operate. Call it $\tilde{A}_{13}$.

$$\tilde{A}_{13} = \begin{pmatrix} 1 & -1 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{pmatrix} \tag{2.7}$$

Now use row operations to “knock out” a_{12} by turning it into a zero. We can turn the a_{12} element into a zero by adding the second row to the first row to obtain $\tilde{A}_{12}$.

$$\tilde{A}_{12} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{pmatrix} \tag{2.8}$$

The answers are then $a_1 = 1, a_2 = 2, a_3 = -2$.

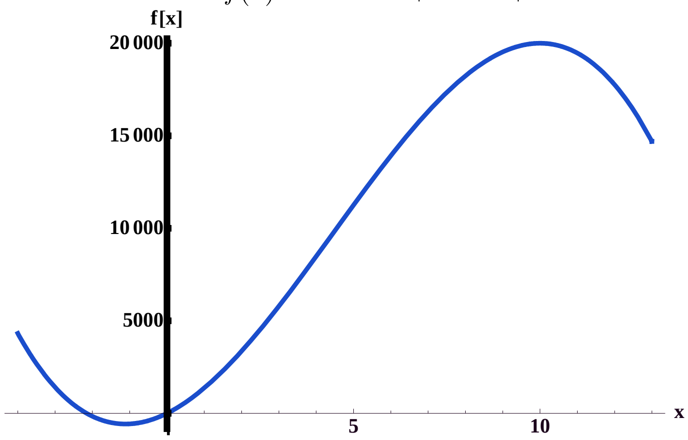
Problem 3. [30 Points]

You may skip part a or part b.

For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise. Also find the points of inflection for each function.

a. $f(x) = -30x^3 + 400x^2 + 1000x$

FIGURE 1. $f(x) = -30x^3 + 400x^2 + 1000x$



Find the first and second derivatives of f.

$$\begin{aligned} f(x) &= -30x^3 + 400x^2 + 1000x \\ \frac{df(x)}{dx} &= -90x^2 + 800x + 1000 \\ \frac{d^2 f(x)}{dx^2} &= -180x + 800 \end{aligned} \tag{3a.1}$$

Set the first derivative of f equal to zero and solve for the critical values of x.

$$\begin{aligned} \frac{df(x)}{dx} &= -90x^2 + 800x + 1000 = 0 \\ &\Rightarrow -10(9x^2 - 80x - 100) = 0 \\ &\Rightarrow -10(9x + 10)(x - 10) = 0 \\ \Rightarrow (9x + 10) &= 0 \text{ or } (x - 10) = 0 \\ \Rightarrow x &= \frac{-10}{9} \text{ or } x = 10 \end{aligned} \tag{3a.2}$$

Now evaluate the second derivative of f(x) at each critical value of x. First for $x = \frac{-10}{9}$

$$\begin{aligned}
\frac{d^2 f(x)}{dx^2} &= -180x + 800 \\
\frac{d^2 f(x)}{dx^2}\left(\frac{-10}{9}\right) &= -180\left(\frac{-10}{9}\right) + 800 \\
&= 200 + 800 = 1000 \\
&\Rightarrow x = \frac{-10}{9} \text{ is a minimum}
\end{aligned} \tag{3a.3}$$

Now for $x = 10$

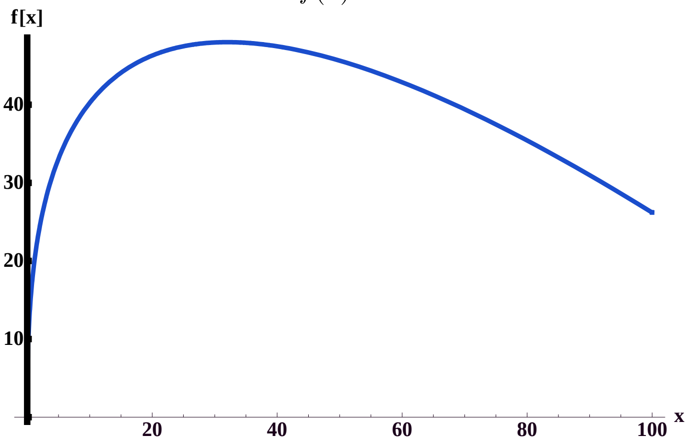
$$\begin{aligned}
\frac{d^2 f(x)}{dx^2} &= -180x + 800 \\
\frac{d^2 f(x)}{dx^2}(10) &= -180(10) + 800 \\
&= -1800 + 800 = -1000 \\
&\Rightarrow x = 10 \text{ is a maximum}
\end{aligned} \tag{3a.4}$$

We obtain the inflection point by setting the second derivative equal to zero.

$$\begin{aligned}
\frac{d^2 f(x)}{dx^2} &= -180x + 800 = 0 \\
&\Rightarrow -180x = -800 \\
&\Rightarrow x = \frac{800}{180} = \frac{80}{18} = \frac{40}{9}
\end{aligned} \tag{3a.5}$$

b. $f(x) \;=\; 20x^{2/5} - x$

FIGURE 2. $f(x) \;=\; 20x^{2/5} - x$



Find the first and second derivatives of f.

$$\begin{aligned} f(x) &= 20x^{2/5} - x \\ \frac{df(x)}{dx} &= 8x^{-3/5} - 1 \\ \frac{d^2 f(x)}{dx^2} &= -\frac{24}{5}x^{-8/5} \end{aligned} \tag{3b.1}$$

Set the first derivative of f equal to zero and solve for the critical values of x.

$$\begin{aligned} \frac{df(x)}{dx} &= 8x^{-3/5} - 1 = 0 \\ \Rightarrow x^{-3/5} &= \frac{1}{8} \\ \Rightarrow x &= \left(\frac{1}{8}\right)^{-5/3} = (8)^{5/3} \\ &= \left(2^3\right)^{5/3} = 2^5 = 32 \end{aligned} \tag{3b.2}$$

Now evaluate the second derivative of f(x) at the critical value of x.

$$\begin{aligned}\frac{d^2 f(x)}{dx^2} &= -\frac{24}{5}x^{-8/5} \\ \frac{d^2 f(x)}{dx^2}(32) &= -\frac{24}{5}(32)^{-8/5} \\ &= -\frac{24}{5}\left(2^5\right)^{-8/5} = -\frac{24}{5}(2)^{-8} \\ &= -\frac{24}{5\times 2^8} = -\frac{3\times 2^3}{5\times 2^8} \\ &= -\frac{3}{5\times 2^5} = -\frac{3}{160} \\ &\Rightarrow x = 32 \text{ is a maximum}\end{aligned} \tag{3b.3}$$

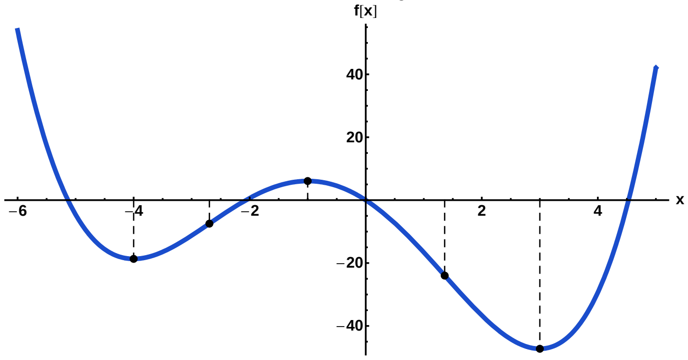
We obtain the inflection point by setting the second derivative equal to zero.

$$\begin{aligned}\frac{d^2 f(x)}{dx^2} = -\frac{24}{5}x^{-8/5} &= 0 \\ \Rightarrow x^{-8/5} &= 0 \\ \Rightarrow \frac{1}{x^{8/5}} &= 0\end{aligned} \tag{3b.4}$$

⇒ There is no inflection point as there is no value of x that makes this equation zero.

c. $f(x) = \frac{x^4}{4} + \frac{2}{3}x^3 - \frac{11}{2}x^2 - 12x$
Hint: The inflection points are $x = \frac{-2 \pm \sqrt{37}}{3}$, but you still need to show how to get them. You will only lose one for not showing this, however.

FIGURE 3. $f(x) = \frac{x^4}{4} + \frac{2}{3}x^3 - \frac{11}{2}x^2 - 12x$



Find the first and second derivatives of the function.

$$\begin{aligned} f'(x) &= x^3 + 2x^2 - 11x - 12 \\ f''(x) &= 3x^2 + 4x - 11 \end{aligned} \tag{3c.1}$$

Set the first derivative equal to zero.

$$f'(x) = x^3 + 2x^2 - 11x - 12 = 0 \tag{3c.2}$$

If there are integer roots to this equation,we can factor the cubic if we know one of them. We can guess that x = -4 is a root of this equation so we know that we must have $(x+4)(x+a)(x+b) = x^3 + 2x^2 - 11x - 12$. So if we divide $x^3 + 2x^2 - 11x - 12$ by (x+4) we can obtain a quadratic represented by (x+a)(x+b). Hopefully we can factor this expression. So let's begin.

If we divide $x^3 + 2x^2 - 11x - 12$ by (x+4) we first obtain

$$\begin{array}{r} x^2 \\ x+4 \overline{) \; x^3 + 2x^2 - 11x - 12} \\ \underline{-x^3 - 4x^2} \end{array}$$

$$\begin{array}{r} x^2 \\ x+4 \overline{) \; x^3 + 2x^2 - 11x - 12} \\ \underline{-x^3 - 4x^2} \\ -2x^2 - 11x \end{array}$$

Continuing we obtain

$$
\begin{array}{r|l}
 & x^2 \quad - 2x \\
\hline
x+4 & x^3 + 2x^2 - 11x - 12 \\
 & \underline{- x^3 - 4x^2} \\
 & \quad - 2x^2 - 11x \\
 & \quad \underline{2x^2 \quad + 8x}
\end{array}
$$

and then

$$
\begin{array}{r|l}
 & x^2 \quad - 2x \\
\hline
x+4 & x^3 + 2x^2 - 11x - 12 \\
 & \underline{- x^3 - 4x^2} \\
 & \quad - 2x^2 - 11x \\
 & \quad \underline{2x^2 \quad + 8x} \\
 & \qquad\qquad - 3x - 12
\end{array}
$$

$$
\begin{array}{r|l}
 & x^2 \quad - 2x \quad - 3 \\
\hline
x+4 & x^3 + 2x^2 - 11x - 12 \\
 & \underline{- x^3 - 4x^2} \\
 & \quad - 2x^2 - 11x \\
 & \quad \underline{2x^2 \quad + 8x} \\
 & \qquad\qquad - 3x - 12
\end{array}
$$

$$
\begin{array}{r}
x^2 \quad - 2x \quad - 3 \\
x+4 \overline{\big) \; x^3 + 2x^2 - 11x - 12} \\
\underline{- x^3 - 4x^2 \qquad\qquad} \\
- 2x^2 - 11x \qquad \\
\underline{2x^2 \quad + 8x \qquad} \\
- 3x - 12 \\
\underline{3x + 12} \\
0
\end{array}
$$

We can now factor the remaining term as follows

$$x^2 - 2x - 3 \; = (x-3)(x+1) \tag{3c.3}$$

We then have

$$
\begin{aligned}
(x+4)(x-3)(x+1) \; &= 0 \\
\Rightarrow x = -4, x = 3, x = -1
\end{aligned} \tag{3c.4}
$$

We can check each of the roots in the second derivative.

$$
\begin{aligned}
f''(-4) \; &= 3(-4)^2 + 4(-4) - 11 \\
&= 48 - 16 - 11 \; = \; 21
\end{aligned} \tag{3c.5}
$$

So x = -4 is a minimum point.

$$
\begin{aligned}
f''(-1) \; &= 3(-1)^2 + 4(-1) - 11 \\
&= 3 - 4 - 11 \; = \; -12
\end{aligned} \tag{3c.6}
$$

So x = -1 is a maximum point.

$$
\begin{aligned}
f''(3) \; &= 3(3)^2 + 4(3) - 11 \\
&= 27 + 12 - 11 \; = \; 28
\end{aligned} \tag{3c.7}
$$

So x = 3 is a minimum point.
To find the points of inflection we set the second derivative equal to zero.

$$f''(x) \; = 3x^2 + 4x - 11 \; = \; 0 \tag{3c.8}$$

And unfortunately it does not factor so we obtain

$$\begin{aligned} x &= \frac{-4 \pm \sqrt{16-(4)(3)(-11)}}{6} \\ &= \frac{-4 \pm \sqrt{16+132}}{6} \\ &= \frac{-4 \pm \sqrt{148}}{6} \\ &= \frac{-4 \pm \sqrt{4 \times 37}}{6} \\ &= \frac{-4 \pm 2\sqrt{37}}{6} \\ &= \frac{-2 \pm \sqrt{37}}{3} \end{aligned} \tag{3c.9}$$

Problem 4 (11 points)**.**
Solve the following systems of equations for x_1 and x_2.

$$\frac{27}{2} x_1^{-3/4} x_2^{2/3} - 8 = 0 \tag{4.1}$$

$$36 x_1^{1/4} x_2^{-1/3} - 27 = 0 \tag{4.2}$$

Rearrange the first equation 4.1 to obtain

$$\begin{aligned} x_1^{-3/4} x_2^{2/3} &= \frac{8}{\frac{27}{2}} = \frac{16}{27} \\ \Rightarrow x_1^{3/4} x_1^{-3/4} x_2^{2/3} &= \frac{16}{27} x_1^{3/4} \\ \Rightarrow x_2^{2/3} &= \frac{16}{27} x_1^{3/4} \\ \Rightarrow x_2 &= \left(\frac{16}{27}\right)^{3/2} \left(x_1^{3/4}\right)^{3/2} \\ &= \left(\frac{16}{27}\right)^{3/2} x_1^{9/8} \end{aligned} \tag{4.1.a}$$

Rearrange the second equation 4.2 slightly to obtain

$$x_1^{1/4} x_2^{-1/3} = \frac{27}{36} = \frac{3}{4} \tag{4.2$'$}$$

Now substitute x_2 from equation 4.1.a into equation 4.2′ to obtain

$$
\begin{aligned}
x_1^{1/4}\left(\left(\frac{16}{27}\right)^{3/2} x_1^{9/8}\right)^{-1/3} &= \frac{3}{4} \\
\Rightarrow x_1^{1/4}\left(\frac{16}{27}\right)^{-1/2} x_1^{-3/8} &= \frac{3}{4} \\
\Rightarrow x_1^{-1/8}\left(\frac{16}{27}\right)^{-1/2} &= \frac{3}{4} \\
\Rightarrow x_1^{-1/8} &= \frac{3}{4}\left(\frac{16}{27}\right)^{1/2} \\
\Rightarrow x_1 &= \left(\frac{3}{4}\left(\frac{16}{27}\right)^{1/2}\right)^{-8} \\
&= \left(\frac{3}{4}\right)^{-8}\left(\frac{16}{27}\right)^{-4} \\
&= 3^{-8}(2^{-2})^{-8}(2^4)^{-4}(3^{-3})^{-4} \\
&= 3^{-8}2^{16}2^{-16}3^{12} \\
&= 3^4 = 81
\end{aligned}
\tag{4.2.a}
$$

Now substitute x_1 from equation 4.2.a into equation 4.1.a to obtain

$$
\begin{aligned}
x_2 &= \left(\frac{16}{27}\right)^{3/2} x_1^{9/8} \\
&= \left(\frac{16}{27}\right)^{3/2} (81)^{9/8} \\
&= (2^4)^{3/2}(3^{-3})^{3/2}(3^4)^{9/8} \\
&= 2^6 3^{-9/2} 3^{36/8} \\
&= 2^6 3^{-9/2} 3^{9/2} = 2^6 = 64
\end{aligned}
$$

Problem 5. [20 points]

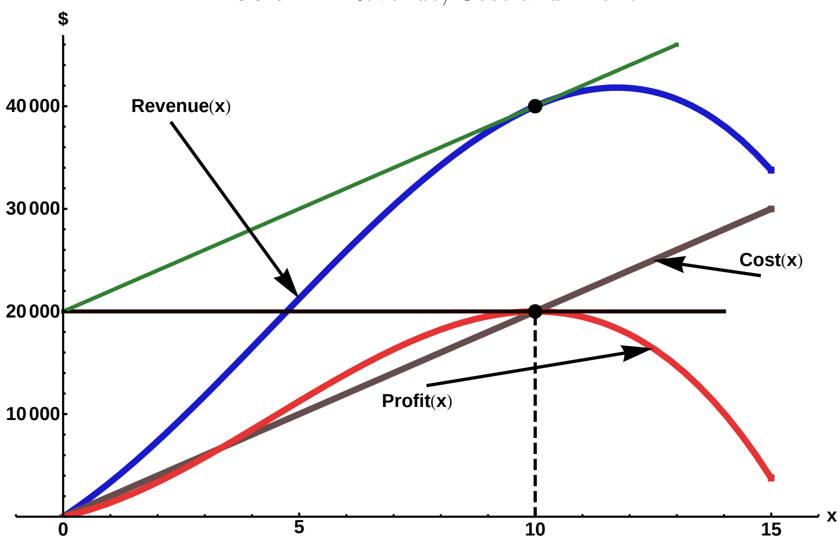
In the following problem you are given a production function for a firm where y is the level of output and x is the level of the variable input. You are given the price (p) of the output and the price (w) of the single variable input.

$$output\ price = p = 10$$

$$input\ price = w = 2000$$

$$y = output = f(x) = 300x + 40x^2 - 3x^3$$

FIGURE 4. Revenue, Cost and Profit



a. Write down an equation that represents profit for the firm.

For this example, profit is given by

$$\begin{aligned} Profit &= 10(300x + 40x^2 - 3x^3) - 2000x \\ &= 3000x + 400x^2 - 30x^3 - 2000x \\ &= -30x^3 + 400x^2 - 1000x \end{aligned} \tag{5.1}$$

b. Maximize this function by taking its derivative with respect to the variable input x and setting the resulting equation equal to zero.

$$\begin{aligned} Profit \;= \pi = &\; -30x^3 + 400x^2 + 1000x \\ \frac{d\,\pi}{dx} = &\; -90x^2 + 800x + 1000 \\ \frac{d^2\,\pi}{dx^2} = &\; -180x + 800 \end{aligned} \tag{5.2}$$

Now set the derivative of profit equal to zero and solve for x as follows.

$$\begin{aligned} \frac{d\pi}{dx} &= -90x^2 + 800x + 1000 = 0 \\ &\Rightarrow -10(9x^2 - 80x - 100) = 0 \\ &\Rightarrow -10(9x + 10)(x - 10) = 0 \\ \Rightarrow (9x + 10) &= 0 \text{ or } (x - 10) = 0 \\ \Rightarrow x &= \frac{-10}{9} \text{ or } x = 10 \end{aligned} \tag{5.3}$$

c. If you identify more than one critical value from setting the first derivative of profit equal to zero, show which ones, if any, maximize profit.

First for $x = \frac{-10}{9}$

$$\begin{aligned} \frac{d^2\,\pi}{dx^2} &= -180x + 800 \\ \frac{d^2\,\pi}{dx^2}\left(\frac{-10}{9}\right) &= -180\left(\frac{-10}{9}\right) + 800 \\ &= 200 + 800 = 1000 \\ \Rightarrow x = &\frac{-10}{9} \text{ is a minimum} \end{aligned} \tag{5.4}$$

Now for $x = 10$

$$\begin{aligned} \frac{d^2\,\pi}{dx^2} &= -180x + 800 \\ \frac{d^2\,\pi}{dx^2}(10) &= -180\,(10) + 800 \\ &= -1800 + 800 = -1000 \\ \Rightarrow x = &10 \text{ is a maximum} \end{aligned} \tag{5.5}$$

d. Explain in words why the value of the marginal product for this firm is equal to the price of the single variable input at the profit maximizing level of input use. You can use the following information in explaining this phenomenon. Say something about the benefits of using an input not being less than the cost of the input.

$$
\begin{aligned}
Output =& y = f(x) \\
MP =& \text{Marginal Product} = \frac{d\,f(x)}{dx} = f'(x) = \frac{\Delta y}{\Delta x} \\
Revenue =& pf(x) \\
Cost =& wx \\
Profit = \pi =& Revenue - Cost = pf(x) - wx \\
\frac{d\pi}{dx} =&
\end{aligned}
\tag{5.6}
$$

If we take the derivative of equation (5.6) with respect to x we obtain

$$
\begin{aligned}
\pi =& pf(x) - wx \\
\frac{d\pi}{dx} =& pf'(x) - w = 0 \\
\Rightarrow pf'(x) =& w
\end{aligned}
\tag{5.7}
$$

The right hand side of equation 5.7 is the cost of an input or the price we must pay to obtain one more unit of that input. We sometimes call this marginal factor cost (MFC). The left hand side of equation 5.7 is the price of the output multiplied by the marginal product of the input. It is the marginal value product (MVP) of the input or the revenue we obtain from using one more unit of the input. If the marginal value product of using one more unit of the input is greater than its cost, i.e. MVP $\geq$ w, we should continue applying more units of the input up until the point where MVP = w or MVP = MFC.